76153A,S PLASMA PHYSICS

Lecturer: Kalevi Mursula

University of Oulu

Department of Physical Sciences

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The most important source books:

(K) Koskinen, H., Johdatus plasmafysiikkaan ja sen avaruussovellutuksiin (Limes ry, 2001): A new book to be tested here as a text book.

(B) Bittencourt, J. A., Fundamentals of Plasma Physics (Pergamon Press, 1986): A complete text book but contains little of space plasma physics.

(P) Parks, G. K., Physics of Space Plasmas (Addison Wesley, 1991): Modern and versatile in space plasma physics. Many exercises. Unfortunately many errors.

(C) Chen, F., Introduction to Plasma Physics and Controlled Fusion, Volume 1: Plasma Physics (2. edition; Plenum Press): Concise and physical. A good text book.

Other books (books using cgs units noted):

Baumjohann, W., and R. Treumann, Basic Space Plasma Physics (Imperial College Press, 1996): A modern introduction.

Boyd, T. J. M., and J. J. Sanderson, Plasma Dynamics (Nelson, 1969): Old but good. Many exercises. (cgs)

Goldston, R. J., and P. H. Rutherford, Introduction to Plasma Physics (IOP Physics Publishing Ltd., 1995): Quite readable.

Kivelson, M. G., and C. T. Russell (eds.), Introduction to Space Physics (Cambridge University Press, 1995): Versatile and a lot of space physics observations. However, not very consistent and therefore not a good text book for beginners.

Krall, N. A., and A. W. Trivelpiece, Principles of Plasma Physics (San Francisco Press, 1986, a reproduction of the original text published by McGraw-Hill, 1973): A theoretical classic. (cgs)

Nicolson, D. R., Introduction to Plasma Theory (John Wiley &Sons, 1983): A fluent introduction to plasma theory. (cgs)

Sturrock, P. A. Plasma Physics, An Introduction to the Theory of Astrophysical, Geophysical & Laboratory Plasmas (Cambridge University Press, 1994): Fundamental, with many plasma and astrophysical examples. (cgs)

Treumann, R., and W. Baumjohann, Advanced Space Plasma Physics (Imperial College Press, 1996): An advanced text book of plasma waves, instabilities and nonlinear phenomena.

1 Introduction

1.1 Background and history (B, K)

Plasma physics is a young branch of physics, even younger in many respects than the basic theories of modern physics, the theory of relativity and quantum mechanics. The modern formalism of plasma physics was mainly developed only after World War 2. Still plasma physics is not often regarded as a part of modern physics because it is mainly based on classical physics, in particular, on classical electrodynamics (Maxwell equations) and classical mechanics.

The term "plasma" is Greek and means "molded" or "shaped". This term was coined in 1929 by Langmuir and Tonks while doing experiments with gas discharges. However, plasma phenomena have been studied even in laboratory conditions much earlier. Already in 1879 Crookes used the term "fourth state of matter" to describe hot, ionized gas. This name is still frequently used although it is not quite correct as we will later discuss in more detail.

However, the early history of plasma physics until 1950's mostly consists of observations that are relevant to space plasma physics. Presently, space plasma physics is an important part of plasma physics although the laboratory plasma physics has also grown to a large scale science.

Space plasma physics is also a part of space physics which studies the near-Earth space by direct and remote observations. The near-Earth space in space physics consists of the Sun, solar wind, interplanetary (and sometimes interstellar) space and the magnetospheres and ionospheres of the planets and their moons. More recently, the results of space plasma physics have also been applied to study more remote plasma objects, like other stars, stellar accretion disks etc.

Auroras are an important visual phenomena related to space plasma processes that have fascinated people for tens of thousands of years. Documented notes that can be interpreted as auroras exist since more than two thousand years. The name "aurora" is actually a misnomer from Latin, meaning (the Goddess of) dawn. As we now know, auroras mainly occur around (pre)midnight, and have nothing to do with normal solar light appearing at dawn. The term aurora was probably coined by Galileo Galilei. Aurora borealis and aurora australis refer to auroras appearing in the northern and southern hemisphere, respectively.

Other important early studies related to space physics are, e.g., the observations of the behaviour of the Earth's magnetic field. In 1730 Celcius and Hjorter noticed that during strong auroras the compass needle changed its direction. This observation connected auroras and changes in the magnetic field, and has later evolved to an important method of monitoring space plasma processes, especially space currents. After the development of the first scientific magnetometer by Gauss in about 1832, the Earth's magnetic field and its short-term and long-term changes are nowadays being monitored by hundreds of different types of magnetometers all over the world, and more recently even in space.

It was soon understood that the changes in the magnetic field are due to electric currents in space above the auroral regions. Norwegian researchers Birkeland and Størmer studied auroras with several optical instruments and showed that most of auroral light was emitted at the height of about 100–500 km. Later, the altitude of electric currents was located sligthly above 100 km.

Birkeland is mainly known for his Terrella experiment in 1896 where he directed a beam of (newly found) electrons toward a sphere which was magnetized in a dipolar form, like the Earth. The beam was diverted by the magnetic field to high latitudes, colliding the surface of the sphere mainly on oval-formed regions around the magnetic poles. These are the analogues of auroral ovals. The Terrella experiment suggested that auroras are formed by charged particles which follow the magnetic field lines and collide with the atoms and molecules in the upper atmosphere, causing them to radiate (auroral) light.

Charged particles can easily move along (but not perpendicular to) the magnetic field, thus quickly stabilizing any potential differences in that direction. Therefore, it was long considered an absolute paradigm that there are no field-aligned electric fields or related currents. Such electric fields could accelerate particles to energies needed to create auroral light. However, based on his studies of ionospheric currents, Birkeland suggested in 1913 that such field-aligned currents might exist in the ionosphere above auroral regions. This bold suggestion met with great opposition still long after Birkeland's time since no theoretical mechanism had yet been invented which could create such currents. Birkeland's suggestion had to await satellite observations in 1960's and 1970's which finally proved that field-aligned electric fields and currents not only can exist but are nearly a permanent (although not stable) feature in the high-latitude space.

Subsequently it was suggested that the particles causing auroras and magnetic disturbances are of solar origin. Chapman and Ferraro constructed in 1930's a model of magnetic storms in which particle clouds are intermittently emitted by erupting solar disturbances. However, in 1951 Biermann proved by cometary observations that the Sun emits continuously, rather than intermittently, a stream of particles which has an average speed of about 400 km/s. Later, in 1960's this was verified by direct satellite measurements. This stream is now called the solar wind. Soon thereafter in 1958 Parker formulated the first realistic theory how a supersonic solar wind can be formed as an extension of solar corona. In 1957 Hannes Alfvén proposed that solar wind is magnetized, i.e., carries a magnetic field with it. This field is now generally called the interplanetary magnetic field (IMF). Again, satellite observations were needed before the existence of the IMF could definitely be proven and its properties be studied.

In 1958 Gold suggested the name "magnetosphere" (magnetic sphere) for the region where the Earth's magnetic field prevents the flow of solar wind. Since the magnetic energy density rather than kinetic energy density dominates in the magnetosphere, it may also be regarded as a kind of magnetic bubble in the solar wind. The name magnetosphere remained although subsequent satellite measurements showed that the magnetosphere is far from the form of a sphere. Rather, it has a cometary shape with a far-streching tail and a slightly flattened nose. Satellite observations also found out the surprisingly complicated and dynamic structure of the magnetosphere which consists of several plasma regions with different types of particle populations. The first special region was found in 1958 by Van Allen who flew a Geiger counter on board the Explorer-1 satellite and detected regions of very energetic particles. These regions have since then been called the radiation belts, or Van Allen belts. Later the thin boundaries between the different magnetospheric plasma regions, e.g., the magnetopause, the bow shock and plasmapause, were observed.

Also, our knowledge of the Sun has dramatically increased during the last century. Some 100 years ago, there was a great problem related to the energy source of the Sun and other stars. Lord Kelvin had noted that the only possible source of energy known at that time, the Sun's gravitational potential, could only procude energy for about 20 million years. On the other hand, geologists had already shown conclusively that the Earth is much older. This problem was only solved after the invention of nuclear forces and the theory of relativity.

Now we know that the energy comes from the fusion of light nuclei to heavier nuclei (in the Sun mainly hydrogen to helium) where a small amount of mass is transformed to energy. The matter in the Sun's internal layers is in a state of hot plasma whence nuclear reactions are able to proceed. The most important problem in the laboratory plasma physics nowadays is to have this thermonuclear process continuing in a controlled manner long enough so that more energy can be extracted from the process than is needed to maintain it. Despite intensive research and technological development, reaching a state of an economically viable fusion reactor has proven to be extremely difficult, and is still at least 15–20 years ahead.

Another important observation was made by Hale in 1908 that the Sun has a magnetic field. The surface fields are strongest in sunspots. Later observations have shown that the solar magnetic field is very complicated and irregular in small scales. On a larger scale, the Sun is varing fairly regularly, changing its overall magnetic polarity every 11 years so that the magnetic cycle, so called Hale cycle, is about 22 years. However, it is also known that the Sun occasionally enters into exceptionally quiet periods (so called great minima) where nearly all magnetic variability disappears. The magnetic field is generated at the bottom of the convection layer where the rigid rotation of the inner core is changed to the differentially rotating outer convection layer. A better understanding of the solar dynamo and its dynamics is one of the main challenges of solar physics.

Outside the study of the solar system, the development of short wavelength radio techniques since 1930's opened a new method (radio astronomy) to study the universe through the Earth's atmosphere and ionosphere. A large fraction of the observed radio waves is produced by the bremsstrahlung of hot charged particles. However, this mechanism was soon found to be insufficient to explain all radio waves coming from the universe. In 1959 Ginzburg suggested that another mechanism, the synchotron radiation of relativistic electrons in strong magnetic fields, is responsible, e.g., for the radiation coming from the 1054 supernova remnant, the Crab Nebula. Later, this suggestion was verified by the measurement of the polarization of the radiation. Since then, the importance of magnetic fields in different systems of the universe has greatly increased.

In order to avoid the absorption of large frequency bands of electromagnetic radiation in the Earth's atmosphere and ionosphere, a number astronomical satellites have been launched during the last 10-15 years. These satellites have been fundamental for astronomy. It has been understood that many of the new astronomical observations relate to phenomena where plasma processes are important. Plasma astrophysics can be expected to become one of the leading branches of plasma physics in near future.

1.2 Basic properties of plasma (B)

A substance is a plasma if it contains sufficiently many free charged particles (e.g., electrons and ions) so that their mutual electromagnetic interactions have an essential effect on the dynamics of the system and cause the system to behave collectively.

Note that plasmas are macroscopically neutral. This means that they contain, on macroscopic scales, equally many positive and negative charges.

Collective behaviour may already appear when the relative fraction of charged particles (the so called ionization degree) is rather small, of about 0.1% of all particles. If the ionization degree is higher, of about 1%, plasma is nearly perfectly conducting.

In order for collective behaviour to appear, certain conditions, the so called plasma conditions, have to be satisfied. These will be discussed later in more detail.

As mentioned above, plasmas are often called the fourth state of matter, in addition to the solid, liquid and gaseous states of matter. This is to some extend justified, especially in the case of a gaseous plasma. However, there is no clear phase transition or a latent heat related to the formation of plasma which, instead, proceeds gradually. Also, this naming is somewhat restrictive and even erroneous, since plasma behaviour can be observed in liquid (electrolytes) and even in solid substances. (These are, however, beyond the present course where we only deal with gaseous plasmas).

Production of plasma

One needs energy to form plasma (free electrons and ions) from neutral gas, just like energy is needed in any other changes of the state of matter. This energy can be given, e.g., in the form of heat, i.e., by increasing the average energy of gas particles. Then the number of those gas particles rises whose kinetic energy is sufficient to ionize atoms or molecules in their mutual collisions. Accordingly, the number of free electrons and ions increases. When the heat is sufficiently high, the ionization degree may rise above the level of plasma formation.

Note: A gas in thermodynamic equilibrium always contains some fraction of atoms (or molecules) which have a sufficiently high kinetic energy to cause ionization when colliding. Therefore even a ”neutral” gas always has a small fraction of free electrons and ions. Most often this fraction is so small that the gas is far from the plasma state. Only when the heat of the gas approaches the lowest ionization potential energy of the gas atoms (or molecules) the ionization degree grows sufficient to make the gas a plasma. The relative fraction of ionization in a gas in thermodynamic equilibrium is described by the so called Saha equation.

The ionization degree can also be raised by other means, e.g., by strong electric fields or by sufficiently energetic photons. The latter is called photoionization. E.g., the E-layer of the Earth's ionosphere is mainly produced by the photoionization by the solar UV- and X-ray radiation. If the ionizing source fades away, the ionization degree decreases to the level which corresponds to the thermal equilibrium of the gas. This proceeds by recombination where electrons and ions collide with each other and combine back to neutral atoms (or molecules). Note, e.g., that there is a large diurnal variation in the low ionosphere because of the corresponding diurnal variation in solar radiation.

Collective behaviour

The collective behaviour required from plasmas is due to the long-range electric interaction between the particles described by the Coulomb potential $\phi = \dfrac{Q}{4\pi\epsilon_0 r}$. With this interaction, each plasma particle interacts simultaneously with several other plasma particles.

The dynamics of the charged particles in the plasma is formed by the external and internal (due to other plasma particles) electromagnetic fields, as well as by the collisions with the neutrals and other plasma particles.

If there are so many neutral particles that the dynamics of charged particles is essentially affected by their collisions with the neutrals, the plasma is said to be weakly ionized. This is, e.g., the case in most of the Earth's ionosphere. On the other hand, if the collisions with the neutrals have no significant effect on the dynamics of plasma particles and can be forgotten completely, plasma is said to be strongly ionized. This is, e.g., the case in most of the Earth's magnetosphere.

Other plasma properties

Electrons have a much better mobility than ions due to their much smaller mass ($m_p/m_e = 1.67 \cdot 10^{-27}$ kg$/9.1 \cdot 10^{-31}$ kg $= 1835$). Because of the good mobility of electrons, plasma is a good conductor of heat and electricity.

If plasma particles are set to motion by some reason (e.g., external electric field, density gradients etc.), the difference in mobility between electrons and ions gives rise to an electric field called polarization electric field.

In a magnetic field $\bar{B}$, the properties of plasma are considerably different along the magnetic field and perpendicular to it. This is due to the Lorentz force

$$\bar{F} = q\bar{v} \times \bar{B},$$

which makes the particles move in helical orbits (so called cyclotron or Larmor orbits) in the plane perpendicular to the magnetic field. However, the motion along the magnetic field is unaffected by the Lorentz force.

Plasma is quite an amorphous substance and sustains several different wave phenomena which may be transverse or longitudinal, have low or high frequency, and propagate along or perpendicular to the magnetic field. One type of magnetospheric waves, the so called whistler, has a dramatic history. It could be rather often heard in VLF radios that were used, e.g., by the military. Because of its very special signal with decreasing pitch, it can be easily recognized among noise and other program but remained long unexplained. One strong candidate was that they were due to flying grenades. Now we know that whistler waves are produced by lightnings in the opposite hemisphere and that the special signal with a decreasing pitch is due to wave dispersion during wave propagation.

Waves and particles may interact with each other. Some plasma particles may resonate with the waves and absorb energy from them, whence particles are accelerated and waves are damped. Wave energy may also be lost by dissipative processes like particle collisions. On the other hand, the opposite may also happen and energy may be transformed from particles to growing waves. (In fact, this is the way all the waves are produced.) When particle populations are appropriate for wave production, they must be somehow unstable, out of thermal balance. One then talks about instabilities. There are several types of instabilities and each of them may produce only certain types of waves.

Charged plasma particles also experience acceleration while moving in helical orbits in magnetic fields and colliding with each other and neutral particles. Accordingly, they radiate cyclotron radiation, as well as bremsstrahlung radiation. These are important issues for several astrophysical objects with strong magnetic fields.

1.3 Plasma conditions (B)

We will now present the most basic and important concepts of plasma physics, and the conditions that need to be fulfilled in order to have plasma.

1.3.1 Debye length and Debye shielding

As mentioned above, plasma is macroscopically neutral. However, this does not exclude the possibility that charged particles may significantly deviate from an even distribution on a microscopical scale.

The critical distance how far the charge imbalance may extend, is defined by one of the most important parameters of plasma physics called the Debye length λ_D. (The name comes from Peter J. W. Debye, 1884–1966, Nobel prize winner in chemistry in 1936).

We will now present a heuristic derivation for λ_D. Let us study a plasma where the density and temperature of electrons and ions are the same: $n_e = n_i = n$ and $T_e = T_i = T$. (Note that this implies that ions are singly charged).

Let us now assume that at some point in plasma there is region which only contains electrons. Let this region be a sphere with radius R (see Figure 1.1). There are N_e electrons in the sphere which make an electric potential whose value on the surface of the sphere is

$$\phi = \frac{q}{4\pi\epsilon_0 R} = \frac{-e \cdot N_e}{4\pi\epsilon_0 R} = \frac{-en_e\frac{4}{3}\pi R^3}{4\pi\epsilon_0 R} = -\frac{e}{3\epsilon_0}n_e R^2 \tag{1.1}$$

This potential attracts positive ions. Let us assume that one ion with an average (thermal)

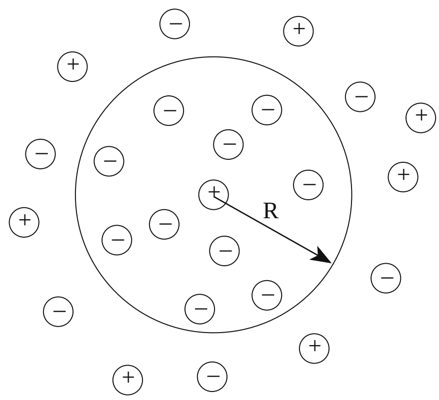

Figure 1.1. Debye sphere

kinetic energy is at the center of the sphere. It can surpass the potential well only if

$$E_{kin} = \frac{3}{2}k_B T \geq e \mid \phi \mid = \frac{e^2}{3\epsilon_0} n_e R^2, \tag{1.2}$$

from which

$$R \leq \frac{3}{\sqrt{2}} \sqrt{\frac{\epsilon_0 k_B T}{e^2 n_e}}.$$

The Debye length (of electrons) is determined from this as follows

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{e^2 n_e}}. \tag{1.3}$$

One often uses the following form of presentation:

$$\lambda_D^{-2} = \frac{1}{\epsilon_0} \frac{e^2 n_e}{k_B T}. \tag{1.4}$$

One can scale the Debye length numerically as follows

$$\lambda_D = 69.0 \text{ m} \sqrt{\frac{\mathrm{T/K}}{\mathrm{n_e/m^{-3}}}} = 7.43 \text{ m} \sqrt{\frac{\mathrm{k_B T/eV}}{\mathrm{n_e/cm^{-3}}}}, \tag{1.5}$$

where we have used the following constants:

- – unit charge $e = 1.6 \cdot 10^{-19} C$ $(C = As)$
- – permittivity of vacuum (also called the dielectric constant) $\epsilon_o = 8.85 \cdot 10^{-12}\ F/m$ $(F = \frac{C}{V} = \frac{As}{V})$
- – Boltzmann constant $k_B = 1.38 \cdot 10^{-23}\ J/K$ $(J = Nm = \frac{kgm^2}{s^2} = VAS)$.

Note a very useful relation:

$$\frac{k_B T}{\mathrm{eV}} = \frac{1}{11600} \frac{T}{\mathrm{K}} \tag{1.6}$$

This means that the thermal energy of 1 eV corresponds to the temperature of about 11600 K ! (Prove this !)

So the Debye length determines, e.g., how far the charge imbalance due to thermal motion can extend. It also defines the impact length of an external electric potential in plasma, the so called shielding length (varjostuspituus). If, e.g., a conducting surface is put into plasma, a so called plasma sheath (plasmavaippa) is formed around it where charge imbalance can exist and whose thickness is of the order of λ_D.

If L is a typical length scale of plasma, it is clear that the so called first plasma condition

$$L >> \lambda_D \tag{1.7}$$

has to be valid. Otherwise, plasma is not necessarily macroscopically neutral anywhere.

The so called Debye sphere is a plasma sphere with a radius λ_D. The Debye sphere around any single plasma particle contains those other plasma particles with which the particle can be in a collective interaction.

The number of electrons in a Debye sphere is

$$N_D = n_e \frac{4}{3} \pi {\lambda_D}^3 = \frac{4\pi}{3} \left(\frac{\epsilon_0 k_B T}{e^2 \cdot n_e^{1/3}} \right)^{3/2} . \tag{1.8}$$

Note that $N_D \to 0$ as $n_e \to \infty$, whence also $\lambda_D \to 0$.

In order to really have a collective interaction, several particles must exist inside the Debye sphere:

$$N_D >> 1. \tag{1.9}$$

This is the second plasma condition. One often calls the related combination

$$\Lambda = n_e {\lambda_D}^3 \tag{1.10}$$

the plasma parameter. Thus, an equivalent form of the second plasma condition is

$$\Lambda >> 1. \tag{1.11}$$

This is also sometimes called the plasma approximation.

Note also that some books define the plasma parameter as an inverse to above:

$$g = \frac{1}{\Lambda} = \frac{1}{n_e {\lambda_D}^3} \tag{1.12}$$

whence the plasma approximation reads $g << 1$.

Note that since the average distance between electrons is ${n_e}^{-1/3}$, the plasma approximation implies that this distance must be $<< \lambda_D$. Accordingly, the two first plasma conditions can be written as the following set of inequalities:

$$\frac{1}{n_e^{-1/3}} << \lambda_D << L. \tag{1.13}$$

Note that $g^{2/3}$ also gives the ratio of the average potential energy between two electrons and the average kinetic energy of electrons. (To be proven in exercises).

The macroscopic neutrality of plasma can also be presented in the following form:

$$n_e = \sum_i n_i, \tag{1.14}$$

where the densities are not local, microscopic densities but average densities which have been calculated over a macroscopic plasma region, much larger than a Debye sphere.

Similarly to electrons, one can define a separate Debye length for any type of charged particles. Sometimes, the Debye length is defined as a combined effect of all charged particles

$$\lambda_D^{-2} = \frac{1}{\epsilon_0} \sum_\alpha \frac{n_\alpha q_\alpha^2}{k_B T_\alpha}, \tag{1.15}$$

where the sum is takes over all charged particle types indexed by α.

If the particles are in equilibrium they follow the Boltzmann distribution which, in the presence of an electric potential ϕ, takes the following form:

$$n_\alpha(\bar{r}) = n_{0\alpha} exp(\frac{-q_\alpha \phi}{k_B T_\alpha}), \tag{1.16}$$

where $n_{0\alpha}$ are densities without the potential. One can fairly easily calculate that the Coulomb potential caused by a test plasma particle with charge q_T is transformed, in the case of a small potential, to the following form:

$$\phi = \frac{q_T}{4\pi\epsilon_0 r} exp(\frac{-r}{\lambda_D}). \tag{1.17}$$

This is called the shielding potential. It shows that the electric field is constrained (shielded) to essentially remain within the distance of the Debye length.

1.3.2 Plasma frequency (B, K)

Plasma has the tendency that it starts oscillating after being disturbed. Let us study a simple plasma consisting of electrons and positive ions with equal initial equilibrium densities $n_e = n_i = n_0$. We also assume that the particle velocities can initially be neglected. (This is called the cold plasma assumption).

Let us now disturb the system briefly by a small external electric field $\mathbf{E}_1$ (Subscript 1 is used to denote first-order approximation). The electrons, due to their excellent mobility, react immediately to the electric field and their density is changed to

$$n_e = n_0 + n_1(\mathbf{r}, t) \tag{1.18}$$

while the ions remain (nearly) static. Accordingly, charges are separated and an internal (polarization) electric field is formed within the plasma.

After the external field is switched off, the electrons are accelerated by the internal electric field, trying to restore charge neutrality. However, due to their kinetic energy given by the electric field, the electrons move beyond their initial equilibrium locations and an internal electric field of opposite direction is formed, which again accelerates the electrons back toward their equilibrium locations etc. This leads to an oscillation of electrons. We will now derive the frequency of this oscillation.

The number of electrons is conserved and therefore the electron density n_e obeys a similar equation which expresses the conservation of the electric charge. This equation is called the continuity equation and reads for the electron density as follows:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u_e}) = 0 \tag{1.19}$$

where $\mathbf{u_e} = \mathbf{u}_1$ is the velocity of electrons caused by the disturbance. In the first order of perturbation this equation reads

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{u}_1 = 0 \tag{1.20}$$

The electric field causes the force $\mathbf{F} = q\mathbf{E}_1$. Accordingly, the equation of motion of electrons is

$$m_e \frac{\partial \mathbf{u}_1}{\partial t} = -e\mathbf{E}_1. \tag{1.21}$$

On the other hand, the field $\mathbf{E}_1$ is determined by the disturbed density according to the Maxwell's first law (Coulomb law):

$$\nabla \cdot \mathbf{E}_1 = -en_1/\epsilon_0. \tag{1.22}$$

(We will review the Maxwell equations later in Ch. 2).

By applying the partial time derivative $\partial/\partial t$ to Eq. (1.20) we obtain, with the help of Eqs. (1.21) and (1.22)

$$\frac{\partial^2 n_1}{\partial t^2} + \left(\frac{n_0 e^2}{\epsilon_0 m_e}\right) n_1 = 0\,. \tag{1.23}$$

This describes an oscillation (standing wave) whose angular frequency ω_{pe} is

$$\omega_{pe}^2 = \frac{n_0 e^2}{\epsilon_0 m_e}. \tag{1.24}$$

The angular frequency ω_{pe} or the corresponding oscillation frequency $f_{pe} = \omega_{pe}/2\pi$ is called the (electron) plasma frequency. (Note that this indeterminate situation leads to an ambiguity by a factor of 2π. One has to check always which of the two parameters is meant.)

The oscillation described by Eq. (1.23) is called the plasma oscillation or Langmuir oscillation. In this cold plasma approximation it is a static (non-propagating) oscillation.

However, if a finite temperature of electrons is taken into account, Langmuir oscillation is changed to a propagating plasma wave, also called the Langmuir wave. (We will describe this later in Ch. 10 in more detail.)

Plasma frequency is inversely proportional to the square root of the mass of the moving particle. The disturbing electric field also causes the ions to move but the ion plasma frequency is much slower than the electron plasma frequency. Therefore the assumption of stable ions that was made in the beginning is reasonable. Normally by plasma frequency one indeed means the plasma frequency of electrons, not ions unless specifically mentioned.

Plasma frequency is among the most important plasma parameters. It is proportional to the plasma (electron) density and can be given numerically as follows for the angular frequency

$$\omega_{pe} = 56.4\ \frac{\mathrm{rad}}{\mathrm{s}} \left(\frac{n_e}{\mathrm{m}^{-3}}\right)^{1/2}. \tag{1.25}$$

or for the oscillation frequency

$$f_{pe} = \frac{\omega_e}{2\pi} \approx 9.0\ Hz \left(\frac{n_e}{\mathrm{m}^{-3}}\right)^{1/2}. \tag{1.26}$$

1.3.3 Collisions in plasma (B, K)

The collisions of electrons with ions and neutral particles damp their collective plasma oscillations. In order for plasma oscillations to occur before damping becomes dominant, the so called third plasma condition

$$f_{pe} > \nu_{en} \tag{1.27}$$

must be valid where ν_{en} denotes the collision frequency between electrons and neutrals. Otherwise no collective phenomena appears and the system of charged particles (not: plasma !) is dominated by the dynamics of the neutral particles.

In practice, Eq. (1.27) restricts the number density of neutrals with respect to that of charged particles. E.g., the ionospheric plasma is only partly ionized and neutral atoms and molecules form most of the total mass. Ionization degree decreases when going downwards from the E layer until it is so small that charged particles have no effect in overall dynamics. This boundary between the ionosphere and the neutral atmosphere (thermosphere) is approximately at the altitude of 50–60 km.

On the other hand, ionization degree increases when moving upwards from the E layer (at about 100 km's altitude), and in the magnetosphere it is practically 100 %. (There are neutral particles even in the magnetosphere but due to the sparse density and the implied long mean free path, neutral and charged particles hardly collide). However, since particle densities decrease with altitude, the total ionization has a maximum in the ionospheric F layer at the altitude of about 200–300 km.

Let us now study the Coulomb collisions between charged particles in the plasma. If the cross section of the collisions is σ and particle density is n, the mean free path is

$$l_{mfp} = 1/(n\sigma). \tag{1.28}$$

Similarly, the collision frequency ν_c can be easily calculated from the mean free path and the average velocity v:

$$\nu_c = v/l_{mfp} = n\sigma v. \tag{1.29}$$

For Coulomb collisions it is sufficient to study collisions with a small momentum transfer, i.e., small angle collisions. This is because every plasma particle interacts with several other particles most of which are rather far away, and the further away the other particle is, the smaller momemtum transfer it experiences on an average. On the other hand, there are only few very closely located particles with which the collision is, on an average, stronger.

The collision cross section for the electron-ion collision is $\sigma \propto 1/v_0^4$, and the related collision frequency is

$$\nu_c = \nu_{ei} = \frac{2n_0(Ze^2)^2 \ln\Lambda}{\epsilon_0^2 m_e^2 v_0^3}, \tag{1.30}$$

where v_0 and n_0 are the velocity and density far from the collision region. (We will discuss this later in Ch. 4 in more detail). E.g., the mean free path of a typical solar wind particle at 1 AU (1 AU $\approx$ 150 million km) is about 1 AU.

Because of the Debye shielding, each particle mainly reacts with the particles in the Debye sphere, i.e., with as many particles as expressed by the plasma parameter Λ. This is taken into account in Eq. (1.30) by the factor $\ln\Lambda$ which is called the Coulomb logarithm. Its typical values are of the order of 10–20.

According to Eq. (1.30), if plasma temperature increases or plasma density decreases, $\nu_c \to 0$ and plasma becomes collisionless. Note that in either limit $\Lambda \to \infty$ but this increase is much slower and does not dominate in Eq. (1.30). (See also Eq. (1.8)).

Collisionlessness means in practice that the time between two collisions or the mean free path of particles increases greater than the temporal or spatial scale of the phenomena in question. Note also that collisionlessness does not imply that the electromagnetic interactions between the plasma particles would be negligible. However, they can often be treated collectively using average fields, not having to examine the Coulomb collision of individual particles.

For a perfectly conducting plasma the following approximate relations are valid:

$$\begin{aligned} \nu_{ee} &\approx \nu_{ei} \\ \nu_{ii} &\approx \sqrt{m_e/m_i}\,\nu_{ee} \\ \nu_{ie} &\approx (m_e/m_i)\,\nu_{ee}\,. \end{aligned}$$

1.4 Magnetic fields (K)

Plasmas are often immersed within and affected by magnetic fields whose sources may be either external (e.g., due to the Earth's magnetic dipole moment, or the current coils circulating the plasma reactor chamber), or internal (e.g., the magnetospheric currents or the currents driven inside the fusion plasma).

Magnetic field intensities vary greatly from very weak average field intensities in interstellar and interplanetary space of less than 1 pT to very strong fields of up to 10^{12} T in some astronomical objects like pulsars.

The motion of a charged particle in a magnetic field with intensity B consists of a linear motion in the direction of the magnetic field and a circular motion in the plane perpendicular to the magnetic field. (We will study the motion of charged particles in a magnetic field in Ch. 3 in more detail.)

The angular frequency of the circular motion of particle of type α with mass m_α and charge q_α is

$$\omega_{c\alpha} = \frac{|q_\alpha| B}{m_\alpha} \tag{1.31}$$

This is one more fundamental plasma parameter. The corresponding frequency is $f_{c\alpha} = \omega_{c\alpha}/(2\pi)$. Both of these are interchangeably called the cyclotron frequency or the gyrofrequency or the Larmor frequency. (So take care of 2π again!).

Numerically for electrons and protons

$$f_{ce} = 28\ Hz\ B/(nT) \tag{1.32}$$

$$f_{cp} = 1.5\ 10^{-2}\ Hz\ B/(nT) \tag{1.33}$$

Magnetic fields do not only direct the motion of particles but also react to the various disturbances by forming fluctuations in the intensity or direction of the magnetic field. The period of possible magnetic fluctuations varies from very long (upper limit depends on field intensity) to quite short periods related to the cyclotron and plasma frequencies.

1.5 Occurrence of plasmas in nature (B)

Most of the known luminous matter of the universe is in the plasma state. However, the luminous matter is only a fraction of the known gravitating matter which is in an unknown state, and therefore also called the "missing mass". (Thus, the universe is dominated by matter whose nature is still unknown. So little do we know, yet).

Examples of known luminous matter.

1. Sun and other (main sequence) stars
   - Core: very dense ($n \sim 10^{33}$ m^{-3}) and hot ($T > 10^7\ K$) plasma where nuclear reactions take place.
   - Surface: Considerably cooler ($\sim 6000K$) plasma with $B \sim 10^{-4} - 1$ T.
   - Corona: Hot ($T \sim 10^6\ K$) but rather tenuous ($n \sim 10^{13}$ m^{-3}) plasma.
2. Solar wind (interplanetary plasma) at 1 AU
   - Very tenuous ($n_e \sim 5$ cm^{-3}).
   - Fairly hot ($T_i \sim 10^4\ K$, $T_e \sim 5 \cdot 10^4\ K$).
   - Moves anti-sunward at $v \sim 300 - 1000 km/s$.
   - Carries with it a magnetic field whose average intensity is $B \sim 5$ nT and which has the form of a spiral.
3. The magnetospheres of the Earth and other planets
   - Several regions with different types of plasma.
   - Mostly rather tenuous plasma ($n_e \sim 0.1 - 10 cm^{-3}$).
   - Most often completely ionized.
   - Most common ions: H^+, He^+, He^{++} and O^+. Other ions in special environment like in Jovian and Saturnian magnetospheres.
4. The ionospheres of the Earth and other planets
   - Rather cool plasma ($T \sim 100 - 1000\ K$).
   - Only partially ionized plasma.
   - Many different ion types, even negative ions in some regions.
5. Interstellar and intergalactic plasma
   - Extermely tenuous ($n_{IS} \sim 10^{-3} - 10^{-1}$ cm^{-3}, $n_{IG} \leq 10^{-3}$ cm^{-3}).
   - Rather cold ($T_{IS} \sim 100\ K$, $T_{IG} \sim 10^5\ K$).
   - Small magnetic field intensities ($B_{IS} \sim 10^{-10}$T, $B_{IG} \leq 10^{-12}$T).
   - Enormously large regions.
6. Other astrophysical objects
   - Neutron stars, and pulsars: Extremely dense (up to $n \sim 10^{48}$ m^{-3}) nuclear matter in core (neutrons, or possibly quark-gluon plasma). Extremely strong magnetic fields ($B \sim 10^8 - -10^{12}$ T) even at the surface.
   - Quasars: Extremely energetic radiators at most wavelengths. Possibly formed by matter circulating a black hole. Matter falling into the black hole radiates.
   - etc...

1.6 Applications of plasma physics (B)

1. Historically the first (unintentional) application was the use of ionospheric plasma in long-range radio communications. (The ionosphere reflects sufficiently long-period radio waves with $\omega \leq \omega_{pe}$, so that the signal can propagate beyond the horizon.)

2. The most common application is the electric discharge in tenuous gases which is used in to create light in fluorescent lamps.

3. The magnetohydrodynamic generator can be used to transform part of the kinetic energy of plasma directly into electricity, avoiding the greater losses related to any thermal conversion process. A schematic view of the generator principle is depicted in Fig. 1.2. When plasma is brought into a magnetic field perpendicular to the field lines, the ions and electrons are directed by the Lorentz force in opposite directions and arrive at opposite electrodes. Therefore, an electromotive force (voltage) is formed. Loading this force one can use part of the kinetic energy of plasma as electric current and power.

4. Plasma rocket engine acts opposite to the magnetohydrodynamic generator, transforming electricity into kinetic energy. A schematic view of the engine principle is depicted in Fig. 1.3. In perpendicular electric and magnetic fields plasma is guided in a direction which is opposite to either field at a velocity

$$\bar{v}_E = \frac{\bar{E} \times \bar{B}}{B^2}. \tag{1.34}$$

   Releasing some plasma out, pushes the rocket in the opposite direction. This plasma rocket engine principle has already been used in long-distance space probes.

5. The dream: Controlled fusion reactor. Principle: $4 \times m_H > m_{He} \Rightarrow$ energy is released in fusion. This is the energy source of all main sequence stars.

   However, there are three main, increasingly difficult problems in controlled fusion.

   - Sufficiently high temperature. The repulsive Coulomb force prevents protons and other positively charged nuclei to unite. Therefore the temperature must be sufficiently high ($T \geq 10^8\ K$, $k_B T \geq 10$ keV) so that the average kinetic energy of ions could surpass the repulsive Coulomb wall.
   - Sufficient energy gain. The reactor should produce more energy than is used to form, heat and store the plasma, or lost to radiation and other losses. The minimum level of sufficient energy gain is described by the so called Lawson condition:

$$n\tau > (n\tau)_{min} \sim 10^{20}\mathrm{m}^{-3}\mathrm{s} \ \ \text{(for a D} - \text{T plasma)}, \tag{1.35}$$

     where n is plasma density and τ plasma lifetime. The two most important and successful fusion reactor types are the torus formed Tokamak reactor and laser compression by strong laser pulses. The former has a long τ and reasonable n, while the latter has a large n at the expense of a very short τ. Both types have briefly reached the Lawson condition.

- Economical viability. This is the only remaining but still a very hard problem. Despite brief intervals of reaching the Lawson condition, the present fusion reactors are still quite far from becoming economically viable, i.e., producing significant amounts of energy. Even further out is the level of economical competitiveness although this may change in the future as mankind moves away from oil-based society. Anyway, an economically viable fusion reactor is still at least 15-20 more years ahead.

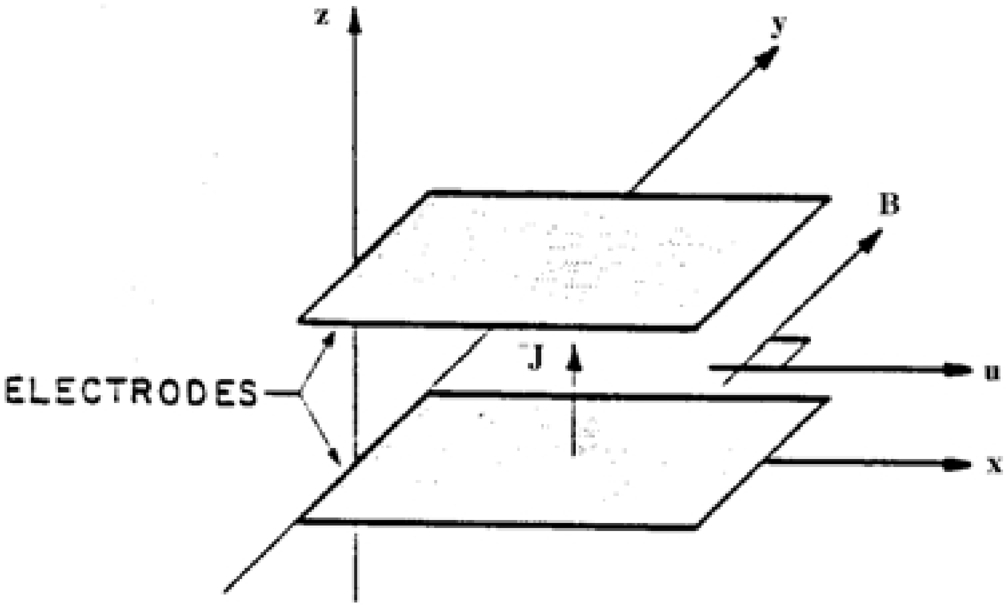

Figure 1.2. Principle of the magnetohydrodynamic generator

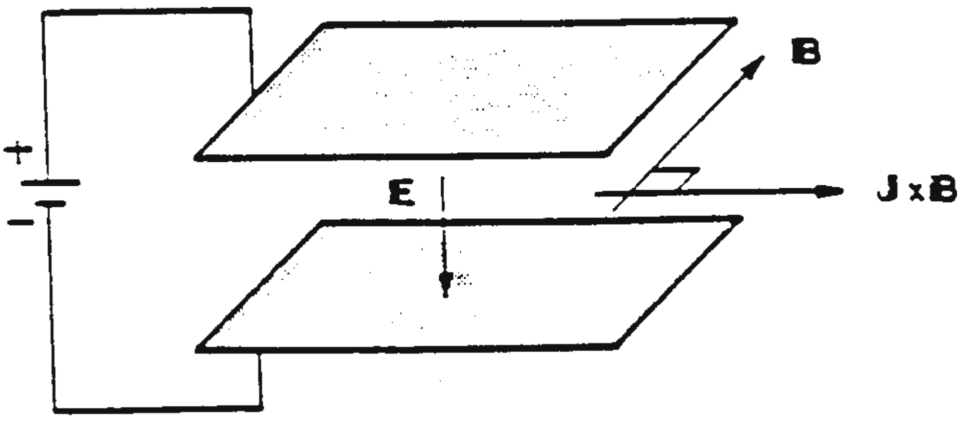

Figure 1.3. Principle of the plasma rocket engine