Self-Adjusting Compensating Thermal Lens to Balance the Thermally Induced Lens in Solid-State Lasers

R. Weber, Thomas Graf, and Heinz P. Weber, *Fellow, IEEE*

***Abstract*—The thermal lens is a critical issue, particularly in high-power diode-pumped solid-state laser rods. A self-adjusting scheme for compensation of the thermally induced lens is presented. The requirements for such an element and its influence on the resonator are discussed. With an appropriate compensating element and a suitable resonator design, constant beam parameters are expected to be achieved over a pump range of several kilowatts.**

***Index Terms*—Compensation of thermal lens, high-power solid-state lasers, negative thermal lens, thermal lens.**

I. Introduction

HIGH-POWER diode-pumped solid-state lasers are believed to be the workhorse for the next generation of lasers in industrial applications. Multimode output powers in the range of several hundred watts will soon be commercially available. Such lasers are usually designed to be operated at the maximum pump and, therefore, output power. This is necessary because the thermally induced lenses inside the laser cavity, particularly in the active material, limit the useful pump power range.

The thermal lens results from the locally different temperatures inside optical materials. This inhomogeneous temperature distribution is due to the heating of the material, which results from absorption of either the pump power or the laser power itself. In general, thermal lensing consists of three parts: first, the temperature dependence of the refractive index, in this paper referred to as the dn/dT-part; second, the axial expansion of the optical material leading to curved end faces, referred to as the end-effect; and third, the location deformation (strain) of the material due to thermally induced stress and its influence on optics, which is locally different for different polarization directions, and is referred to as birefringence. While the reduction of the resulting depolarization caused by thermal birefringence was demonstrated successfully [1], [2], compensation for the phase front distortions originating from the temperature dependence of the refractive index and the expansion of the material is addressed in alternative active medium designs [3] but is not solved yet for rod-lasers.

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The authors are with the Institute of Applied Physics, University of Bern, CH-3012 Bern, Switzerland.

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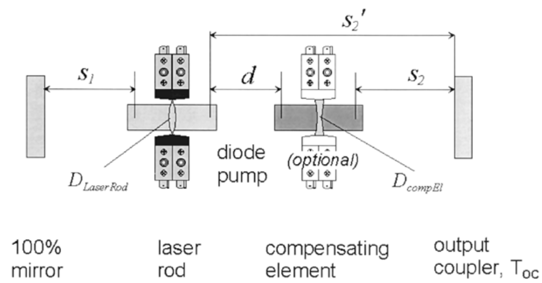

Fig. 1. Schematic of the intracavity compensation scheme with the two thermal lenses of the laser rod ($D_{\rm LaserRod}$) and the compensating element ($D_{\rm CompEl}$).

Moreover, it would be desirable to maintain constant beam parameters over a wide range of pump powers. This can be achieved by adapting the resonator to the respective pump power level. Although active mirrors [4] or resonator length adjustments [5] are possible means, they require sophisticated mechanical arrangements and/or electronic control. In addition, they usually do not allow one to compensate for the aberrations.

Another possibility for compensating for the thermally induced lens is to take advantage of the effect itself and to use a heated optical element as a compensating element. As first proposed by Koch [6], the end effect as well as the dn/dT-part have the potential to be used in a compensating element. For an end-pumped system, the formation of a convex curvature in an end mirror induced by absorption of the pump light was discussed extensively in [6].

For high-power lasers, transverse pumping is preferred due to its simpler scalability. In this case, a compensating element, in most cases with a negative dn/dT, which is placed inside the resonator (Fig. 1), seems to be more promising than the scheme with the heated end mirror. Actually, optical glasses with a strong negative dn/dT, with an absolute value comparable to that of Nd:YAG, such as the phosphate laser glasses Schott LG-760 or Hoya LHG-8 [7], are available. In the situation of transverse pumping, it might be difficult to use the same pump source for the laser rod and to heat the compensating element simultaneously. An additional transverse pump source for the compensating element could solve this problem, requiring a pump distribution, which should be as equal as possible to the one in the laser rod. On the other hand, a compensating element, which is heated by the intracavity power itself, shows many advantages. First, no additional pump source is required. Second, the thermal lens is directly correlated to the output power and aberrations will be inherently and passively compensated.

In this paper, the requirements for such compensating elements are discussed for diode-pumped solid-state lasers with output powers in the range of several hundred watts. Although this is not our primary objective, schemes with an actively pumped compensating element are possible as well and are inherently included in the following considerations. The compensating element should compensate for the phase front distortions produced in the gain medium as exactly as possible. Therefore, detailed knowledge of the thermally induced lens in the active medium is crucial. An analytical approach to determining the requirements for such a compensating element and its influence on the laser properties is presented. In addition, the influence of the compensating element on the resonator properties is discussed and an optimized resonator design is proposed.

II. Thermal Considerations

In order to compensate for the thermal lens in the laser rod, the thermal lens generated in heated optical materials has to be calculated. This allows one to determine appropriate parameters for the compensating element. Good insight into the basic physical behavior is achieved with the simple analytical approach to describe the thermally induced lenses as presented in [8], [7], and [9]. Using this approach, homogeneously heated rods are assumed for both the active medium and the compensating element. The thermal conductivity is assumed to be temperature-independent. For the following, the total dioptric power of the thermal lens is given by the sum of the contributions from the temperature dependence of the refractive index (dn/dT-effect) and the surface curvature (end-effect), denoted as $D_{dn/dT}$ and D_{end}, respectively.

The nature of birefringence makes it necessary to compensate for it separately, e.g., with the scheme described in [1], [2]. Thus, it will not be included in the following considerations.

Homogeneous heating of a rod produces a temperature difference between its circumference and the center, which is independent of the cooling, given by [7]

$$\Delta T = \frac{1}{4 \cdot \pi \cdot k} \cdot \frac{P_{\mathrm{heat}}}{\ell_{\mathrm{heat}}} \tag{1}$$

where k is the heat conductivity, ℓ_{heat} the length over which the element is heated (which may be shorter than the rod length of the gain material), and P_{heat} is the total power converted to heat. Together with the coefficient of the temperature dependence of the refractive index dn/dT and the expansion coefficient α_{exp}, this temperature difference is responsible for the thermal lens. Assuming a rod geometry for both the laser material and the compensating element, the two contributions to the thermal lens in heated rods with a radius R_{Rod} are given by [7]

$$D_{dn/dT} = \Delta T \cdot \frac{2 \cdot dn/dT \cdot \ell_{\mathrm{heat}}}{R_{\mathrm{Rod}}^2} = \frac{dn/dT}{2 \cdot \pi \cdot k} \cdot \frac{P_{\mathrm{heat}}}{R_{\mathrm{Rod}}^2} \tag{2}$$

and

$$\begin{aligned} D_{\mathrm{end}} &= \Delta T \cdot \frac{4 \cdot \alpha_{\mathrm{exp}} \cdot (n_0 - 1)}{R_{\mathrm{Rod}}} \\ &= \frac{\alpha_{\mathrm{exp}} \cdot (n_0 - 1)}{\pi \cdot k} \cdot \frac{P_{\mathrm{heat}}}{\ell_{\mathrm{heat}} \cdot R_{\mathrm{Rod}}} \end{aligned} \tag{3}$$

if the element is heated up to its total length. Here n_0 is the undistributed refractive index. The contribution of the end-effect is reduced if the rod is much longer than its diameter and/or the heated zone is shorter than the element. The end-effect can be neglected, if the unheated zone at the rod ends is longer than about one rod radius. It is noted that the laser medium as well as the compensating medium is assumed to be a rod.

Apart from material and geometrical parameters, (2) and (3) contain the heat source P_{heat}. In the case of the laser rod, the heating is given by the fraction of the pump power P_{pump} that is absorbed and converted to heat

$$\begin{aligned} P_{\mathrm{heat,LaserRod}} &= \eta_{\mathrm{transt}} \cdot \eta_{\mathrm{abs}} \cdot \eta_{heat} \cdot P_{\mathrm{pump}} \\ &= \eta_h \cdot P_{\mathrm{pump}} \end{aligned} \tag{4}$$

where η_{trans} and η_{abs} are the transfer and absorption efficient, respectively. The heat conversion factor η_{heat} is about 35% for Nd:YAG under lasing conditions [10].

If an external pump heats the compensating element, the heating is described by the same formula as the heating of the laser rod. In the scheme proposed in this paper, the heating of the compensating element is assumed to result from weak absorption of the circulating intracavity power. Therefore, the heated volume is given by the mode size at the location of this element. For the following analytical approach, the compensating element is assumed to be placed next to the active medium in a multimode laser, i.e., the beam size in the compensating element has about the same size as the laser rod (and the compensating element). Furthermore, heating takes place over the whole length of the element.

The heating of the compensating element results from absorption of the circulating intracavity power P_{cir}, which depends on the output power P_{out} as

$$\begin{aligned} P_{\mathrm{heat,compEl}} &= 2 \cdot \alpha_{\mathrm{abs}} \cdot \ell_{\mathrm{compEl}} \cdot P_{\mathrm{cir}} \\ &= 2 \cdot \alpha_{\mathrm{abs}} \cdot \ell_{\mathrm{compEl}} \cdot \frac{P_{\mathrm{out}}}{T_{\mathrm{OC}}} \end{aligned} \tag{5}$$

where α_{abs} is the absorption coefficient at the laser wavelength in the compensating element, ℓ_{compEl} is its length, and T_{OC} is the transmission of the output coupler. Because of the comparatively high saturation intensities, the absorption coefficient is assumed to be independent of the incident intensity. A short investigation of the rate equations shows that the absorption from the lower laser level in an Nd-doped glass (like the gain) saturates with the saturation intensity $h\nu(\sigma\tau)^{-1}$, where $h\nu$ is the photon energy, σ is the absorption (or stimulated emission) cross section, and τ is the lifetime of the upper laser level. Due to the small cross section and the offset of the peak absorption with respect to 1064 nm, the saturation intensity of the absorption in LG-760 is more than an order of magnitude higher than the gain saturation intensity of Nd:YAG.

In order to treat the optical behavior of this passively heated compensating element, it is necessary to model the output characteristics of a simple multimode laser. The output power P_{out} of such a laser is related to the pump power with the laser threshold P_{th} and the slope efficiency η_{slope} by

$$P_{\mathrm{out}} = \eta_{\mathrm{slope}} \cdot (P_{\mathrm{pump}} - P_{\mathrm{th}}). \tag{6}$$

Both the slope efficiency and the threshold power are functions of the total round-trip loss introduced by the compensating element due to the absorption given by $L_{\text{compEl}} = 2\alpha_{\text{abs}} \cdot \ell_{\text{compEl}}$. For homogeneously pumped rods and reasonably low output coupler transmission, the slope efficiency and the threshold power are given by [7]

$$\eta_{\text{slope}} = \frac{T_{\text{OC}}}{T_{\text{OC}} + L_{\text{int}} + L_{\text{compEl}}} \cdot \eta_{\text{tot}},$$
$$P_{\text{th}} = \frac{T_{\text{OC}} + L_{\text{int}} + L_{\text{compEl}}}{2} \cdot \frac{A_{\text{LaserRod}} \cdot I_s}{\eta_{\text{tot}}} \tag{7}$$

where L_{int} is the sum of the passive internal losses inside the laser resonator, A_{LaserRod} is the rod cross section, I_s is the saturation intensity, and η_{tot} is the product of transfer efficiency, absorption efficieny, Stokes efficiency, quantum efficiency, and mode overlap. Introducing a model of the laser in the above way also allows one to analyze the influence of the compensating element on the laser performance.

A. Resulting Thermal Lenses

With the above heating of the thermally active materials and (2), (3), and (1), the dioptric powers of the thermal lenses in the laser rod (subscript *LaserRod*), which is not heated up to its very end, and the compensating element (subscript *compEl*) are found to be

$$D_{\text{LaserRod}} = \frac{dn/dT_{\text{LaserRod}}}{2 \cdot \pi \cdot k_{\text{LaserRod}}} \cdot \frac{\eta_h \cdot P_{\text{pump}}}{R^2_{\text{LaserRod}}} \tag{8}$$

and

$$D_{\text{compEl}} = \left(\frac{dn/dT_{\text{compEl}}}{2 \cdot R_{\text{compEl}}} + \frac{\alpha_{\text{exp}} \cdot (n_0 - 1)}{\ell_{\text{compEl}}} \right) \cdot \frac{L_{\text{compEl}} \cdot \eta_{\text{slope}} \cdot (P_{\text{pump}} - P_{\text{th}})}{\pi \cdot k_{\text{compEl}} \cdot R_{\text{compEl}} \cdot T_{\text{OC}}}. \tag{9}$$

These are the two lenses which are produced inside the cavity and which should comepnsate for each other.

A discussion of the behavior of resonators including two such elements will be given later in this paper.

B. Is the Intracavity Compensation Scheme Realistic?

As the laser performance should not be degraded too much, the overall absorption inside the compensating element should not exceed the percent range. Therefore, a first estimation is made in this section to clarify whether or not the proposed scheme is realistic. For this purpose, the sum of the dioptric powers of the two lenses is set to be zero, i.e., $D_{\text{therm,LaserRod}} + D_{\text{therm,compEl}} = 0$ (ideal compensation), and combined with (8) and (9). The result is then solved for L_{compEl}. For this first estimate, several assumptions are made. The laser is assumed to operate well above threshold, i.e., the threshold power is set to $P_{\text{th}} = 0$. Furthermore, η_{slope} is assumed to be independent of L_{compEl}, which is allowed if this additional loss is small [see (7)]. With a length of the compensating element of a few centimeters, the end effect is about ten times weaker than the dn/dT-part and can be neglected. It is noted that, with these assumptions, the situation where the compensating element is heated by an additional pump source is also described. Finally, additional realistic assumptions for the values of the involved quantities ($dn/dT_{\text{rod}} \approx -dn/dT_{\text{compEl}}$, $\eta_{\text{heat}} \approx \eta_{\text{slope}}$, and $R_{\text{compEl}} \approx R_{\text{rod}}$) yield a simple result for the required absorption loss inside the compensating element

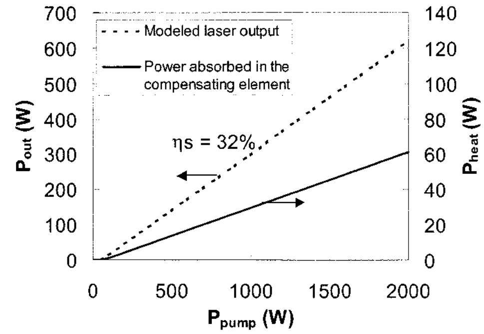

Fig. 2. Performance of the modeled laser and the power absorbed in the compensating element.

$$L_{\text{compEl}} \approx T_{OC} \frac{k_{\text{compEl}}}{k_{\text{LaserRod}}}. \tag{10}$$

As the heat conductivity of technical glass is typically ten times smaller than that of Nd:YAG ($\sim$10 W$\cdot$m$^{-1}\cdot$K^{-1}) and typical output coupler transmissions are about 10%, one gets a required overall absorption in the compensating element of 1%. This number is very encouraging and shows that the principle of intracavity compensation using optical materials with negative thermal dispersion is feasible without significant influence on the laser performance. With this additional loss, the output power is just reduced by about 10%. (Note that, in the case of an actively pumped compensating element, the ratio of laser pump power to compensation pump power is given by the ratio of the heat conductivities, i.e., $(\eta_{h,\text{compEl}} \cdot P_{p,\text{compEl}})/(\eta_{h,\text{LaserRod}} \cdot P_{p,\text{LaserRod}}) = k_{\text{compEl}}/k_{\text{LaserRod}}$ without taking advantage of the power increase inside the cavity due to the reduced output coupling.)

C. A More Detailed Analytical Analysis

A more comprehensive analysis is necessary to describe the influence of a compensating element inside a laser cavity. For this purpose, a high-power transverse diode-pumped laser was modeled using (6) and (7) and taking the numerical values for Nd:YAG as given in [7]: $I_s = 2.9$ kW/cm^2, $k_{YAG} = 10$ W/mK, and $dn/dT_{\text{YAG}} = 7.3 \cdot 10^{-6}$ K^{-1}. The rod has a diameter of 4 mm and 80% of the pump power is assumed to be absorbed. The effective fraction of pump power converted to heat is, therefore, $\eta_h = \eta_{\text{heat}} \cdot \eta_{\text{abs}} = 30\%$, and $\eta_{\text{tot}} = 42\%$.

The following values were assumed for the compensating element: $dn/dT_{\text{compEl}} = -7.8 \cdot 10^{-6}$ K^{-1} and $\alpha_{\text{exp}} = 12.7 \cdot 10^{-6}$ K^{-1}. (Similar values are found for the Schott laser glass LG760, for example).

The efficiencies involved were chosen to reproduce typical laser performances such as described in [11], [12], and [9]. Fig. 2 shows the modeled performance of this laser and the amount of power absorbed in the compensating element with 1% of absorption and 10.7% of output coupling. (These numbers are explained later in the paper).

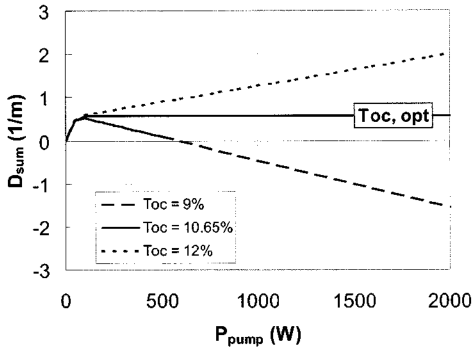

Fig. 3. Sum of the dioptric powers of the two thermal lenses for different output coupler transmissions.

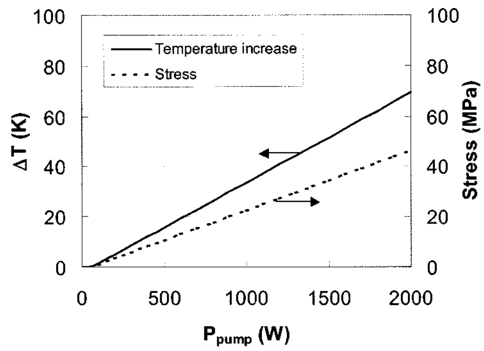

Fig. 4. Calculated temperature increase and maximum tensile stress in the compensating element.

The resulting slope efficiency of $\eta_{\text{slope}} \approx 32\%$ and the threshold power of $P_{\text{th}} \approx 60$ W include the additional 1% of loss inside the compensating element.

The required thermal lens in the compensating element is created by bulk absorption. As the absorption coefficient usually is a material constant, a constant value of 0.0015 cm^{-1} is assumed for the following considerations. This corresponds to a double-pass absorption of 1% for a length of the compensating element of 7 cm. According to (6)–(9), T_{OC} and ℓ_{compEl} are the parameters which allow for adjusting the thermal lens in the compensating element. The radius of the compensating element is preferably the same as the radius of the rod to ensure optimum compensation of the aberrations.

First, the influence of T_{OC} is investigated. For this purpose, the sum of the dioptric powers of the lenses in the laser rod and the compensating element is analyzed, i.e., $D_{\text{sum}} = D_{\text{LaserRod}} + D_{\text{compEl}}$. Fig. 3 shows this sum for three different output coupler transmissions.

An optimum value for T_{OC} can be found where the sum of the dioptric powers of the two lenses is positive but remains constant above the laser threshold. This optimum output coupling is found by differentiating the sum of (8) and (9) with respect to P_{pump}. With the above assumptions, this calculation neglects with sufficient accuracy the dependence of the threshold power on the output coupling. The result is set to zero and solved for T_{OC}, giving an optimum output coupling of

$$T_{\text{OC,opt}} = -\left(\frac{dn/dT_{\text{compEl}}}{2 \cdot R_{\text{compEl}}} + \frac{\alpha_{\exp} \cdot (n_0 - 1)}{\ell_{\text{compEl}}}\right) \cdot \frac{2 \cdot L_{\text{compEl}} \cdot \eta_{\text{tot}} \cdot k_{\text{LaserRod}} \cdot R^2_{\text{LaserRod}}}{dn/dT_{\text{LaserRod}} \cdot \eta_h \cdot k_{\text{compEl}} \cdot R_{\text{compEl}}} - L_{\text{compEl}} - L_{\text{int}}. \tag{11}$$

The length of the compensating element is important, as the positive thermal lens due to expansion tends to compensate for the negative lens caused by the negative dn/dT. Increasing the length of the compensating element reduces this effect.

Due to the absorption of 1%, about 10% of the output power is converted to heat inside the compensating element. This heating causes a significant temperature rise and, therefore, also stress. The temperature rise in the compensating element can be calculated from (1). The maximum tensile stress $\sigma_{\max}$ occurring at the rod circumference is given by [7]

$$\sigma_{\max} = \frac{1}{2} \cdot \Delta T_{\text{compEl}} \cdot \frac{\alpha_{\exp} \cdot E_{\text{compEl}}}{1 - \nu_{\text{compEl}}} \tag{12}$$

where E_{compEl} and ν_{compEl} are the modulus of elasticity and the Poisson's ratio, respectively.

Fig. 4 shows plots of the temperature increase in the center of the compensating element and the maximum tensile stress (with $E_{\text{compEl}} = 74$ GPa and $\nu_{\text{compEl}} - 0.3$).

For a compensating element with a length of 7 cm, a temperature rise of 70 K causes a maximum tensile stress of $\sigma_{\max} \approx$ 50 MPa at 2000 W of pump power. Both values are significantly below the critical value for stress fracture. The temperature rise is a function of the absorbed pump power per unit length. Therefore, increasing the length of the compensating element leads to a reduced temperature increase and to lower stress. (Obviously, this also requires a decreased absorption per unit length in the compensating element). Therefore, suitable values for the absorption coefficient and the length of the compensating element have to be chosen carefully.

III. Resonator Considerations

A. Influence of the Compensating Element on the Resonator

After having discussed the magnitude of the thermal lenses, the question is how such a compensating element will influence the resonator behavior. It is again assumed that the thermal lens in the laser rod and in the compensating element (at pump powers high above threshold) have the same absolute value but the opposite sign, i.e., $D_{\text{compEl}} = -D_{\text{LaserRod}}$.

The straightforward approach is to just insert the compensating element into the resonator, as was sketched in Fig. 1. With a resonator having a length of 240 mm, a length of the two thermally active media of 70 mm, each at a distance of 40 mm from one of the mirrors, the resonator behavior was calculated using the ABCD-matrix formalism as described in [13]–[15]. The thermal lenses inside the rod and the compensating element were modeled with thin lenses between two pieces of material with the approximate refractive index [16]. Fig. 5 shows the radius of the fundamental mode at the location of the thermal lens in the laser rod as a function of the pump power. The same laser

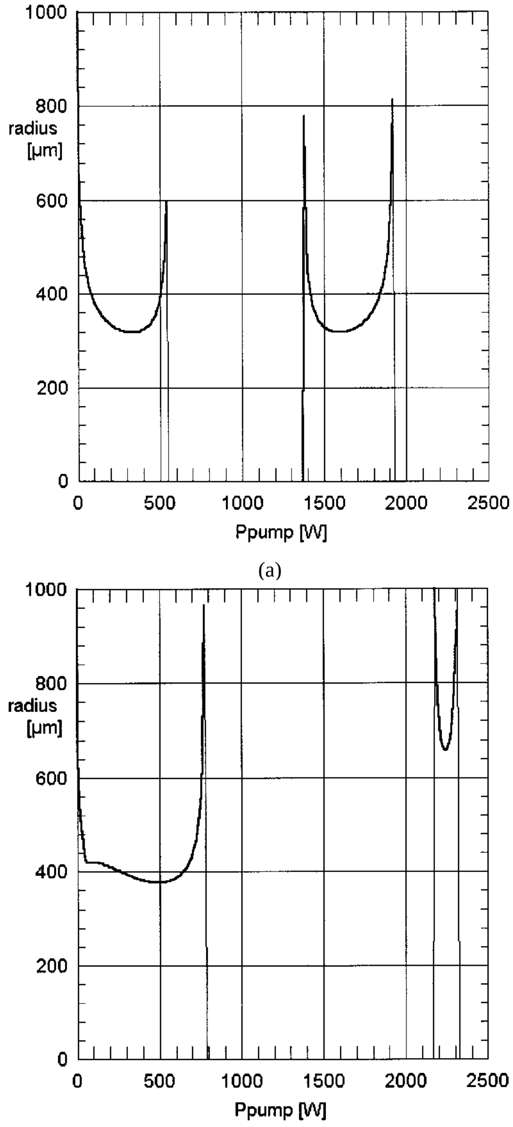

Fig. 5. Stability with increase. (a) Without compensating element, $\Delta P \approx 550$ W, $M^2 \approx 20$. (b) With compensating element, $\Delta P \approx 800$ W, $M^2 \approx 12$.

performance was assumed as in Fig. 2. The lens in the compensating element is set to zero until laser threshold is reached. The plot in (a) shows the performance without the compensating element and the plot (b) shows the behavior with the compensating element.

It is clearly seen that, by simply inserting the compensating element, the first stability range can be increased by almost a factor of 1.5. In addition, the mode size in the rod is increased due to the compensating element, yielding an improved M^2 factor, which was calculated by comparing the radii of the fundamental mode and the laser rod [15]. Furthermore, the mode size is approximately constant over a significant part of the stability range. This provides almost constant beam parameters in this range of operation, which is very favorable. The second stability range is still present with the compensating lens but is much smaller.

Fig. 3 shows that the sum of the dioptric powers of the two thermal lenses is constant when the pump power exceeds the threshold at about 100 W. If the two lenses are placed exactly at the same place (or optically superimposed with an appropriate relay optics), this leads to constant resonator conditions above threshold. For more general cases, the two lenses and the space in between have to be treated as a lens system. The total dioptric power D_{System} of this lens system is given by

$$D_{\mathrm{System}} = D_{\mathrm{LaserRod}} + D_{\mathrm{compEL}} - d \cdot D_{\mathrm{LaserRod}} \cdot D_{\mathrm{compEl}} \tag{13}$$

where d is the distance bewteen the principal planes of the two lenses. The ideal optical superposition of the two lenses requires additional optical elements in the resonator and will be discussed later. The simplest approach is to just place the laser rod and the compensating element in the resonator, as closely spaced as possible. The following section will discuss how much the first stability range is expanded in this simple case.

B. Rigorous Treatment of Resonator Behavior

The g-parameter formalism (as described in [15]) is very convenient to compare the widths of the stability ranges of various resonator configurations including resonators with thermal lenses. For a plane-wave resonator containing only the laser rod with a thermal lens of dioptric power D_{LaserRod}, the g-parameters read

$$g1 = 1 - s_2' \cdot D_{\mathrm{LaserRod}}$$

and

$$g2 = 1 - s_1 \cdot D_{\mathrm{LaserRod}} \tag{14}$$

where s_1 and s_2' are the distances from the principal planes of the thermal lens to the resonator mirrors according to Fig. 1. (The principal planes are located at a distance of $\ell_{\mathrm{LaserRod}}/(2n_{\mathrm{YAG}})$ from the end faces inside the laser rod [15].)

The complexity is increased significantly if the lens system consisting of the two thermal lenses is placed inside the laser cavity. The dioptric power of the laser rod cannot simply be replaced by the dioptric power of the lens system. In a lens system, the location of the principal planes is a function of both the dioptric power of each lens and the dioptric power of the whole lens system. Therefore, the distances between the mirrors and the principal planes of the lens system are a function of the pump power too. The derivation of the g-parameters for a lens sytem in terms of the fixed distances s_1 and s_2 is described in the Appendix. For a plane–plane resonator containing our lens system with the two thermal lenses, the g-parameters are given by

$$g1 = 1 - s_2 \cdot D_{\mathrm{System}} - d \cdot D_{\mathrm{LaserRod}}$$

and

$$g2 = 1 - s_1 \cdot D_{\mathrm{System}} - d \cdot D_{\mathrm{compEl}} \tag{15}$$

where s_1 and s_2 have again been taken from the principal planes of the lenses as shown in Fig. 1. D_{System} is the total dioptric power of the lens system according to (13).

As already stated, the thermal lens in the laser rod and in the compensating element have the same absolute value but the opposite sign. This absolute value of the lenses is, in the following, abbreviated by the positive term D_{therm}. In (15), D_{System} can be substituted with (13). Together with $D_{\mathrm{LaserRod}} = D_{\mathrm{therm}}$ (for the case of Nd:YAG) and $D_{\mathrm{compEl}} = -D_{\mathrm{therm}}$, the following g-parameters result for the plane–plane resonator containing an additional compensating element:

$$g1 = 1 - s_2 \cdot d \cdot D_{\mathrm{therm}}^2 - d \cdot D_{therm}$$

and

$$g2 = 1 - s_1 \cdot d \cdot D_{\mathrm{therm}}^2 + d \cdot D_{\mathrm{therm}}. \tag{16}$$

The width of the first stability range of the two configurations is now compared using (14) and (16). In Fabry–Perot resonators, the stability limits are reached for g-parameter products which are either $g1 \cdot g2 = 0$ or $g1 \cdot g2 = 1$. These conditions are used to determine the dioptric power of the thermal lenses at the two first stability limits D_{Limit0} and D_{Limit1}. These two limits define the width of the first stability range given by $D_{\mathrm{Limit1}} - D_{\mathrm{Limit0}}$. A plane–plane resonator without additional optics and without pumping (i.e., without thermal lenses) has a g-parameter product of $g1 \cdot g2 = 1$. Thus, the lower stability limit is $D_{\mathrm{Limit0}} = 0$ in both cases. This means that the width of the first stability range is just given by D_{Limit1}. This first limit is reached when

$$g1 \cdot g2 = 0. \tag{17}$$

Equation (17) has two solutions, either $g1 = 0$ or $g2 = 0$. The first stability limit is reached for the weakest dioptric power that fulfills the condition (17).

The stability limit for the rod-only resonator described with (14) is considered first. In order to compare resonators of equal physical length, in the rod-only resonator, extra space is left where the compensating element will be placed. This means that the distance s_2' is given by $s_2' = d + \ell_{\mathrm{compEl}} \cdot (1 - 1/n_{\mathrm{compEl}}) + s_2$. With this and the condition $s_1 \le s_2'$, the dioptric power at the first stability limit is given by

$$D_{\mathrm{Limit1}}^{\mathrm{uncomp}} = \frac{1}{d + \ell_{\mathrm{compEl}} \cdot (1 - 1/n_{\mathrm{compEl}}) + s_2} \quad \text{(rod-only)}. \tag{18a}$$

For the resonator with the compensating element, described by (16), $g1 = 0$ and $g2 = 0$ yield quadratic equations as a solution for the dioptric power at the stability limits. D_{Limit1} has to be positive by definition in our case. Together with the (arbitrary) condition $s_1 \le s_2$, the first stability limit is given by

$$D_{\mathrm{Limit1}}^{\mathrm{comp}} = \frac{-d + \sqrt{d^2 + 4 \cdot s_2 \cdot d}}{2 \cdot s_2 \cdot d} \quad \text{(with compensating element)}. \tag{18b}$$

The ratio of the dioptric powers at the first stability limit $R = D_{\mathrm{Limit1}}^{\mathrm{comp}} / D_{\mathrm{Limit1}}^{\mathrm{uncomp}}$ is shown in Fig. 6. This ratio represents the increase of the width of the stability range as a function of s_2 for different distances d. In addition, the pump power at the first stability limit for $d = 60$ mm is shown and is represented by thin lines. (The pump power is related, to good approximation, to the themal lens by $D_{\mathrm{therm}} = P_{\mathrm{pump}}/(f^* \cdot \pi \cdot R_{\mathrm{Rod}}^2)$, where f^* is known as the specific focal length having an approximate value of typically 6500 W/mm in transversely diode-pumped Nd:YAG rods.) The solid line describes the configuration with the compensating element, the dashed line the rod-only resonator.

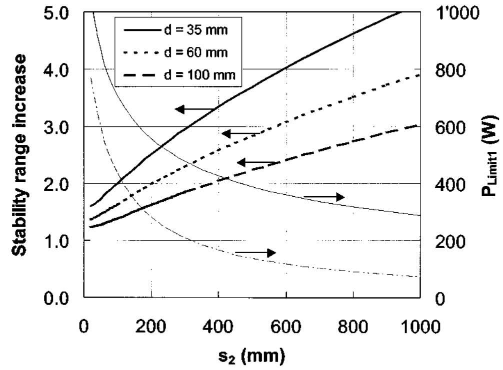

Fig. 6. Increase of the width of the first stability range as a function of s_2 for different d. The thin lines are the pump power at the first stability limit for $d = 60$ mm for the "normal" resonator (dashed line) and the compensated system (solid line).

It is seen that the compensating element significantly enlarges the first stability range. The enlargement increases with increasing length s_2, which corresponds to an increased mode size in the laser rod. Furthermore, the effect is more pronounced as the distance d between the rod and the compensating element is reduced. This qualitative behavior is independent of s_1 as long as the condition $s_1 < s_2$ holds. ($s_2 < s_1$ just means that all the indices have to be interchanged in the above derivation.)

C. Imaging Resonator

The above discussion implies a significant improvement by just placing an additional compensating element into the resonator. However, the thermal lens is not compensated over the whole pump range, as the two lenses are located at different positions inside the resonator and, therefore, encounter different beam properties. Hence, for ideal compensation, the two lenses should be placed exactly at the same location. Since this is not possible physically, it has to be implemented optically with an imaging resonator. This concept was used successfully in the case of birefringence as described in [1] and [2]: two additional lenses of equal focal length f are placed inside the resonator. The arrangement of such a resonator is shown in Fig. 7(a). The distance between the lenses is twice their focal length. The distance between the principal planes of the thermal lenses and the imaging lenses equals one focal length. The transfer matrix of the lens system between the two thermal lenses is given by the identity matrix with negative sign. Hence, according to the ABCD law, the beam parameter (q-parameter) in the laser rod is reproduced inside the compensating element. With this, the two thermal lenses are optically superimposed.

Two lenses with a focal length of 50 mm were used to calculate the fundamental-mode radius inside the laser rod shown in Fig. 7(b). All other resonator parameters were kept the same as in the previous exmaples. The result achieved this way is that the mode size remains constant over the whole pump power range above the laser threshold. A constant calculated beam propagation product of $M^2 \approx 13$ is achieved. Increasing the distance between the mirrors and the thermal lenses, or combining this arrangement with telescopic resonators [5], [16], would permit

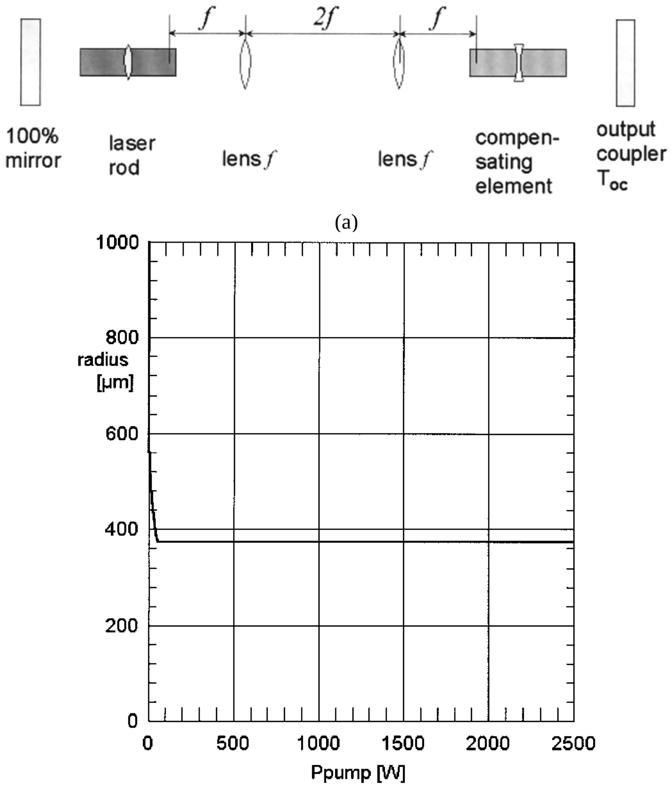

Fig. 7. (a) Imaging resonator arrangement with (b) calculated fundamental mode size in the laser rod.

the achievement of fundamental mode operation over a virtually infinite pump power range.

IV. Conclusions

A passive and self-adaptive compensation scheme for the thermal lens in solid-state laser rods is presented in this paper. An additional rod, preferably glass, with a negative temperature dependence of the refractive index is placed inside the resonator. The heating of this compensating element through weak absorption of the intracavity power causes a negative thermal lens. As the heating is from the laser radiation itself, the lens is self-adjusting if the correct parameters are chosen.

Although simple assumptions were made to obtain the results presented in this paper, the concept of the passively heated compenating element is shown to be very promising. A significant increase of the stability range is achieved by simply placing a compensating element inside the laser resonator. Optimum results are achieved if the compensating element is combined with an imaging optical system. In addition, this compensation scheme has the potential to also correct for the nonspherical aberrations introduced by the thermal lens in the laser rod.

Although optical glasses with the required dn/dT and thermal conductivity are readily available, the major problem at present is to get the glasses with the correct absorption. According to information from glass manufacturers, adding a weak absorption to the available glasses that have a negative dn/dT is possible without principal problems. Unfortunately, it is actually very expensive to get a small amount of spatially manufactured optical quality glass. First, experiments will therefore be carried out with the laser glass LG760 from Schott. In this Nd-doped glass, passive heating is provided by absorption from the lower laser level, and the population of this energy level can be temperature controlled. Since in this case the temperature increase caused by the absorption will also influence the amount of absorption, the system will be more complex.

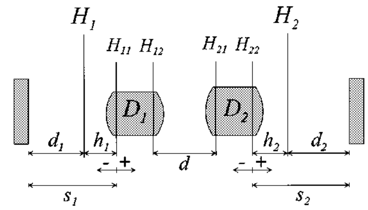

Fig. 8. Plane–plane resonator containing a lens system of thick lenses.

Appendix

g-Parameters of a Plane–Plane Resonator Containing a Lens System

The g-parameters in a resonator containing a thick lens of dioptric power D are given by [15]

$$g1 = 1 - \frac{L^*}{R_1} - D \cdot d_2 \quad \text{and} \quad g2 = 1 - \frac{L^*}{R_2} - D \cdot d_1 \tag{19}$$

where R_1 and R_2 are the radii of curvature of the two resonator mirrors, d_1 and d_2 are the distances of the mirrors to the principal planes of the entire lens system, and $L^* = d_1 + d_2 - D \cdot d_1 \cdot d_2$.

The situation of a resonator containing a lens sytem of thick lenses is shown in Fig. 8.

H_{ij} and H_i denote the locations of the principal planes of the single lenses and the lens system, respectively. The distances between the principal planes of the single lenses and the principal planes of the lens system according to Fig. 8 are given by [17]

$$h_1 = \frac{D_2 \cdot d}{D_{\text{System}}} \quad \text{and} \quad h_2 = -\frac{D_1 \cdot d}{D_{\text{System}}}. \tag{20}$$

A plane–plane resonator is assumed for the following, i.e., $R_1 = \infty$ and $R_2 = \infty$. In this case, (19) reads

$$g1 = 1 - D_{\text{System}} \cdot d_2 \quad \text{and} \quad g2 = 1 - D_{\text{System}} \cdot d_1 \tag{21}$$

where the total dioptric power of the lens system is given by $D_{\text{System}} = D_1 + D_2 \cdot d \cdot D_1 \cdot D_2$.

Unfortunately, the distances d_1 and d_2 are not fixed but are functions of the dioptric power of the lens system.

Considering the sign convention, shown in Fig. 8, the variable distances d_1 and d_2 in (21) can be replaced by $d_1 = s_1 + h_1$ and $d_2 = s_2 - h_2$. In addition, by substituting h_i with (20), one finally obtains for the g-parameters of a plane–plane resonator containing a lens system

$$\begin{aligned} g1 &= 1 - s_2 \cdot D_{\text{System}} - D_1 \cdot d \quad \text{and} \\ g2 &= 1 - s_1 \cdot D_{\text{System}} - D_2 \cdot d. \end{aligned} \tag{22}$$

We note that the distances s_1 and s_2 are the fixed distances from the resonator mirrors to the principal planes of the single lenses. The g-parameters described with (22) allow the convenient treatment of resonators with a lens system of thermal lenses with varying dioptric power.

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R. Weber, photograph and biography not available at the time of publication.

Thomas Graf was born in Lugano, Switzerland, in1966. He received the M.Sc. and Ph.D. degrees from the University of Bern, Bern, Switzerland, in 1993 and 1996, respectively.

As a Post-Doctoral Research Associate, he was engaged in research on high-power and solid-state lasers with high beam quality, beamshaping for diode-laser bars, and thermodynamics of optical systems at the University of Bern until 1997. He then joined the University of Strathclyde, Glasgow, U.K., where he was engaged in research on nonlinear optics and passively modelocked multiwatt all-solid-state lasers. In 1999, he was appointed Head of the High-Power Lasers and Material Science Group of the Laser Department of the Institute of Applied Physics, University of Bern. he is currently interested in high-power all-solid-state laser systems and adaptive resonators.

Dr. Graf was awarded the Applied Physics Prize from the Swiss Physical Society in 1998 and received the Physics Award of the Faculty of Science of the University of Bern for his dissertation in 1996.

Heinz P. Weber (M'72–SM'89–F'89), for a photograph and biography, see p. 626 of the May 2000 issue of this JOURNAL.