LECTURES IN GENERAL PHYSICS

Electrostatic

Principles and Applications

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Al-Azhar University - Gaza

Part Two

LECTURES IN

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Principles and Applications

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محاضرات في الفيزياء العامة
الجزء الثاني

الكهربية الساكنة
أساسيات وتطبيقات

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المقدمــة

منذ نشأة الخليقة على سطح الأرض شرع الإنسان يتساءل عن كيفية وجود الأشياء وعن سبب وجودها. هذا التساؤل كان دافعه الطبيعة الفضولية لدى البشر الذين متعهم الله بنعمة العقل والتفكير. ولكن بسبب جهل الإنسان القديم وخوفه فإنه كان يعزو ظواهر الطبيعة إلى وجود قوى خارقة مجهولة، فعبدها ظانا أنها المسئولة عن بقائه. ولكن مع مزيد من الملاحظات والاكتشافات ودافع الحاجة إلى الاختراع والابتكار أدرك أن الطبيعة تحكمها قوانين مترابطة تربط بين نشاطاته كإنسان وعلاقته بالعالم الجامد والعالم الحي على حد سواء. وعلم الفيزياء هو من العلوم التي تهتم بدراسة هذه القوانين وتسخيرها لخدمة البشرية. وفي أجزاء كتاب "محاضرات في الفيزياء العامة" سنتناول جزءاً أساسياً من علم الفيزياء التي يدرسها الطالب في المرحلة الجامعية.

الجزء الثاني من كتاب محاضرات في الفيزياء العامة يتناول شرح مبادئ الكهربية الساكنة وتطبيقاتها. وقد راعيت في عرض الموضوعات سهولة العبارة ووضوح المعنى. وتم التركيز على حل العديد من الأمثلة بعد كل موضوع لمزيد من التوضيح على ذلك الموضوع، وفي نهاية كل فصل تم حل العديد من المسائل المتنوعة التي تغطي ذلك الفصل، هذا بالإضافة إلى المسائل في نهاية كل فصل للطالب ليحلها خلال دراسته. تم الاعتماد على اللغة العربية وخصوصاً في الفصول الأربعة الأولى في توضيح بعض المواضيع وكذلك في التعليق على حلول الأمثلة وللاستفادة من هذا الكتاب ينصح باتباع الخطوات التالية:

☞ حاول حل الأمثلة المحلولة في الكتاب دون الاستعانة بالنظر إلى الحل الموجود.

☞ اقرأ صيغة السؤال للمثال المحلول عدة مرات حتى تستطيع فهم السؤال جيداً.

☞ حدد المعطيات ومن ثم المطلوب من السؤال.

☞ حدد الطريقة التي ستوصلك إلى إيجاد ذلك المطلوب على ضوء المعطيات والقوانين.

☞ قارن حلك مع الحل الموجود في الكتاب مستفيداً من أخطائك.

يحتوي الكتاب على ثمانية فصول، خصصت الفصول الخمسة الأولى منها لغرض المفاهيم الأساسية للكهربية الساكنة والتي هي القوى الكهربي والمجال الكهربي والجهد الكهربي ومن خلال قانون كولوم وقانون جاوس سنتمكن من إيجاد القوى الكهربية المتبادلة بين الشحنات وحساب المجال الكهربي الناتج من شحنة أو مجموعة من الشحنات (سواءً ذات توزيع منفصل أو متصل). وخصصت الفصول الثلاثة الباقية للتطبيقات المعتمدة على الكهربية الساكنة مثل المكثف الكهربائي ودوائر التيار المستمر.

آمل أن أكون قد قدمت لأبنائنا الدارسين من خلال هذا العمل المتواضع ما يعينهم على فهم واستيعاب هذا الفرع من فروع المعرفة.

والله من وراء القصد

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Contents

Chapter (4) Electric Flux

Chapter (5) Electric Potential

Multiple Choice Questions   110

Chapter (6) Capacitors

Chapter (7) Current and Resistance

Chapter (8) Direct Current Circuits

Appendices

Part 1

Principle of Electrostatic

Chapter 1

Introduction to Electrostatic

مقدمة عن علم الكهربية الساكنة

Introduction to Electrostatic

1.1 Understanding Static Electricity

1.2 Properties of electrostatic

1.2.1 Electric charge

1.2.2 Conductor and insulator

1.2.3 Positive and negative charge

1.2.4 Charge is conserved

1.2.5 Charge and matter

1.2.6 Charge is Quantized

Introduction to Electrostatic
مقدمة عن علم الكهربية الساكنة

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اكتشفت الكهربيـة الساكنة منذ 600 سنة قبل المـيلاد عنـدما لاحـظ عـالم يونـاني انجـذاب قصاصـات مـن الـورق إلـى سـاق ذُلك بالصـوف. ومـن ثـم توالـت التجـارب إلـى يومنـا هـذا لتكشف المزيد من خصائص الكهربية الساكنة ولتصبح الكهرباء عنصراً أساسياً في حياتنا العملية. في هذا الفصل سندرس باختصار بعض خصائص الكهربية الساكنة.

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We have all seen the strange device, known as a *Van De Graaff Generator*, that makes your hair stand on end. The device looks like a big aluminum ball mounted on a pedestal, and has the effect pictured on the right. Have you ever wondered what this device is, how it works, why it was invented, Surely it wasn't invented to make children's hair stand on end... Or have you ever shuffled your feet across the carpet on a dry winter day and gotten the shock of your life when you touched something metal? Have you ever wondered about static electricity and static cling? If any of these questions have ever crossed your mind, then here we will be amazingly interesting as we discuss Van de Graaff generators and static electricity in general.
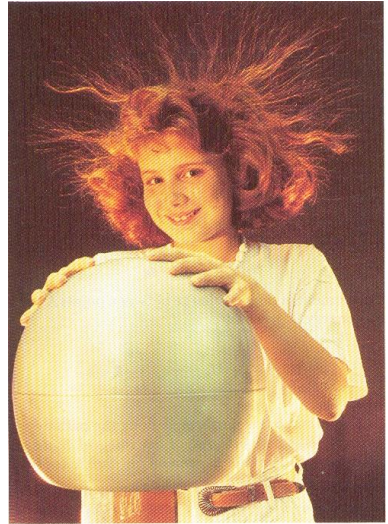
1.1 Understanding Static Electricity

To understand the Van de Graaff generator and how it works, you need to understand static electricity. Almost all of us are familiar with static electricity because we can see and feel it in the winter. On dry winter days, static electricity can build up in our bodies and cause a spark to jump from our bodies to pieces of metal or other people's bodies. We can see, feel and hear the sound of the spark when it jumps.

In science class you may have also done some experiments with static electricity. For example, if you rub a glass rod with a silk cloth or if you rub a piece of amber with wool, the glass and amber will develop a static charge that can attract small bits of paper or plastic.

To understand what is happening when your body or a glass rod develops a static charge, you need to think about the atoms that make up everything we can see. All matter is made up of atoms, which are themselves made up of charged particles. Atoms have a nucleus consisting of neutrons and protons. They also have a surrounding "shell" which is made up electrons. Typically matter is neutrally charged, meaning that the number of electrons and protons are the same. If an atom has more electrons than protons, it is

negatively charged. Likewise, if it has more protons than electrons, it is positively charged. Some atoms hold on to their electrons more tightly than others do. How strongly matter holds on to its electrons determines its place in the **Triboelectric Series**. If a material is more apt to give up electrons when in contact with another material, it is more positive on the Triboelectric Series. If a material is more to "capture" electrons when in contact with another material, it is more negative on the Triboelectric Series.

The following table shows you the Triboelectric Series for many materials you find around the house. Positive items in the series are at the top, and negative items are at the bottom:

- Human Hands (usually too moist though) (very positive)
- Rabbit Fur
- Glass
- Human Hair
- Nylon
- Wool
- Fur
- Lead
- Silk
- Aluminum
- Paper
- Cotton
- Steel (neutral)
- Wood
- Amber
- Hard Rubber
- Nickel, Copper
- Brass, Silver
- Gold, Platinum
- Polyester
- Styrene (Styrofoam)
- Saran Wrap
- Polyurethane
- Polyethylene (like scotch tape)
- Polypropylene
- Vinyl (PVC)
- Silicon
- Teflon (very negative)

The relative position of two substances in the Triboelectric series tells you how they will act when brought into contact. Glass rubbed by silk causes a charge separation because they are several positions apart in the table. The

same applies for amber and wool. The farther the separation in the table, the greater the effect.

When two non-conducting materials come into contact with each other, a chemical bond, known as adhesion, is formed between the two materials. Depending on the triboelectric properties of the materials, one material may "capture" some of the electrons from the other material. If the two materials are now separated from each other, a charge imbalance will occur. The material that captured the electron is now negatively charged and the material that lost an electron is now positively charged. This charge imbalance is where "static electricity" comes from. The term "static" electricity is deceptive, because it implies "no motion", when in reality it is very common and necessary for charge imbalances to flow. The spark you feel when you touch a doorknob is an example of such flow.

You may wonder why you don't see sparks every time you lift a piece of paper from your desk. The amount of charge is dependent on the materials involved and the amount of surface area that is connecting them. Many surfaces, when viewed with a magnifying device, appear rough or jagged. If these surfaces were flattened to allow for more surface contact to occur, the charge (voltage) would most definitely increase. Another important factor in electrostatics is humidity. If it is very humid, the charge imbalance will not remain for a useful amount of time. Remember that humidity is the measure of moisture in the air. If the humidity is high, the moisture coats the surface of the material providing a low-resistance path for electron flow. This path allows the charges to "recombine" and thus neutralize the charge imbalance. Likewise, if it is very dry, a charge can build up to extraordinary levels, up to tens of thousands of volts!
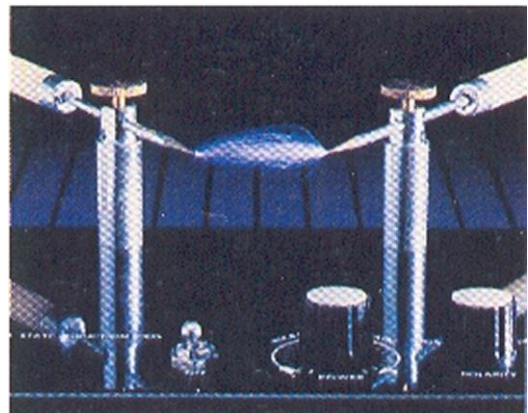
Think about the shock you get on a dry winter day. Depending on the type of sole your shoes have and the material of the floor you walk on, you can build up enough voltage to cause the charge to jump to the doorknob, thus leaving you neutral. You may remember the old "Static Cling" commercial. Clothes in the dryer build up an electrostatic charge. The dryer provides a

low moisture environment that rotates, allowing the clothes to continually contact and separate from each other. The charge can easily be high enough to cause the material to attract and "stick" to oppositely charged surfaces (your body or other clothes in this case). One method you could use to remove the "static" would be to lightly mist the clothes with some water. Here again, the water allows the charge to leak away, thus leaving the material neutral.

It should be noted that when dirt is in the air, the air will break down much more easily in an electric field. This means that the dirt allows the air to become ionized more easily. Ionized air is actually air that has been stripped of its electrons. When this occurs, it is said to be **plasma**, which is a pretty good conductor. Generally speaking, adding impurities to air improves its conductivity. You should now realize that having impurities in the air has the same effect as having moisture in the air. Neither condition is at all desirable for electrostatics. The presence of these impurities in the air, usually means that they are also on the materials you are using. The air conditions are a good gauge for your material conditions, the materials will generally break down like air, only much sooner.
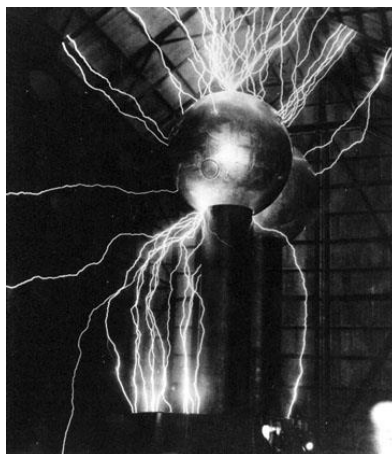
[Note: Do not make the mistake of thinking that electrostatic charges are caused by friction. Many assume this to be true. Rubbing a balloon on your head or dragging your feet on the carpet will build up a charge. Electrostatics and friction are related in that they both are products of adhesion as discussed above. Rubbing materials together can increase the electrostatic charge because more surface area is being contacted, but friction itself has nothing to do with the electrostatic charge]

For further information see appendix A (Understanding the Van de Graaff generator)

1.2 Properties of electrostatic

1.2.1 Electric charge

If a rod of ebonite is rubbed with fur, or a fountain pen with a coat-sleeve, it gains the power to attract light bodies, such as pieces of paper or tin foil. The discovery that a body could be made attractive by rubbing is attributed to Thales (640-548 B.C). He seems to have been led to it through the Greeks' practice of spinning silk with an amber spindle; the rubbing of the spindle cause the silk to be attracted to it. The Greek world of amber is *electron,* and a body made attractive by rubbing is said to be ***electrified*** or ***charged***. The branch of electricity is called ***Electrostatics***.

1.2.2 Conductor and insulator

قليل من التقدم الملحوظ في مجال الكهربية الساكنة بعد Thales حتى القرن 16 حين قام العالم Gilbert بشحن ساق من الزجاج بواسطة الحرير، ولكنه لم يتمكن من شحن أي نوع من المعادن مثل النحاس أو الحديد، وبذلك أستنتج أن شحن هذا النوع من الأجسام مستحيل. ولكن بعد حوالي 100 عام (1700) ثبت أن استنتاجه خاطئ وأن الحديد يمكن شحنه بواسطة الصوف أو الحرير ولكن بشرط أن يكون ممسوكا بقطعة من البلاستيك.

وبعد عدة تجارب وجد أن الشحنة المكتسبة يمكن أن تنتقل من الحديد إلى يد الإنسان ثم إلى الأرض وبالتالي تأثيرها سوف يختفي تماما إلا إذا عزل الحديد عن يد الإنسان بواسطة البلاستيك أثناء الدلك. وبالتالي فإن المواد قسمت حسب خواصها الكهربية إلى ثلاثة أقسام هي الموصلات Conductors والعوازل Insulators وأشباه الموصلات Semiconductors.
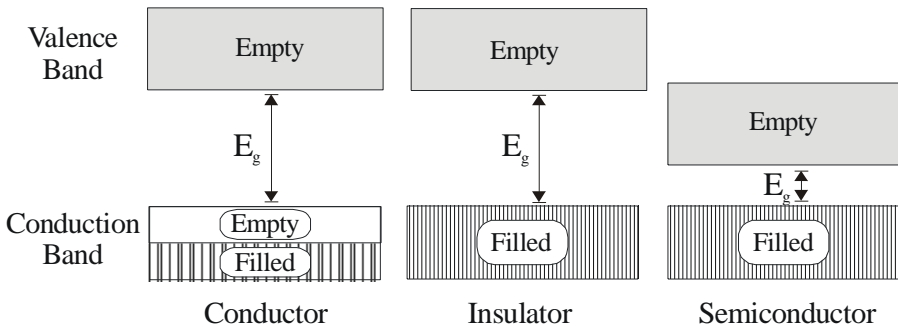
Figure 1.1

بصفة عامة تكون الشحنة الكهربية في الموصلات حرة الحركة لوجود شاغر بينما في العوازل فإن الشحنة مقيدة.

يتضح في الشكل 1.1 أنه في المواد الصلبة solid الإلكترونات لها طاقات موزعة على مستويات طاقة محددة Energy level. هذه المستويات مقسمة إلى حزم طاقة تسمى Energy Bands المسافات بين حزم الطاقة لا يمكن أن يوجد فيها أي إلكترونات. وهناك نوعان من حزم الطاقة أحدهما يعرف بحزمة التكافؤ Valence Band والأخرى حزمة التوصيل Conduction Band ويسمى الفراغ بين الحزمتين بـ Energy Gap E_g. وتعتمد خاصية التوصيل الكهربائي على الشواغر في حزمة التوصيل حتى تتمكن الشحنة من الحركة، وبالتالي فإن المادة التي تكون بهذه الخاصية تكون موصلة للكهرباء بينما في المواد العوازل كالبلاستيك أو الخشب فإنه تكون حزمة التوصيل مملوءة تماما، ولكي ينتقل أي إلكترون من حزمة التكافؤ إلى حزمة التوصيل يحتاج إلى طاقة كبيرة حتى يتغلب على Energy Gap E_g وبالتالي سيكون عازلاً لعدم توفر هذه الطاقة له.

توجد حالة وسط بين الموصلات والعوازل تسمى semiconductor وفيها تكون حزمة التوصيل قريبة نوعا ما من حزمة التكافؤ المملوءة تماما وبالتالي يستطيع إلكترون من القفز بواسطة اكتساب طاقة حرارية Absorbing thermal energy ليقفز إلى حزمة التوصيل.

1.2.3 Positive and negative charge

بواسطة التجارب يمكن إثبات أن هناك نوعين مختلفين من الشحنة. فمثلاً عن طريق دلك ساق من الزجاج بواسطة قطعة من الحرير وتعليقها بخيط عازل. فإذا قربنا ساقاً آخر مشابها تم دلكه بالحرير أيضا من الساق المعلق فإنه سوف يتحرك في اتجاه معاكس، أي أن الساقين يتنافران *Repel*. وبتقريب ساق من البلاستيك تم دلكه بواسطة الصوف فإن الساق المعلق سوف يتحرك باتجاه الساق البلاستيك أي أنهما يتجاذبان *Attract*.

Like charge repel one another and unlike charges attract one another as shown in figure 1.1 where a suspended rubber rod is negatively charged is attracted to the glass rod. But another negatively charged rubber rod will repel the suspended rubber rod.
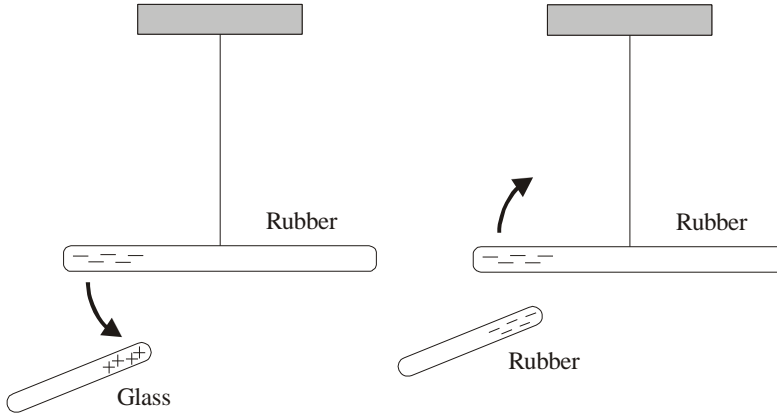
Figure 1.2
Unlike charges attract one another and like charge repel one another

وقد سمى العالم الأمريكي Franklin الشحنة التي تتكون على البلاستيك *Negative* سالبة واستنتج أن الشحنات المتشابهة تتنافر والشحنات المختلفة تتجاذب.

1.2.4 Charge is conserved

النظرة الحديثة للمواد هي أنها في الحالة العادية متعادلة Normal. هذه المواد تحتوي على كميات متساوية من الشحنة تنتقل من واحد إلى الأخر أثناء عملية الدلك (الشحن)، كما هو الحالة في دلك الزجاج بالحرير، فإن الزجاج يكتسب شحنة موجبة من الحرير بينما يصبح الحرير مشحوناً بشحنة سالبة، ولكن كلاً من الزجاج والحرير معاً متعادل كهربياً. وهذا ما يعرف بالحفاظ على الشحنة Conservation of electric charge.

1.2.5 Charge and Matter

القوى المتبادلة المسؤولة عن التركيب الذرى أو الجزئي أو بصفة عامة للمواد هي مبدئيا قوى كهربائية بين الجسيمات المشحونة كهربياً، وهذه الجسيمات هي البروتونات والنيوترونات والإلكترونات.

وكما نعلم فإن الإلكترون شحنته سالبة, وبالتالي فإنه يتجاذب مع مكونات النواة الموجبة، وهذه القوى هي المسؤولة عن تكوين الذرة Atom. وكما أن القوى التي تربط الذرات مع بعضها البعض مكونة الجزيئات هي أيضا قوى تجاذب كهربية بالإضافة إلى القوى التي تربط بين الجزيئات لتكون المواد الصلبة والسائلة.

الجدول (1) التالي يوضح خصائص المكونات الأساسية للذرة من حيث قيمة الشحنة والكتلة:

Particle	Symbol	Charge	Mass
Proton	p	1.6×10^{-19}C	1.67×10^{-27}Kg
Neutron	n	0	1.67×10^{-27}Kg
Electron	e	-1.6×10^{-19}C	9.1×10^{-31}Kg

Table 1.1

ويجب أن ننوه هنا أن هناك نوعاً آخر من القوى التي تربط مكونات النواة مع بعضها البعض وهى القوى النووية، ولولاها لتفتت النواة بواسطة قوى التجاذب بين الإلكترون والبروتون. وتدرس هذه القوى في مقرر الفيزياء النووية.

1.2.6 Charge is Quantized

في عهد العالم Franklin's كان الاعتقاد السائد بأن الشحنة الكهربية شيء متصل كالسوائل مثلا. ولكن بعد اكتشاف النظرية الذرية للمواد غيرت هذه النظرة تماماً حيث تبين أن الشحنة الكهربية عبارة عن عدد صحيح من الإلكترونات السالبة أو البروتونات الموجبة، وبالتالي فإن أصغر شحنة يمكن الحصول عليها هي شحنة إلكترون مفرد وقيمتها 1.6×10^{-19}c. وعملية الدلك لشحن ساق من الزجاج هي عبارة عن انتقال لعدد صحيح من الشحنة السالبة إلى الساق. وتجربة ميليكان تثبت هذه الخاصية.

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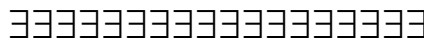
Chapter 2

Coulomb's Law

قانون كولوم

Coulomb's law

2.1 Coulomb's Law

2.2 Calculation of the electric force

2.2.1 Electric force between two electric charges

2.2.2 Electric force between more than two electric charges

2.3 Problems

Coulomb's law

قانون كولوم

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القوى الموجودة في الطبيعة هي نتيجة لأربع قوى أساسية هي: القوى النووية والقوى الكهربية والقوى المغناطيسية وقوى الجاذبية الأرضية. وفي هذا الجزء من المقرر سوف نركز على القوى الكهربية وخواصها. حيث أن القوة الكهربية هي التي تربط النواة بالإلكترونات لتكون الذرة، هذا بالإضافة إلى أهمية الكهرباء في حياتنا العملية. وقانون كولوم موضوع هذا الفصل هو أول قانون يحسب القوى الكهربية المتبادلة بين الشحنات الكهربية.

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2.1 Coulomb's Law

In 1785, Coulomb established the fundamental law of *electric force* between two stationary, charged particles. Experiments show that an electric force has the following properties:

(1) The force is *inversely proportional* to the square of separation, r^2, between the two charged particles.

$$F \propto \frac{1}{r^2} \qquad (2.1)$$

(2) The force is *proportional* to the product of charge q_1 and the charge q_2 on the particles.

$$F \propto q_1 q_2 \qquad (2.2)$$

Figure 2.1

(3) The force is *attractive* if the charges are of opposite sign and *repulsive* if the charges have the same sign.

We can conclude that

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\therefore F = K \frac{q_1 q_2}{r^2} \qquad (2.3)$$

where K is the coulomb constant $= 9 \times 10^9$ N.m^2/C^2.

The above equation is called ***Coulomb's law***, which is used to calculate the force between electric charges. In that equation F is measured in Newton (N), q is measured in unit of coulomb (C) and r in meter (m).

The constant K can be written as

$$K = \frac{1}{4\pi\varepsilon_o}$$

where ε_o is known as the *Permittivity constant of free space*.

$$\varepsilon_o = 8.85 \times 10^{-12} \text{ C}^2/\text{N.m}^2$$

$$K = \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} = 9 \times 10^9 N.m^2/C^2$$
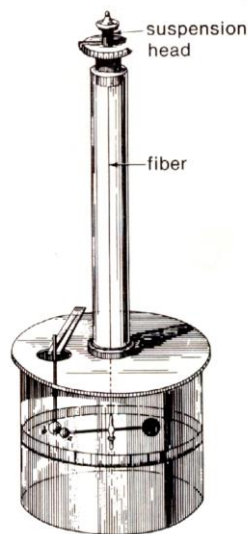
2.2 Calculation of the electric force

القوى الكهربية تكون ناتجة من تأثير شحنة على شحنة أخرى أو من تأثير توزيع معين لعدة شحنات على شحنة معينة q_1 على سبيل المثال، ولحساب القوة الكهربية المؤثرة على تلك الشحنة نتبع الخطوات التالية:–

2.2.1 Electric force between two electric charges

في حالة وجود شحنتين فقط والمراد هو حساب تأثير القوى الكهربية لشحنة على الأخرى. الحالة في الشكل(Figure 2.2(a تمثل شحنات متشابهة إما موجبة أو سالبة حيث القوة المتبادلة هي قوة تنافر *Repulsive force*.
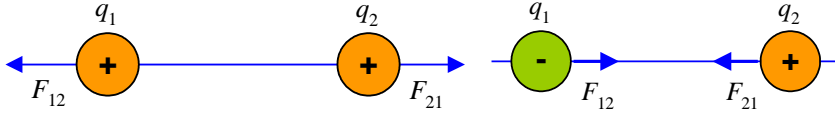
Figure 2.2(a)   *Figure 2.2(b)*

لحساب مقدار القوة المتبادلة نسمى الشحنة الأولى q_1 والثانية q_2. فإن القوة المؤثرة على الشحنة q_1 نتيجة الشحنة q_2 تكتب F_{12} وتكون في اتجاه التنافر عن q_2. وتحسب مقدار القوة من قانون كولوم كالتالي:

$$F_{12} \ = \ K \ \frac{q_1 q_2}{r^2} \ = F_{21} \qquad \text{مقداراً}$$

$$\vec{F}_{12} = -\vec{F}_{21} \qquad \text{واتجاها}$$

أي أن القوتين متساويتان في المقدار ومتعاكستان في الاتجاه.

كذلك الحال في الشكل (Figure 2.2(b والذي يمثل شحنتين مختلفتين، حيث القوة المتبادلة قوة تجاذب *Attractive force*. وهنا أيضا نتبع نفس الخطوات السابقة وتكون القوتان متساويتين في المقدار ومتعاكستين في الاتجاه أيضاً.

$$\vec{F}_{12} = -\vec{F}_{21}$$

لاحظ اتجاه أسهم القوة على الرسم.

Example 2.1

Calculate the value of two equal charges if they repel one another with a force of 0.1N when situated 50cm apart in a vacuum.

Solution

$$F = K \ \frac{q_1 q_2}{r^2}$$

Since $q_1 = q_2$

$$0.1 = \frac{9 \times 10^9 \times q^2}{(0.5)^2}$$

$$q = 1.7 \text{x} 10^{-6} \text{C} = 1.7 \mu \text{C}$$

وهذه هي قيمة الشحنة التي تجعل القوة المتبادلة تساوي 0.1N.

2.2.2 Electric force between more than two electric charges
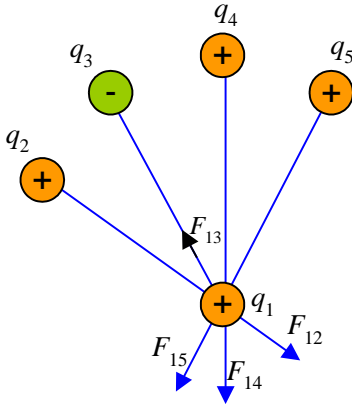
في حالة التعامل مع أكثر من شحنتين والمراد حساب القوى الكهربية الكلية The resultant electric forces المؤثرة على شحنة q_1 كما في الشكل Figure 2.3 فإن هذه القوة هي F_1 وهى الجمع الاتجاهي لجميع القوى المتبادلة مع الشحنة q_1 أي أن

Figure 2.3

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} \qquad (2.4)$$

ولحساب قيمة واتجاه F_1 نتبع الخطوات التالية:-

1) حدد متجهات القوة المتبادلة مع الشحنة q_1 على الشكل وذلك حسب إشارة الشحنات وللسهولة نعتبر أن الشحنة q_1 قابلة للحركة وباقي الشحنات ثابتة.

2) نأخذ الشحنتين q_1&q_2 أولا حيث أن الشحنتين موجبتان. إذاً q_1 تتحرك بعيدا عن الشحنة q_2 وعلى امتداد الخط الواصل بينهما ويكون المتجه F_{12} هو اتجاه القوة المؤثرة على الشحنة q_1 نتيجة

الشحنة q_2 وطول المتجه يتناسب مع مقدار القوة. وبالمثل نأخذ الشحنتين $q_1\&q_3$ ونحدد اتجاه القوة F_{13} ثم نحدد F_{14} وهكذا.

3) هنا نهمل القوى الكهربية المتبادلة بين الشحنات q_4 & q_3 & q2 لأننا نحسب القوى المؤثرة على q_1.

4) لحساب مقدار متجهات القوة كل على حده نعوض في قانون كولوم كالتالي:–

$$F_{12} = K \ \frac{q_1 q_2}{r^2}$$

$$F_{13} = K \ \frac{q_1 q_3}{r^2}$$

$$F_{14} = K \ \frac{q_1 q_4}{r^2}$$

5) تكون محصلة هذه القوى هي F_1 ولكن كما هو واضح على الشكل فإن خط عمل القوى مختلف ولذلك نستخدم طريقة تحليل المتجهات إلى مركبتين كما يلي

$$F_{1x} = F_{12x} + F_{13x} + F_{14x}$$
$$F_{1y} = F_{12y} + F_{13y} + F_{14y}$$

- مقدار محصلة القوى

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} \qquad\qquad (2.5)$$

- واتجاهها

$$\theta = \tan^{-1} \frac{F_y}{F_x} \qquad\qquad (2.6)$$

نتبع هذه الخطوات لأن القوة الكهربية كمية متجهة، والأمثلة التالية توضح تطبيقاً على ما سبق ذكره.

Example 2.2

In figure 2.4, two equal positive charges $q=2\times10^{-6}$C interact with a third charge $Q=4\times10^{-6}$C. Find the magnitude and direction of the resultant force on Q.
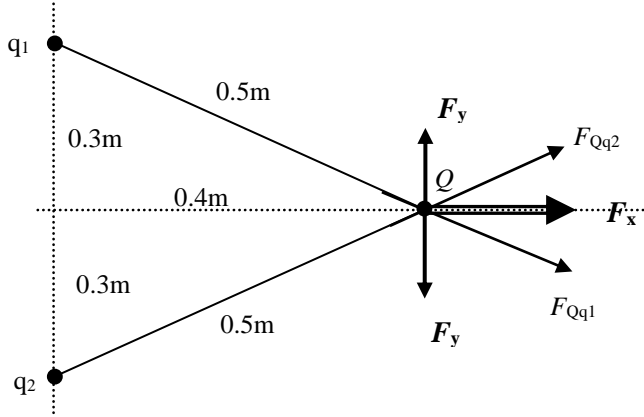
Figure 2.4

Solution

لإيجاد محصلة القوى الكهربية المؤثرة على الشحنة Q نطبق قانون كولوم لحساب مقدار القوة التي تؤثر بها كل شحنة على الشحنة Q. وبما أن الشحنتين q_1&q_2 متساويتان وتبعدان نفس المسافة عن الشحنة Q فإن القوتين متساويتين في مقدار وقيمة القوة

$$F_{Qq1} = K\frac{qQ}{r^2} = 9\times10^9 \frac{(4\times10^{-6})(2\times10^{-6})}{(0.5)^2} = 0.29N = F_{Qq2}$$

بتحليل متجه القوة إلى مركبتين ينتج:

$$F_x = F\cos\theta = 0.29\left(\frac{0.4}{0.5}\right) = 0.23N$$

$$F_y = -F\sin\theta = -0.29\left(\frac{0.3}{0.5}\right) = -0.17N$$

وبالمثل يمكن إيجاد القوة المتبادلة بين الشحنتين q_2 و Q وهي F_{Qq2} وبالتحليل الاتجاهي نلاحظ أن مركبتي y متساويتان في المقدار ومتعاكستان في الاتجاه.

$$\sum F_x = 2 \times 0.23 = 0.46N$$

$$\sum F_y = 0$$

وبهذا فإن مقدار القوة المحصلة هي 0.46N واتجاهها في اتجاه محور x الموجب.

Example 2.3

In figure 2.5 what is the resultant force on the charge in the lower left corner of the square? Assume that $q=1\times10^{-7}$ C and $a = 5$cm
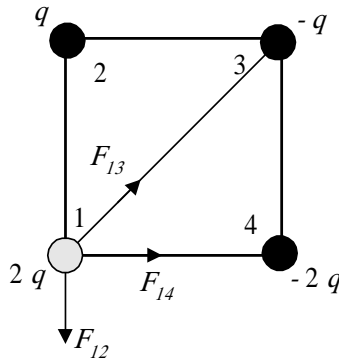
Figure 2.5

Solution

For simplicity we number the charges as shown in figure 2.5, then we determine the direction of the electric forces acted on the charge in the lower left corner of the square q_1

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

$$F_{12} = K\frac{2qq}{a^2}$$

$$F_{13} = K\frac{2qq}{2a^2}$$

$$F_{14} = K\frac{2q2q}{a^2}$$

لاحظ هنا أننا أهملنا التعويض عن إشارة الشحنات عند حساب مقدار القوى. وبالتعويض في المعادلات ينتج أن:

$F_{12} = 0.072$ N,

$F_{13} = 0.036$ N,

$F_{14} = 0.144$ N

لاحظ هنا أننا لا نستطيع جمع القوى الثلاث مباشرة لأن خط عمل القوى مختلف، ولذلك لحساب المحصلة نفرض محورين متعامدين x,y ونحلل القوى التي لا تقع على هذين المحورين أي متجه القوة F_{13} ليصبح

$F_{13x} = F_{13} \sin 45 = 0.025$ N   &

$F_{13y} = F_{13} \cos 45 = 0.025$ N

$F_x = F_{13x} + F_{14} = 0.025 + 0.144 = 0.169$ N

$F_y = F_{13y} - F_{12} = 0.025 - 0.072 = -0.047$ N

الإشارة السالبة تدل على أن اتجاه مركبة القوة في اتجاه محور y السالب.

The resultant force equals

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} \quad = 0.175 \text{ N}$$

The direction with respect to the x-axis equals

$$\theta = \tan^{-1} \frac{F_y}{F_x} = -15.5°$$
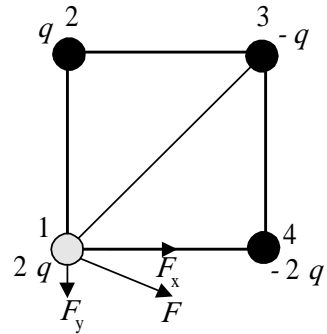
Example 2.4

A charge Q is fixed at each of two opposite corners of a square as shown in figure 2.6. A charge q is placed at each of the other two corners. (a) If the resultant electrical force on Q is Zero, how are Q and q related.
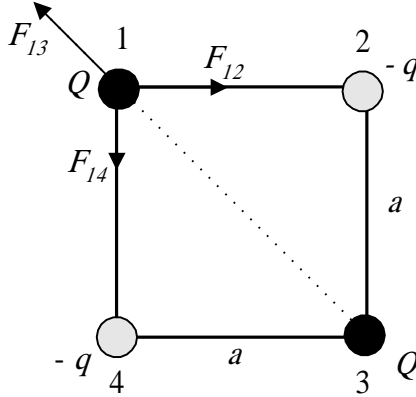
Figure 2.6

Solution

حتى تكون محصلة القوى الكهربية على الشحنة Q نتيجة الشحنات الأخرى مساوية للصفر، فإنه يجب أن تكون تلك القوى متساوية في المقدار ومتعاكسة في الاتجاه عند الشحنة Q رقم (1) مثلا، وحتى يتحقق ذلك نفرض أن كلتي الشحنتين (2) و (4) سالبة و Q (1) و (3) موجبة ثم نعين القوى المؤثرة على الشحنة (1).

نحدد اتجاهات القوى على الشكل (2.6). بعد تحليل متجه القوة F_{13} نلاحظ أن هناك أربعة متجهات قوى متعامدة، كما هو موضح في الشكل أدناه، وبالتالي يمكن أن تكون محصلتهم تساوي صفراً إذا كانت محصلة المركبات الأفقية تساوى صفراً وكذلك محصلة المركبات الرأسية

$$F_x = 0 \implies F_{12} - F_{13x} = 0$$

then

$$F_{12} = F_{13} \cos 45$$
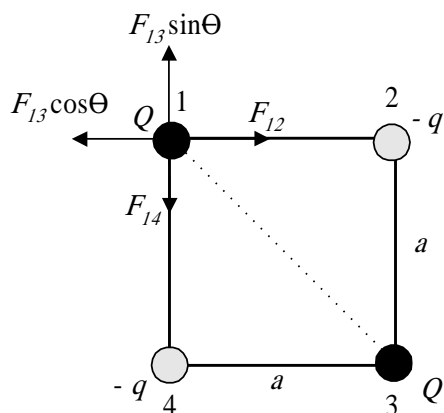
$$K \frac{Qq}{a^2} = K \frac{QQ}{2a^2} \frac{1}{\sqrt{2}} \quad \Rightarrow \quad q = \frac{Q}{2\sqrt{2}}$$

$$F_y = 0 \quad \Rightarrow \quad F_{13y} - F_{14} = 0$$

$$F_{13} \sin 45 = F_{14}$$

$$K \frac{QQ}{2a^2} \frac{1}{\sqrt{2}} = K \frac{Qq}{a^2} \quad \Rightarrow \quad q = \frac{Q}{2\sqrt{2}}$$

$$Q = 2\sqrt{2}\, q$$

وهذه هي العلاقة بين Q و q التي تجعل محصلة القوى على Q تساوى صفر مع ملاحظة أن إشارة q تعاكس إشارة Q أي أن

$$Q = -2\sqrt{2}\; q$$

Example 2.5

Two fixed charges, 1μC and -3μC are separated by 10cm as shown in figure 2.7 (a) where may a third charge be located so that no force acts on it? (b) is the equilibrium stable or unstable for the third charge?
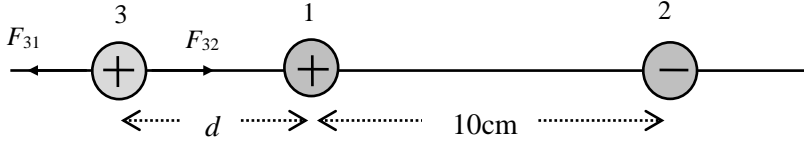
Figure 2.7

Solution

المطلوب من السؤال هو أين يمكن وضع شحنة ثالثة بحيث تكون محصلة القوى الكهربية المؤثرة عليها تساوي صفراً، أي أن تكون في وضع اتزان equilibrium. (لاحظ أن نوع الشحنة ومقدارها لا يؤثر في تعيين نقطة الاتزان). حتى يتحقق هذا فإنه يجب أن تكون القوى المؤثرة متساوية في المقدار ومتعاكسة في الاتجاه. وحتى يتحقق هذا الشرط فإن الشحنة الثالثة يجب أن توضع خارج الشحنتين وبالقرب من الشحنة الأصغر. لذلك نفرض شحنة موجبة q_3 كما في الرسم ونحدد اتجاه القوى المؤثرة عليها.

$$F_{31} = F_{32}$$

$$k\frac{q_3 q_1}{r_{31}^2} = k\frac{q_3 q_2}{r_{32}^2}$$

$$\frac{1\times10^{-6}}{d^2} = \frac{3\times10^{-6}}{(d+10)^2}$$

نحل هذه المعادلة ونوجد قيمة d

(b) This equilibrium is unstable!! Why!!

Example 2.6

Two charges are located on the positive x-axis of a coordinate system, as shown in figure 2.8. Charge $q_1 = 2nC$ is 2cm from the origin, and charge $q_2 = -3nC$ is 4cm from the origin. What is the total force exerted by these two charges on a charge $q_3 = 5nC$ located at the origin?
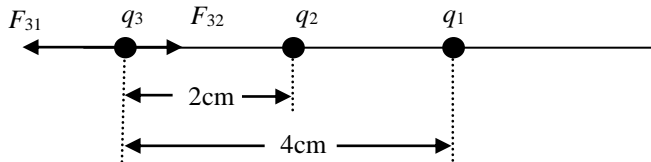
Figure 2.8

Solution

The total force on q_3 is the vector sum of the forces due to q_1 and q_2 individually.

$$F_{31} = \frac{(9 \times 10^9)(2 \times 10^{-9})(5 \times 10^{-9})}{(0.02)^2} = 2.25 \times 10^{-4} N$$

$$F_{32} = \frac{(9 \times 10^9)(3 \times 10^{-9})(5 \times 10^{-9})}{(0.04)^2} = 0.84 \times 10^{-4} N$$

حيث أن الشحنة q_1 موجبة فإنها تؤثر على الشحنة q_3 بقوة تنافر مقدارها F_{31} واتجاهها كما هو موضح في الشكل، أما الشحنة q_2 سالبة فإنها تؤثر على الشحنة q_3 بقوة تجاذب مقدارها F_{32}. وبالتالي فإن القوة المحصلة F_3 يمكن حسابها بالجمع الاتجاهي كالتالي:

$$F_3 = F_{31} + F_{32}$$
$$\therefore F_3 = 0.84 \times 10^{-4} - 2.25 \times 10^{-4} = -1.41 \times 10^{-4} N$$

The total force is directed to the left, with magnitude 1.41×10^{-4}N.

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2.3 Problems

2.1) Two protons in a molecule are separated by a distance of 3.8×10^{-10}m. Find the electrostatic force exerted by one proton on the other.

2.2) A 6.7µC charge is located 5m from a -8.4µC charge. Find the electrostatic force exerted by one on the other.

2.3) Two fixed charges, $+1.0 \times 10^{-6}$C and -3.0×10^{6}C, are 10cm apart. (a) Where may a third charge be located so that no force acts on it? (b) Is the equilibrium of this third charge stable or unstable?

2.4) Each of two small spheres is charged positively, the combined charge being 5.0×10^{-5}C. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart, how is the total charge distributed between the spheres?

2.5) A certain charge Q is to be divided into two parts, q and Q-q. What is the relationship of Q to q if the two parts, placed a given distance apart, are to have a maximum Coulomb repulsion?

2.6) A 1.3µC charge is located on the x-axis at x=-0.5m, 3.2µC charge is located on the x-axis at x=1.5m, and 2.5µC charge is located at the origin. Find the net force on the 2.5µC charge.

2.7) A point charge q_1= -4.3µC is located on the y-axis at y=0.18m, a charge q_2=1.6µC is located at the origin, and a charge q_3=3.7µC is located on the x-axis at x= -0.18m. Find the resultant force on the charge q_1.

2.8) Three point charges of 2µC, 7µC, and –4µC are located at the corners of an equilateral triangle as shown in the figure 2.9. Calculate the net electric force on 7µC charge.
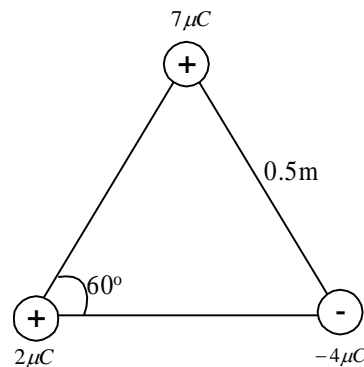
Figure 2.9

2.9) Two free point charges +q and +$4q$ are a distance 1cm apart. A third charge is so placed that the entire system is in equilibrium. Find the location, magnitude and

sign of the third charge. Is the equilibrium stable?

2.10) Four point charges are situated at the corners of a square of sides a as shown in the figure 2.10. Find the resultant force on the positive charge $+q$.
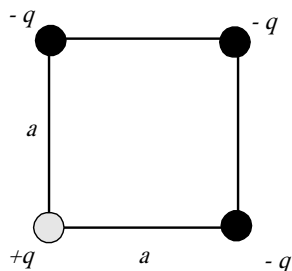
Figure 2.10

2.11) Three point charges lie along the y-axis. A charge $q_1=-9\mu C$ is at y=6.0m, and a charge $q_2=-8\mu C$ is at y=-4.0m. Where must a third positive charge, q_3, be placed such that the resultant force on it is zero?

2.12) A charge q_1 of $+3.4\mu C$ is located at x=+2m, y=+2m and a second charge $q_2=+2.7\mu C$ is located at $x=-4m$, y=-4m. Where must a third charge $(q_3>0)$ be placed such that the resultant force on q_3 will be zero?

2.13) Two similar conducting balls of mass m are hung from silk threads of length l and carry similar charges q as shown in the figure 2.11. Assume that θ is so small that $\tan\theta$ can be replaced by $\sin\theta$. Show that

$$x = \left(\frac{q^2 l}{2\pi\varepsilon_o mg} \right)^{1/3}$$

where x is the separation between the balls (b) If $l=120$cm, $m=10$g and $x=5$cm, what is q?
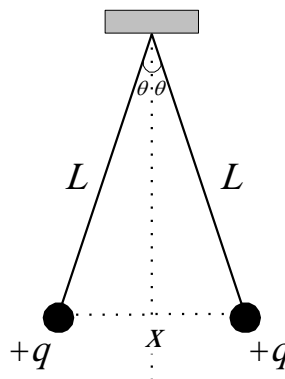
Figure 2.11

Chapter 3

Electric Field

المجال الكهربي

Electric field

Electric field

المجال الكهربي

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في هذا الفصل سنقوم بإدخال مفهوم المجال الكهربي الناشئ عن الشحنة أو الشحنات الكهربية، والمجال الكهربي هو الحيز المحيط بالشحنة الكهربية والذي تظهر فيه تأثير القوى الكهربية. كذلك سندرس تأثير المجال الكهربي على شحنة في حالة أن كون السرعة الابتدائية تساوي صفراً وكذلك في حالة شحنة متحركة.

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3.1 The Electric Field

The gravitational field *g* at a point in space was defined to be equal to the gravitational force *F* acting on a test mass m_o divided by the test mass

$$\vec{g} = \frac{\vec{F}}{m_o}$$ (3.1)

In the same manner, an *electric field* at a point in space can be defined in term of electric force acting on a test charge q_o placed at that point.

3.2 Definition of the electric field

The electric field vector $\vec{E}$ at a point in space is defined as the electric force $\vec{F}$ acting on a positive test charge placed at that point divided by the magnitude of the test charge q_o

$$\vec{E} = \frac{\vec{F}}{q_o}$$ (3.2)

The electric field has a unit of N/C

لاحظ هنا أن المجال الكهربي $\vec{E}$ هو مجال خارجي وليس المجال الناشئ من الشحنة q_o كما هو موضح في الشكل 3.1، وقد يكون هناك مجال كهربي عند أية نقطة في الفراغ بوجود أو عدم وجود الشحنة q_o ولكن وضع الشحنة q_o عند أية نقطة في الفراغ هو وسيلة لحساب المجال الكهربي من خلال القوى الكهربية المؤثرة عليها.
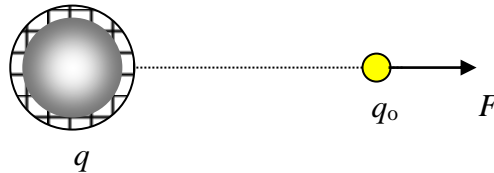
Figure 3.1

3.3 The direction of $\overset{\nu}{E}$

If Q is +ve the electric field at point p in space is radially outward from Q as shown in figure 3.2(a).

If Q is -ve the electric field at point p in space is radially inward toward Q as shown in figure 3.2(b).
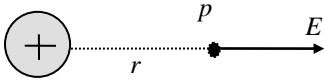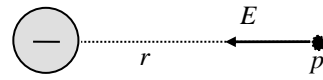
Figure 3.2 (a)   *Figure 3.2 (b)*

يكون اتجاه المجال عند نقطة ما لشحنة موجبة في اتجاه الخروج من النقطة كما في الشكل 3.2(a)، ويكون اتجاه المجال عند نقطة ما لشحنة سالبة في اتجاه الدخول من النقطة إلى الشحنة كما في الشكل 3.2(b).

3.4 Calculating $\overset{\nu}{E}$ due to a charged particle

Consider Fig. 3.2(a) above, the magnitude of force acting on q_o is given by Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Qq_o}{r^2}$$

$$E = \frac{F}{q_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad\qquad (3.3)$$

3.5 To find *E* for a group of point charge

To find the magnitude and direction of the electric field due to several charged particles as shown in figure 3.3 use the following steps

(1) نرقم الشحنات المراد إيجاد المجال الكهربي لها.

(2) نحدد اتجاه المجال الكهربي لكل شحنة على حده عند النقطة المراد إيجاد محصلة المجال عندها ولتكن النقطة p، يكون اتجاه المجال خارجاً من النقطة p إذا كانت الشحنة موجبة ويكون اتجاه المجال داخلاً إلى النقطة إذا كانت الشحنة سالبة كما هو الحال في الشحنة رقم (2).

(3) يكون المجال الكهربي الكلي هو الجمع الاتجاهي لمتجهات المجال
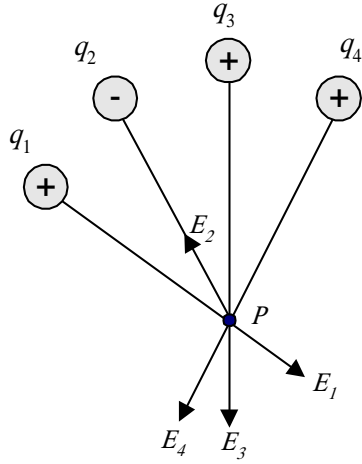
Figure 3.3

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \text{.........} \qquad (3.4)$$

(4) إذا كان لا يجمع متجهات المجال خط عمل واحد نحلل كل متجه إلى مركبتين في اتجاه محوري x و y

(5) نجمع مركبات المحور x على حده ومركبات المحور y.

$$E_x = E_{1x} + E_{2x} + E_{3x} + E_{4x}$$
$$E_y = E_{1y} + E_{2y} + E_{3y} + E_{4y}$$

(6) تكون قيمة المجال الكهربي عند النقطة p هي $E = \sqrt{E_x^2 + E_y^2}$

(7) يكون اتجاه المجال هو $\theta = \tan^{-1} \dfrac{E_y}{E_x}$

Example 3.1

Find the electric field at point *p* in figure 3.4 due to the charges shown.
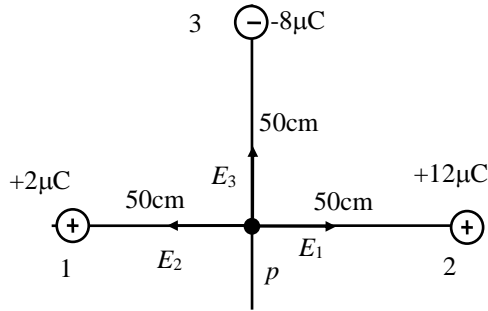
Figure 3.4

Solution

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$E_x = E_1 - E_2 = -36 \times 10^4 \text{N/C}$$

$$E_y = E_3 = 28.8 \times 10^4 \text{N/C}$$

$$E_p = \sqrt{(36 \times 10^4)^2 + (28.8 \times 10^4)^2} = 46.1 \text{N/C}$$

$$\theta = 141°$$
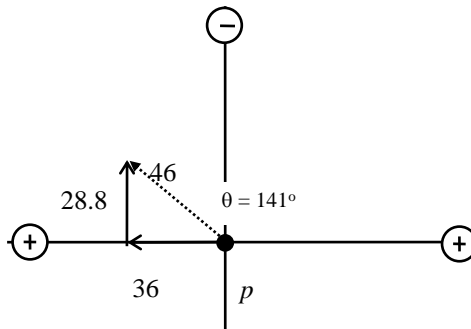
Figure 3.5 Shows the resultant electric field

Example 3.2

Find the electric field due to electric dipole along x-axis at point *p*, which is a distance *r* from the origin, then assume *r>>a*

The electric dipole **is positive charge and negative charge of equal magnitude placed a distance 2*a* apart as shown in figure 3.6**
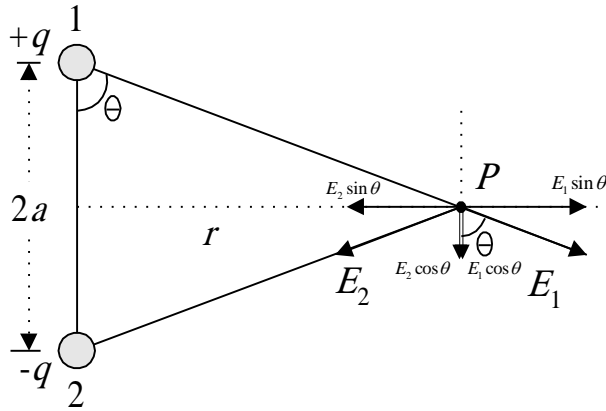
Figure 3.6

Solution

المجال الكلي عند النقطة p هو محصلة المجالين E_1 الناتج عن الشحنة q_1 والمجال E_2 الناتج عن الشحنة q_2 أي أن

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2$$

وحيث أن النقطة p تبعد عن الشحنتين بنفس المقدار، والشحنتان متساويتان إذاً المجالان متساويان وقيمة المجال تعطى بالعلاقة

$$E_1 = \frac{1}{4\pi\varepsilon_o} \ \frac{q_1}{a^2 + r^2} = E_2$$

لاحظ هنا أن المسافة الفاصلة هي ما بين الشحنة والنقطة المراد إيجاد المجال عندها.

نحلل متجه المجال إلى مركبتين كما في الشكل أعلاه

$$E_x = E_1 \sin\theta - E_2 \sin\theta$$

$E_y = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$

$E_p = 2E_1 \cos\theta$

$$E_p = \frac{1}{4\pi\varepsilon_o} \; \frac{q}{a^2 + r^2} \; \cos\theta$$

from the Figure

$$\cos\theta = \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{1}{4\pi\varepsilon_o} \; \frac{q}{a^2 + r^2} \; \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{2aq}{4\pi\varepsilon_o (r^2 + a^2)^{3/2}} \qquad (3.5)$$

The direction of the electric field in the -ve y-axis.

The quantity $2aq$ is called the *electric dipole momentum* (*P*) and has a direction from the -ve charge to the +ve charge

(b) when $r \gg a$

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o r^3} \qquad (3.6)$$

يتضح مما سبق أن المجال الكهربي الناشئ عن electric dipole عند نقطة واقعة على العمود المنصف بين الشحنتين يكون اتجاهه في عكس اتجاه electric dipole momentum وبالنسبة للنقطة البعيدة عن electric dipole فإن المجال يتناسب عكسيا مع مكعب المسافة، وهذا يعنى أن تناقص المجال مع المسافة يكون أكبر منه في حالة شحنة واحدة فقط.

3.6 Electric field lines

The electric lines are a convenient way to visualize the electric filed patterns. The relation between the electric field lines and the electric field vector is this:

(1) The tangent to a line of force at any point gives the direction of $\vec{E}$ at that point.

(2) The lines of force are drawn so that the number of lines per unit cross-sectional area is proportional to the magnitude of $\vec{E}$.

Some examples of electric line of force

Electric field lines due to +ve charge

Electric field lines due to -ve charge

Electric field lines due to +ve line charge

Electric field lines due two surface charge
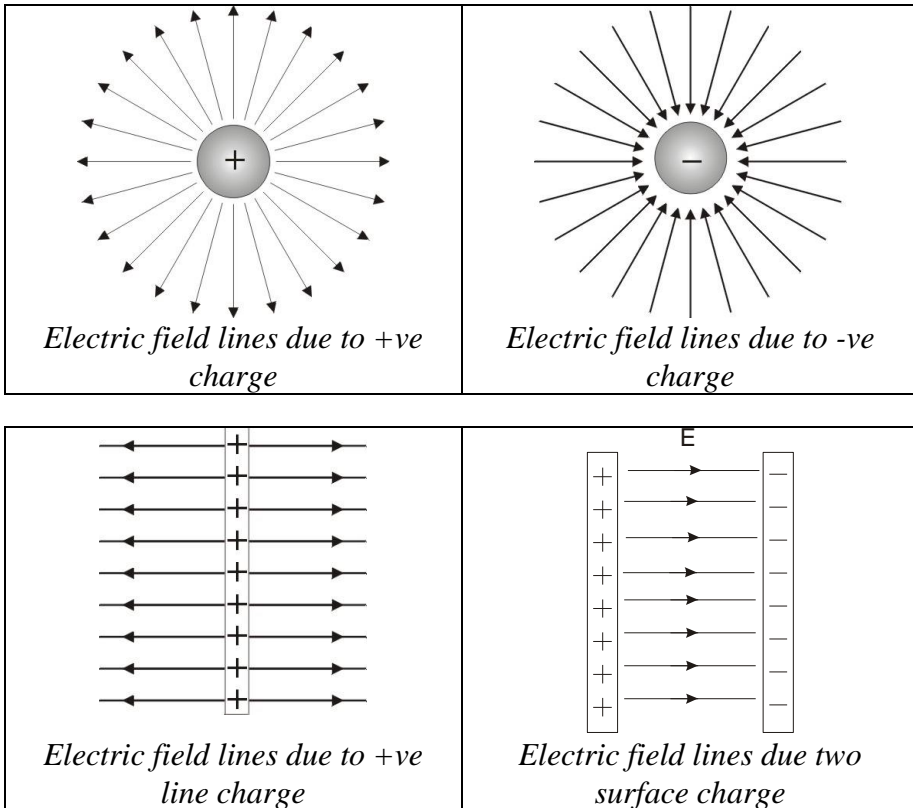
Figure 3.7 shows some examples of electric line of force

Notice that the rule of drawing the line of force:-

(1) The lines must begin on positive charges and terminates on negative charges.

(2) The number of lines drawn is proportional to the magnitude of the charge.

(3) No two electric field lines can cross.

3.7 Motion of charge particles in a uniform electric field

If we are given a field $\overset{\triangleright}{E}$, what forces will act on a charge placed in it?

We start with special case of a point charge in uniform electric field $\overset{\triangleright}{E}$. The electric field will exert a force on a charged particle is given by

$$F = qE$$

The force will produce acceleration

$$a = F/m$$

where m is the mass of the particle. Then we can write

$$F = qE = ma$$

The acceleration of the particle is therefore given by

$$a = qE/m \qquad\qquad (3.7)$$

If the charge is positive, the acceleration will be in the direction of the electric field. If the charge is negative, the acceleration will be in the direction opposite the electric field.

One of the practical applications of this subject is a device called the (*Oscilloscope*) See appendix A (*Cathode Ray Oscilloscope*) for further information.

3.8 Solution of some selected problems

في هذا الجزء سنعرض حلولاً لبعض المسائل التي تغطى موضوع
المجال الكهربي

3.8 Solution of some selected problems

Example 3.3

A positive point charge *q* of mass m is released from rest in a uniform electric field $\overset{\nu}{E}$ directed along the x-axis as shown in figure 3.8, describe its motion.
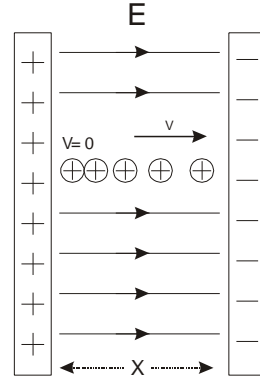
Solution

Figure 3.8

The acceleration is given by

$$a = qE/m$$

Since the motion of the particle in one dimension, then we can apply the equations of kinematics in one dimension

$$x\text{-}x_o = v_0 t + \tfrac{1}{2}\, at^2 \qquad v = v_0 + at \qquad v^2 = v_o^2 + 2a(x\text{-}x_o)$$

Taking $x_o = 0$ and $v_0 = 0$

$$x = \tfrac{1}{2}\, at^2 = (qE/2m)\, t^2$$

$$v = at = (qE/m)\, t$$

$$v^2 = 2ax = (2qE/m)x \qquad\qquad (3.7)$$

Example 3.4

In the above example suppose that a negative charged particle is projected horizontally into the uniform field with an initial velocity v_o as shown in figure 3.9.
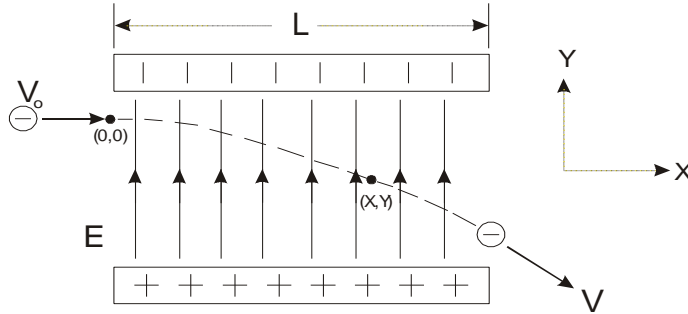
Figure 3.9

Solution

Since the direction of electric field $\overset{\nu}{E}$ in the y direction, and the charge is negative, then the acceleration of charge is in the direction of -y.

$$a = -qE/m$$

The motion of the charge is in two dimension with constant acceleration, with $v_{xo} = v_o$ & $v_{yo} = 0$

The components of velocity after time t are given by

$v_x = v_o$ =constant

$v_y = at = -(qE/m)\, t$

The coordinate of the charge after time t are given by

$x = v_o t$

$y = \tfrac{1}{2}\, at^2 = -1/2\, (qE/m)\, t^2$

Eliminating t we get

$$y = \frac{qE}{2mv_0^2} x^2 \qquad\qquad (3.8)$$

we see that y is proportional to x^2. Hence, the trajectory is parabola.

Example 3.5

Find the electric field due to electric dipole shown in figure 3.10 along x-axis at point _p_ which is a distance _r_ from the origin. then assume _r>>a_

Solution

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2$$

$$E_1 = K \frac{q}{(x+a)^2}$$

$$E_2 = K \frac{q}{(x-a)^2}$$

$$E_p = K \left[\frac{q}{(x-a)^2} - \frac{q}{(x+a)^2} \right]$$

$$E_p = Kq \frac{4ax}{(x^2 - a^2)^2}$$

When $x \gg a$ then

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o x^3} \tag{3.9}$$
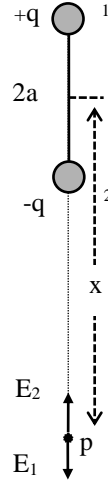
Figure 3.10

لاحظ الإجابة النهائية عندما تكون x أكبر كثيرا من المسافة $2a$ حيث يتناسب المجال عكسيا مع مكعب المسافة.

Example 3.6

What is the electric field in the lower left corner of the square as shown in figure 3.11? Assume that $q = 1 \times 10^{-7}$C and $a = 5$cm.

Solution

First we assign number to the charges (1, 2, 3, 4) and then determine the direction of the electric field at the point p due to the charges.

$$E_1 = \frac{1}{4\pi\varepsilon_o} \frac{q}{a^2}$$

$$E_2 = \frac{1}{4\pi\varepsilon_o} \frac{q}{2a^2}$$

$$E_3 = \frac{1}{4\pi\varepsilon_o} \frac{2q}{a^2}$$

Figure 3.11

Evaluate the value of E_1, E_2, & E_3

$$E_1 = 3.6 \times 10^5 \text{ N/C},$$

$$E_2 = 1.8 \times 10^5 \text{ N/C},$$

$$E_3 = 7.2 \times 10^5 \text{ N/C}$$

Since the resultant electric field is the vector additions of all the fields *i.e.*

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

We find the vector E_2 need analysis to two components

$$E_{2x} = E_2 \cos 45$$

$$E_{2y} = E_2 \sin 45$$

$$E_x = E_3 - E_2\cos 45 = 7.2 \times 10^5 - 1.8 \times 10^5 \cos 45 = 6 \times 10^5 \text{ N/C}$$
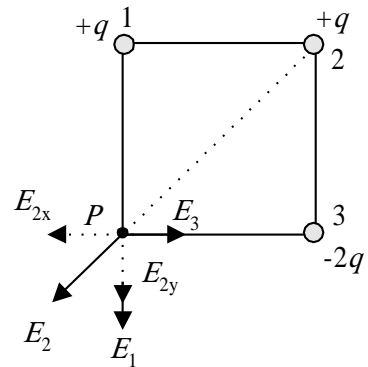
$$E_y = -E_1 - E_2\sin45 = -3.6\times10^5 - 1.8 \times 10^5 \sin45 = -4.8 \times 10^5 \text{ N/C}$$

$$E = \sqrt{E_x^2 + E_y^2} \qquad = 7.7 \times 10^5 \text{ N/C}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} \qquad = -38.6^{\circ}$$

Example 3.7

In figure 3.12 shown, locate the point at which the electric field is zero? Assume $a = 50$cm

Solution
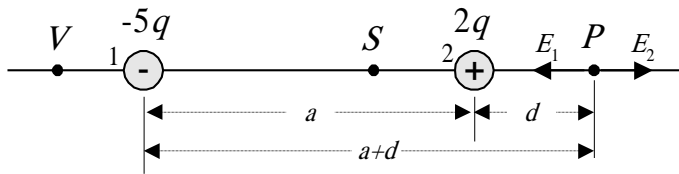
Figure 3.12

To locate the points at which the electric field is zero ($E=0$), we shall try all the possibilities, assume the points S, V, P and find the direction of E_1 and E_2 at each point due to the charges q_1 and q_2.

The resultant electric field is zero only when E_1 and E_2 are equal in magnitude and opposite in direction.

At the point S E_1 in the same direction of E_2 therefore E cannot be zero in between the two charges.

At the point V the direction of E_1 is opposite to the direction of E_2, but the magnitude could not be equal (can you find the reason?)

At the point P the direction of E_1 and E_2 are in opposite to each other and the magnitude can be equal

$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_o} \frac{2q}{(0.5+d)^2} = \frac{1}{4\pi\varepsilon_o} \frac{5q}{(d)^2}$$

$$d = 30\text{cm}$$

لاحظ هنا أنه في حالة الشحنتين المتشابهتين فإن النقطة التي ينعدم عندها المجال تكون بين الشحنتين، أما إذا كانت الشحنتان مختلفتين في الإشارة فإنها تكون خارج إحدى الشحنتين وعلى الخط الواصل بينهما وبالقرب من الشحنة الأصغر.

Example 3.8

A charged cord ball of mass 1g is suspended on a light string in the presence of a uniform electric field as in figure 3.13. When $E=(3i+5j) \times 10^5$N/C, the ball is in equilibrium at $\theta=37^\circ$. Find (a) the charge on the ball and (b) the tension in the string.
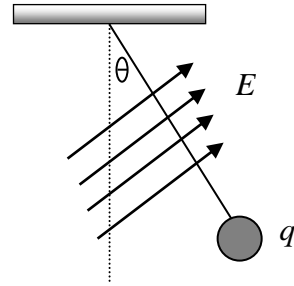
Solution

Figure 3.13

حيث أن الكرة مشحونة بشحنة موجبة فإن القوة الكهربية المؤثرة على الكرة المشحونة في اتجاه المجال الكهربي.

كما أن الكرة المشحونة في حالة اتزان فإن محصلة القوى المؤثرة على الكرة ستكون صفر. بتطبيق قانون نيوتن الثاني $\sum F = ma$ على مركبات x و y.
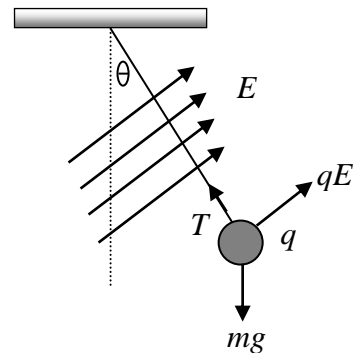
$$E_x = 3 \times 10^5 \text{N/C} \qquad E_y = 5j \times 10^5 \text{N/C}$$

$$\sum F = T + qE + F_g = 0$$

$$\sum F_x = qE_x - T \sin 37 = 0 \qquad\qquad (1)$$

$$\sum F_y = qE_y + T \cos 37 - mg = 0 \qquad\qquad (2)$$

Substitute T from equation (1) into equation (2)

$$q = \frac{mg}{\left(E_y + \dfrac{E_x}{\tan 37}\right)} = \frac{(1 \times 10^{-3})(9.8)}{\left(5 + \dfrac{3}{\tan 37}\right) \times 10^5} = 1.09 \times 10^{-8} \ C$$

To find the tension we substitute for q in equation (1)

$$T = \frac{qEx}{\sin 37} = 5.44 \times 10^{-3} N$$

3.9 The electric dipole in electric field

If an electric dipole placed in an external electric field E as shown in figure 3.14, then a torque will act to align it with the direction of the field.
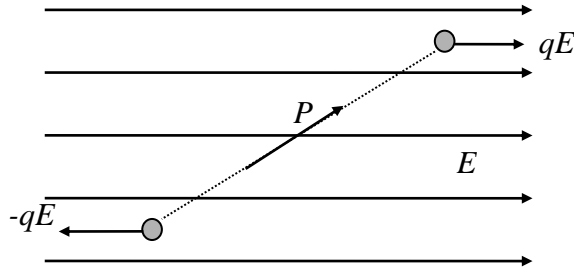
Figure 3.14

$$\vec{\tau} = \vec{P} \times \vec{E} \qquad\qquad (3.10)$$

$$\tau = P\,E\,\sin\theta \qquad\qquad (3.11)$$

where P is the electric dipole momentum, θ the angle between P and E

يكون ثنائي القطب في حالة اتزان equilibrium عندما يكون الازدواج مساويا للصفر وهذا يتحقق
عندما تكون (θ π , $0 =$)
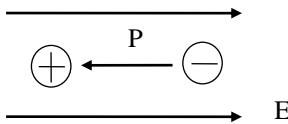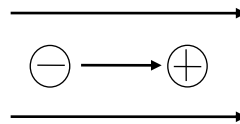
Figure 3.15(ii)   *Figure 3.15(i)*

في الوضع الموضح في الشكل 3.15(i) عندما θ=0 يقال إن الـ dipole في وضع اتزان مستقر
stable equilibrium لأنه إذا أزيح بزاوية صغيرة فانه سيرجع إلى الوضع θ=0، بينما في
الوضع الموضح في الشكل 3.15(ii) يقال إن الـ dipole في وضع اتزان غير مستقر unstable
equilibrium لأن إزاحة صغيرة له سوف تعمل على أن يدور الـ dipole ويرجع إلى الوضع
θ=0 وليس θ π=.

ƎƎƎƎƎƎƎƎƎƎƎƎƎƎƎƎƎƎ

3.10 Problems

3.1) The electric force on a point charge of 4.0µC at some point is 6.9×10^{-4}N in the positive *x* direction. What is the value of the electric field at that point?

3.2) What are the magnitude and direction of the electric field that will balance the weight of (a) an electron and (b) a proton? (Use the data in Table l.)

3.3) A point charge of -5.2µC is located at the origin. Find the electric field (a) on the x-axis at x=3 m, (b) on the y-axis at y= -4m, (c) at the point with coordinates x=2m, y=2m.

3.4) What is the magnitude of a point charge chosen so that the electric field 50cm away has the magnitude 2.0N/C?

3.5) Two point charges of magnitude $+2.0 \times 10^{-7}$C and $+8.5 \times 10^{-11}$C are 12cm apart. (a) What electric field does each produce at the site of the other? (b) What force acts on each?

3.6) An electron and a proton are each placed at rest in an external electric field of 520N/C. Calculate the speed of each particle after 48nanoseconds.

3.7) The electrons in a particle beam each have a kinetic energy of 1.6×10^{-17}J. What are the magnitude and direction of the electric field that will stop these electrons in a distance of 10cm?

3.8) A particle having a charge of -2.0×10^{-9}C is acted on by a downward electric force of 3.0×10^{-6}N in a uniform electric field. (a) What is the strength of the electric field? (b) What is the magnitude and direction of the electric force exerted on a proton placed in this field? (c) What is the gravitational force on the proton? (d) What is the ratio of the electric to the gravitational forces in this case?

3.9) Find the total electric field along the line of the two charges shown in figure 3.16 at the point midway between them.

$-4.7\,\mu C$ $+9\,\mu C$
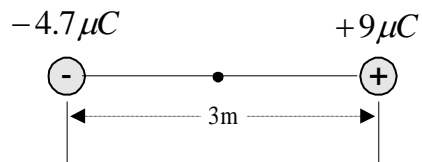
Figure 3.16

3.10) What is the magnitude and direction of an electric field that will balance the weight of (a) an electron and (b) a proton?

3.11) Three charges are arranged in an equilateral triangle as shown in figure 3.17. What is the direction of the force on $+q$?
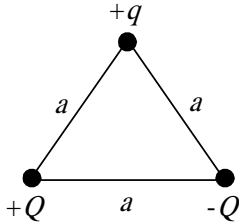
Figure 3.17

3.12) In figure 3.18 locate the point at which the electric field is zero and also the point at which the electric potential is zero. Take $q=1\mu C$ and $a=50cm$.
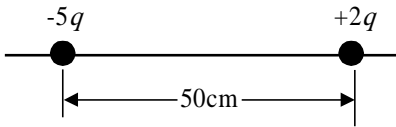
Figure 3.18

3.13) What is E in magnitude and direction at the center of the square shown in figure 3.19? Assume that $q=1\mu C$ and $a=5cm$.
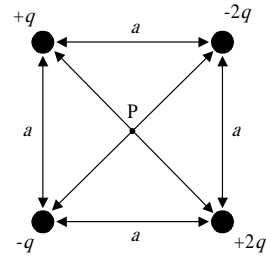
Figure 3.19

3.14) Two point charges are a distance d apart (Figure 3.20). Plot $E(x)$, assuming $x=0$ at the left-hand charge. Consider both positive and negative values of x. Plot E as positive if E points to the right and negative if E points to the left. Assume $q_1=+1.0\times10^{-6}C$, $q_2=+3.0\times10^{-6}C$, and $d=10cm$.
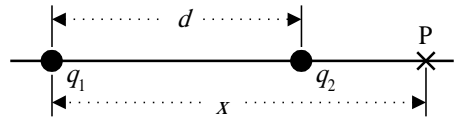
Figure 3.20

3.15) Calculate E (direction and magnitude) at point P in Figure 3.21.
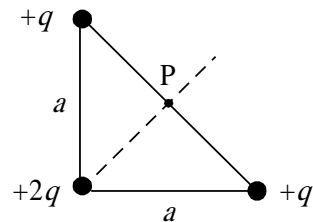
Figure 3.21

3.16) Charges +*q* and -2*q* are fixed a distance *d* apart as shown in figure 3.22. Find the electric field at points A, B, and C.
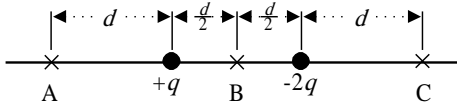
Figure 3.22

3.17) A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0cm away, in a time 1.5×10^{-8}s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field *E*?

Chapter 4

Electric Flux

التدفق الكهربي

Electric Flux

Electric Flux
التدفق الكهربي

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درسنا سابقا كيفية حساب المجال لتوزيع معين من الشحنات باستخدام قانون كولوم. وهنا سنقدم طريقة أخرى لحساب المجال الكهربي باستخدام "قانون جاوس" الذي يسهل حساب المجال الكهربي لتوزيع متصل من الشحنة على شكل توزيع طولي أو سطحي أو حجمي. يعتمد قانون جاوس أساساً على مفهوم التدفق الكهربي الناتج من المجال الكهربي أو الشحنة الكهربائية، ولهذا سنقوم أولاً بحساب التدفق الكهربي الناتج عن المجال الكهربي، وثانياً سنقوم بحساب التدفق الكهربي الناتج عن شحنة كهربية، ومن ثم سنقوم بإيجاد قانون جاوس واستخدامه في بعض التطبيقات الهامة في مجال الكهربية الساكنة.

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4.1 The Electric Flux due to an Electric Field

We have already shown how electric field can be described by lines of force. A line of force is an imaginary line drawn in such a way that its direction at any point is the same as the direction of the field at that point. Field lines never intersect, since only one line can pass through a single point.

The Electric flux (Φ) is a measure of the number of electric field lines penetrating some surface of area *A*.

Case one:

The electric flux for a plan surface perpendicular to a uniform electric field (figure 4.1)

To calculate the electric flux we recall that the number of lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface of area *A* is proportional to the product *EA*. The product of the electric filed E and the surface area *A* perpendicular to the field is called the electric flux Φ.
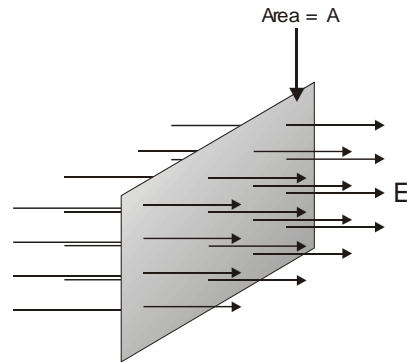
Figure 4.1

$$\Phi = \vec{E}.\vec{A} \qquad (4.1)$$

The electric flux Φ has a unit of N.m^2/C.

Case Two

The electric flux for a plan surface make an angle θ to a uniform electric field (figure 4.2)

Note that the number of lines that cross-area is equal to the number that cross the projected area $A`$, which is perpendicular to the field. From the figure we see that the two area are related by $A`=Acos\theta$. The flux is given by:

$$\Phi = \vec{E}.\vec{A}' = E\,A\,cos\theta$$

$$\Phi = \vec{E}.\vec{A}$$
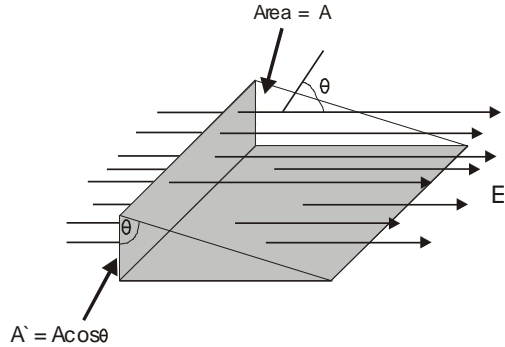
Figure 4.2

Where θ is the angle between the electric field E and the normal to the surface $\vec{A}$.

إذاً يكون الفيض ذا قيمة عظمى عندما يكون السطح عمودياً على المجال أي $\theta = 0$ ويكون ذا قيمة صغرى عندما يكون السطح موازياً للمجال أي عندما $\theta = 90$. لاحظ هنا أن المتجه $\vec{A}$ هو متجه المساحة وهو عمودي دائما على المساحة وطوله يعبر عن مقدار المساحة.

Case Three

In general the electric field is nonuniform over the surface (figure 4.3)

The flux is calculated by integrating the normal component of the field over the surface in question.

$$\Phi = \oint \vec{E}.\vec{A} \qquad (4.2)$$

The **net flux** through the surface is proportional to the **net number of lines** penetrating the surface
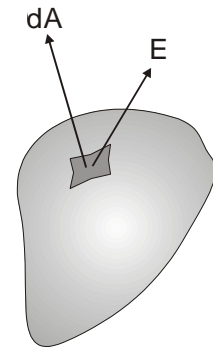
Figure 4.3

والمقصود بـ net number of lines أي عدد الخطوط الخارجة من السطح (إذا كانت الشحنة موجبة) - عدد الخطوط الداخلة إلى السطح (إذا كانت الشحنة سالبة).

Example 4.1

What is electric flux Φ for closed cylinder of radius *R* immersed in a uniform electric field as shown in figure 4.4?
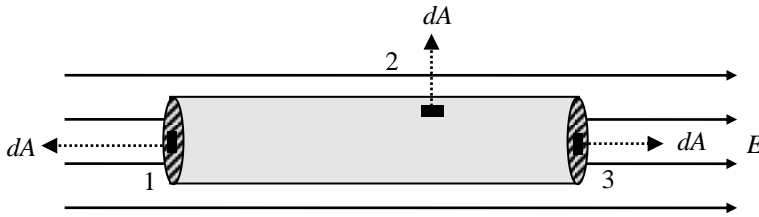
Figure 4.4

Solution

نطبق قانون جاوس على الأسطح الثلاثة الموضحة في الشكل أعلاه

$$\Phi = \oint \vec{E}.d\vec{A} = \oint_{(1)} \vec{E}.d\vec{A} + \oint_{(2)} \vec{E}.d\vec{A} + \oint_{(3)} \vec{E}.d\vec{A}$$

$$= \oint_{(1)} E\cos 180\, dA + \oint_{(2)} E\cos 90\, dA + \oint_{(3)} E\cos 0\, dA$$

Since *E* is constant then

$$\Phi = -EA + 0 + EA = \text{zero}$$

Exercise

Calculate the total flux for a cube immersed in uniform electric field $\vec{E}$.

4.2 The Electric Flux due to a point charge

To calculate the electric flux due to a point charge we consider an imaginary closed spherical surface with the point charge in the center figure 4.5, this surface is called *gaussian surface.* Then the flux is given by

Figure 4.5

$$\Phi = \oint \vec{E}.d\vec{A} = E \oint dA\cos\theta \quad (\theta = 0)$$

$$\Phi = \frac{q}{4\pi\varepsilon_o r^2}\int dA = \frac{q}{4\pi\varepsilon_o r^2} 4\pi r^2$$

$$\Phi = \frac{q}{\varepsilon_o} \quad\quad\quad (4.3)$$
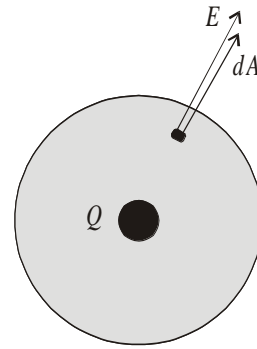
Note that the net flux through a spherical gaussian surface is proportional to the charge *q* inside the surface.

4.3 Gaussian surface

Consider several closed surfaces as shown in figure 4.6 surrounding a charge *Q* as in the figure below. The flux that passes through surfaces S_1, S_2 and S_3 all has a value q/ε_o. Therefore we conclude that the net flux through any closed surface is independent of the shape of the surface.
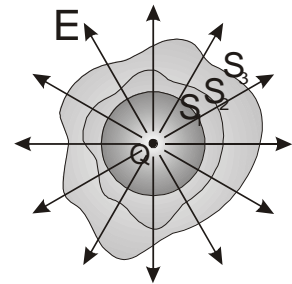
Figure 4.6

Consider a point charge located outside a closed surface as shown in figure 4.7. We can see that the number of electric field lines entering the surface equal the number leaving the surface. Therefore the net electric flux in this case is zero, because the surface surrounds no electric charge.
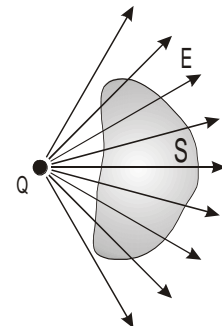
Figure 4.7

Example 4.2

In figure 4.8 two equal and opposite charges of 2*Q* and -2*Q* what is the flux Φ for the surfaces S₁, S₂, S₃ and S₄.
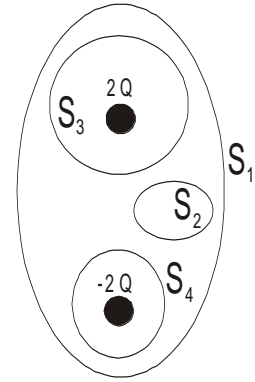
Solution

For S_1 the flux Φ = zero

For S_2 the flux Φ = zero

For S_3 the flux $\Phi = +2Q/\varepsilon_o$

For S_4 the flux $\Phi = -2Q/\varepsilon_o$

Figure 4.8

4.4 Gauss's Law

Gauss law is a very powerful theorem, which relates any charge distribution to the resulting electric field at any point in the vicinity of the charge. As we saw the electric field lines means that each charge q must have q/ε_o flux lines coming from it. This is the basis for an important equation referred to as ***Gauss's law***. Note the following facts:

1. If there are charges q_1, q_2, q_3, ...q_n inside a closed (gaussian) surface, the total number of flux lines coming from these charges will be
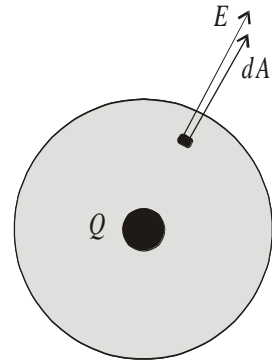
Figure 4.9

$$(q_1 + q_2 + q_3 + ... + q_n)/\varepsilon_o \qquad (4.4)$$

2. The number of flux lines coming out of a closed surface is the integral of $\vec{E}.d\vec{A}$ over the surface, $\oint \vec{E}.d\vec{A}$

We can equate both equations to get Gauss law which state that the net electric flux through a closed gaussian surface is equal to the net charge inside the surface divided by ε_o

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \qquad \textbf{\textcolor{purple}{Gauss's law}} \qquad (4.5)$$

where q_{in} is the total charge inside the gaussian surface.

> **Gauss's law states that the net electric flux through any closed gaussian surface is equal to the net electric charge inside the surface divided by the permittivity.**

4.5 Gauss's law and Coulomb's law

We can deduce Coulomb's law from Gauss's law by assuming a point charge q, to find the electric field at point or points a distance r from the charge we imagine a spherical gaussian surface of radius r and the charge q at its center as shown in figure 4.10.
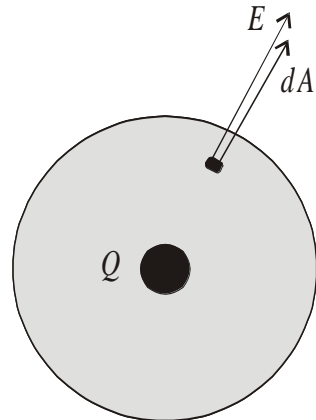
$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$\oint E\cos 0 \, dA = \frac{q_{in}}{\varepsilon_o} \qquad \text{Because} \quad E \quad \text{is}$$

constant for all points on the sphere, it can be factored from the inside of the integral sign, then

$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad EA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad\qquad (4.6)$$

Now put a second point charge q_o at the point, which E is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

4.6 Conductors in electrostatic equilibrium

A good electrical conductor, such as copper, contains charges (electrons) that are free to move within the material. When there is no net motion of charges within the conductor, the conductor is in electrostatic equilibrium.

Conductor in electrostatic equilibrium has the following properties:

1. Any excess charge on an isolated conductor must reside entirely on its surface. *(Explain why?) The answer is when an **excess charge** is placed on a conductor, it will set-up electric field inside the conductor. These fields act on the charge carriers of the conductor (electrons) and cause them to move i.e. current flow inside the conductor. These currents redistribute the **excess charge** on the surface in such away that the internal electric fields reduced to become zero and the currents stop, and the electrostatic conditions restore.*

2. The electric field is zero everywhere inside the conductor. *(Explain why?) Same reason as above*

In figure 4.11 it shows a conducting slab in an external electric field E. The charges induced on the surface of the slab produce an electric field, which opposes the external field, giving a resultant field of zero in the conductor.
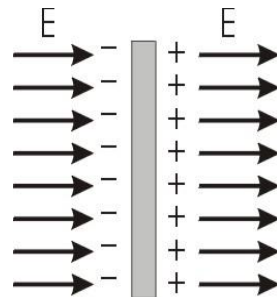
Figure 4.11

Steps which should be followed in solving problems

1. The gaussian surface should be chosen to have the same symmetry as the charge distribution.

2. The dimensions of the surface must be such that the surface includes the point where the electric field is to be calculated.

3. From the symmetry of the charge distribution, determine the direction of the electric field and the surface area vector dA, over the region of the gaussian surface.

4. Write $E.dA$ as $E \ dA \ cos\theta$ and divide the surface into separate regions if necessary.

5. The total charge enclosed by the gaussian surface is $dq = \int dq$, which is represented in terms of the charge density ($dq = \lambda dx$ for line of charge, $dq = \sigma dA$ for a surface of charge, $dq = \rho dv$ for a volume of charge).

4.7 **Applications of Gauss's law**

كما ذكرنا سابقا فإن قانون جاوس يطبق على توزيع متصل من الشحنة، وهذا التوزيع إما أن يكون توزيعاً طولياً أو توزيعاً سطحياً أو توزيعاً حجمياً.   يوجد على كل حالة مثال محلول في الكتاب سنكتفي هنا بذكر بعض النقاط الهامة.

على سبيل المثال إذا أردنا حساب المجال الكهربي عند نقطة تبعد مسافة عن سلك مشحون كما في الشكل 4.12، هنا في هذه الحالة الشحنة موزعة بطريقة متصلة، وغالبا نفترض أن توزيع الشحنة منتظم ويعطى بكثافة التوزيع λ(C/m)، ولحل مثل هذه المشكلة نقسم السلك إلى عناصر صغيرة طول كلا منها dx ونحسب المجال dE الناشئ عند نقطة (p)
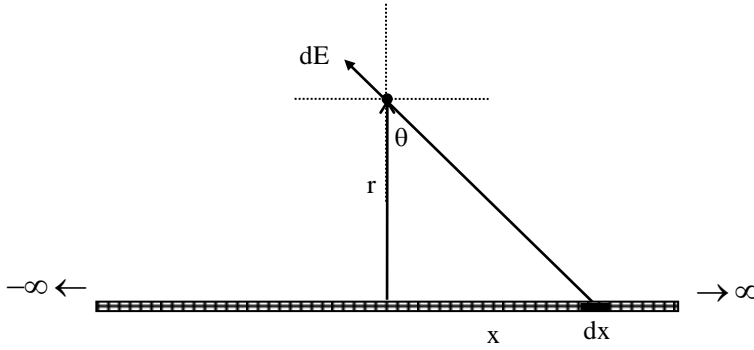
Figure 4.12

$$dE = K \frac{dq}{r^2 + x^2} = K \frac{\lambda dx}{r^2 + x^2}$$

ومن التماثل نجد أن المركبات الأفقية تتلاشى والمحصلة تكون في اتجاه المركبة الرأسية التي في اتجاه y

$$dE_y = dE \cos\theta \qquad E_y = \int dE_y = \int\limits_{-\infty}^{+\infty} \cos\theta \, dE$$

$$E = 2 \int_0^{+\infty} \cos\theta \, dE \qquad \frac{2\lambda}{4\pi\varepsilon_o} \int_0^{+\infty} \cos\theta \frac{dx}{r^2 + x^2}$$

من الشكل الهندسي يمكن التعويض عن المتغير x والمتغير dx كما يلي:

$$x = y\tan\theta \qquad \Rightarrow \qquad dx = y\sec^2\theta \, d\theta$$

$$E = \frac{\lambda}{2\pi\varepsilon_o} \int_0^{\pi/2} \cos\theta \, d\theta \quad \longleftarrow$$

انتبه إلى حدود التكامل

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

لاشك أنك لاحظت صعوبة الحل باستخدام قانون كولوم في حالة التوزيع المتصل للشحنة، لذلك سندرس قانون جاوس الذي يسهل الحل كثيراً في مثل هذه الحالات والتي بها درجة عالية من التماثل.

Gauss's law can be used to calculate the electric field if the symmetry of the charge distribution is high. Here we concentrate in three different ways of charge distribution

	1	2	3
Charge distribution	Linear	Surface	Volume
Charge density	λ	σ	ρ
Unit	C/m	C/m^2	C/m^3

A linear charge distribution

In figure 4.13 calculate the electric field at a distance *r* from a uniform positive line charge of infinite length whose charge per unit length is λ=constant.
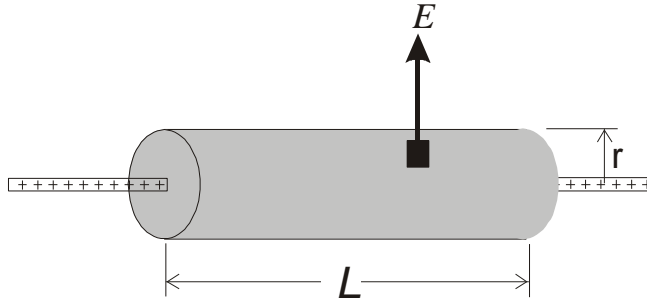
Figure 4.13

The electric field *E* is perpendicular to the line of charge and directed outward. Therefore for symmetry we select a cylindrical gaussian surface of radius r and length *L*.

The electric field is constant in magnitude and perpendicular to the surface.

The flux through the end of the gaussian cylinder is zero since *E* is parallel to the surface.

The total charge inside the gaussian surface is λ*L*.
Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint dA = \frac{\lambda L}{\varepsilon_o}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o}$$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r} \tag{4.7}$$

نلاحظ هنا أنه باستخدام قانون جاوس سنحصل على نفس النتيجة التي توصلنا لها بتطبيق قانون كولوم وبطريقة أسهل.

A surface charge distribution

In figure 4.4 calculate the electric field due to non-conducting, infinite plane with uniform charge per unit area σ.
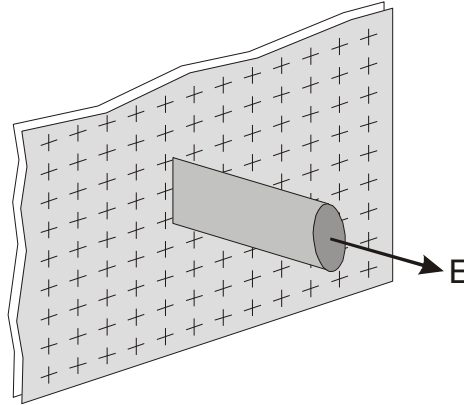
Figure 4.14

The electric field E is constant in magnitude and perpendicular to the plane charge and directed outward for both surfaces of the plane. Therefore for symmetry we select a cylindrical gaussian surface with its axis is perpendicular to the plane, each end of the gaussian surface has area A and are equidistance from the plane.

The flux through the end of the gaussian cylinder is EA since E is perpendicular to the surface.

The total electric flux from both ends of the gaussian surface will be $2EA$. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$\therefore E = \frac{\sigma}{2\varepsilon_o} \qquad\qquad (4.8)$$

An insulated conductor.

ذكرنا سابقا أن الشحنة توزع على سطح الموصل فقط، وبالتالي فإن قيمة المجال داخل مادة الموصل تساوى صفراً، وقيمة المجال خارج الموصل تساوى

$$E = \frac{\sigma}{\varepsilon_o} \qquad\qquad (4.9)$$

لاحظ هنا أن المجال في حالة الموصل يساوى ضعف قيمة المجال في حالة السطح اللانهائي المشحون، وذلك لأن خطوط المجال تخرج من السطحين في حالة السطح غير الموصل، بينما كل خطوط المجال تخرج من السطح الخارجي في حالة الموصل.
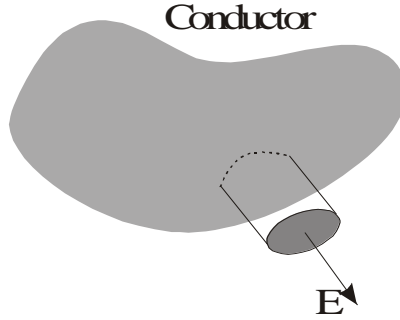
Figure 4.15

في الشكل الموضح أعلاه 4.15 نلاحظ أن الوجه الأمامي لسطح جاوس له فيض حيث أن الشحنة تستقر على السطح الخارجي، بينما يكون الفيض مساوياً للصفر للسطح الخلفي الذي يخترق الموصل وذلك لأن الشحنة داخل الموصل تساوي صفراً.

A volume charge distribution

In figure 4.16 shows an insulating sphere of radius *a* has a uniform charge density ρ and a total charge *Q*.

      1) Find the electric field at point outside the sphere (*r>a*)

      2) Find the electric field at point inside the sphere (*r<a*)

For *r>a*
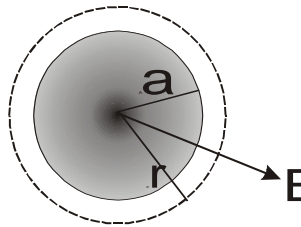
Figure 4.16

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where *r>a*. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.16. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint A = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_o r^2} \quad \text{(for r>a)} \quad\quad\quad (4.10)$$

Note that the result is identical to appoint charge.

<u>For *r<a*</u>
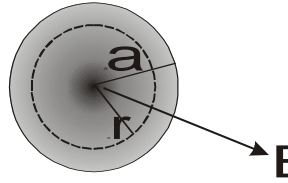
Figure 4.17

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where *r<a*. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.17. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

It is important at this point to see that the charge inside the gaussian surface of volume *V`* is less than the total charge *Q*. To calculate the charge $q_{in}$, we use $q_{in}=\rho V$`, where V`$=4/3\pi r^3$. Therefore,

$$q_{in} = \rho V` = \rho(4/3\pi r^3) \tag{4.11}$$

$$E\oint A = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = \frac{\rho\frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2} = \frac{\rho}{3\varepsilon_o}r \tag{4.12}$$

since $\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

$$\therefore E = \frac{Qr}{4\pi\varepsilon_o a^3} \quad \text{(for } r<a) \tag{4.13}$$

Note that the electric field when *r<a* is proportional to *r*, and when *r>a* the electric field is proportional to $1/r^2$.
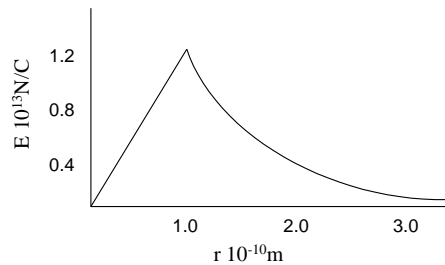
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4.8 Solution of some selected problems

في هذا الجزء سنعرض حلولاً لبعض المسائل التي تغطى استخدام قانون جاوس لإيجاد المجال الكهربي

4.8 Solution of some selected problems

Example 4.3

If the net flux through a gaussian surface is zero, which of the following statements are true?
1) There are no charges inside the surface.
2) The net charge inside the surface is zero.
3) The electric field is zero everywhere on the surface.
4) The number of electric field lines entering the surface equals the number leaving the surface.

Solution

Statements (b) and (d) are true. Statement (a) is not necessarily true since Gauss' Law says that the net flux through the closed surface equals the net charge inside the surface divided by ε_o. For example, you could have an electric dipole inside the surface. Although the net flux may be zero, we cannot conclude that the electric field is zero in that region.

Example 4.4

A spherical gaussian surface surrounds a point charge q. Describe what happens to the: flux through the surface if
1) The charge is tripled,
2) The volume of the sphere is doubled,
3) The shape of the surface is changed to that of a cube,
4) The charge is moved to another position inside the surface;

Solution

1) If the charge is tripled, the flux through the surface is tripled, since the net flux is proportional to the charge inside the surface
2) The flux remains unchanged when the volume changes, since it still surrounds the same amount of charge.
3) The flux does not change when the shape of the closed surface changes.

4) The flux through the closed surface remains unchanged as the charge inside the surface is moved to another position. All of these conclusions are arrived at through an understanding of Gauss' Law.

Example 4.5

A solid conducting sphere of radius *a* has a net charge +2*Q*. A conducting spherical shell of inner radius *b* and outer radius *c* is concentric with the solid sphere and has a net charge –*Q* as shown in figure 4.18. Using Gauss's law find the electric field in the regions labeled 1, 2, 3, 4 and find the charge distribution on the spherical shell.
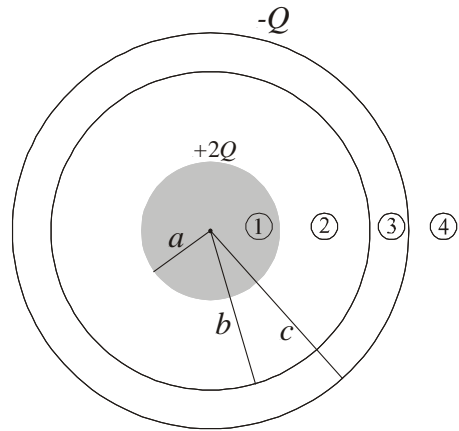
Figure 4.18

Solution

نلاحظ أن توزيع الشحنة على الكرتين لها تماثل كروي، لذلك لتعيين المجال الكهربي عند مناطق مختلفة فإننا سنفرض أن سطح جاوس كروي الشكل نصف قطره *r*.

Region (1) $r < a$
To find the E inside the solid sphere of radius *a* we construct a gaussian surface of radius $r < a$
$E = 0$ since no charge inside the gaussian surface.

Region (2) $a < r < b$
we construct a spherical gaussian surface of radius *r*

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

لاحظ هنا أن الشحنة المحصورة داخل سطح جاوس هي شحنة الكرة الموصلة الداخلية $2Q$ وأن خطوط المجال في اتجاه أنصاف الأقطار وخارجه من سطح جاوس أي $\theta = 0$ و المجال ثابت المقدار على السطح.

$$E\,4\pi r^2 = \frac{2Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2} \qquad a < r < b$$

Region (4) $r > c$

we construct a spherical gaussian surface of radius $r > c$, the total net charge inside the gaussian surface is $q = 2Q + (-Q) = +Q$ Therefore Gauss's law gives

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad r > c$$

Region (3) $b > r < c$

المجال الكهربي في هذه المنطقة يجب أن يكون صفراً لأن القشرة الكروية موصلة أيضا، ولأن الشحنة الكلية داخل سطح جاوس $b<r<c$ يجب أن تساوى صفراً. إذا نستنتج أن الشحنة Q- على القشرة الكروية هي نتيجة توزيع شحنة على السطح الداخلي والسطح الخارجي للقشرة الكروية بحيث تكون المحصلة Q- وبالتالي تتكون بالبحث شحنة على السطح الداخلي للقشرة مساوية في المقدار للشحنة على الكرة الداخلية ومخالفة لها في الإشارة أي $2Q$- وحيث أنه كما في معطيات السؤال الشحنة الكلية على القشرة الكروية هي Q- نستنتج أن على السطح الخارجي للقشرة الكروية يجب أن تكون Q+

Example 4.6

A long straight wire is surrounded by a hollow cylinder whose axis coincides with that wire as shown in figure 4.19. The solid wire has a charge per unit length of $+\lambda$, and the hollow cylinder has a *net* charge per unit length of $+2\lambda$. Use Gauss law to find (a) the charge per unit length on the inner and outer surfaces of the hollow cylinder and (b) the electric field outside the hollow cylinder, a distance *r* from the axis.

Solution

(a) Use a cylindrical Gaussian surface S_1 within the conducting cylinder where $E=0$

Thus $\oint \vec{E}.d\vec{A} = \dfrac{q_{in}}{\varepsilon_o} = 0$

and the charge per unit length on the inner surface must be equal to

$$\lambda_{inner} = -\lambda$$

Also $\qquad \lambda_{inner} + \lambda_{outer} = 2\lambda$

thus $\qquad \lambda_{outer} = 3\lambda$

(b) For a gaussian surface S_2 outside the conducting cylinder

$$\oint \vec{E}.d\vec{A} = \dfrac{q_{in}}{\varepsilon_o}$$

$$E\,(2\pi rL) = \dfrac{1}{\varepsilon_o}\,(\lambda - \lambda + 3\lambda)L$$

$$\therefore E = \dfrac{3\lambda}{2\pi\varepsilon_o r}$$

$+\lambda$

$+2\,\lambda$
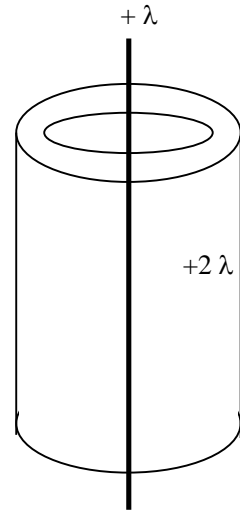
Figure 4.19

Example 4.7

Consder a long cylindrical charge distribution of radius *R* with a uniform charge density ρ. Find the electric field at distance *r* from the axis where *r<R*.

Solution

If we choose a cilindrical gaussian surface of length *L* and radius *r*, Its volume is $\pi r^2 L$, and it enclses a charge $\rho \pi r^2 L$. By applying Gauss's law we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \quad \text{becomes} \qquad E \oint dA = \frac{\rho \pi r^2 L}{\varepsilon_o}$$

$$\Theta \oint dA = 2\pi r L \text{ therefore} \qquad E(2\pi r L) = \frac{\rho \pi r^2 L}{\varepsilon_o}$$

Thus

$$E = \frac{\rho r}{2\varepsilon_o} \qquad \text{radially outward from the cylinder axis}$$

Notice that the electric field will increase as ρ increases, and also the electric field is proportional to r for *r<R*. For thr region outside the cylinder (*r>R*), the electric field will decrese as *r* increases.

Example 4.8

Two large non-conducting sheets of +ve charge face each other as shown in figure 4.20. What is E at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?

Solution

We know previously that for each sheet, the magnitude of the field at any point is

$$E = \frac{\sigma}{2\varepsilon_o}$$

(a) At point to the left of the two parallel sheets

$$E = -E_1 + (-E_2) = -2E$$

$$\therefore E = -\frac{\sigma}{\varepsilon_o}$$

Figure 4.20

(b) At point between the two sheets

$$E = E_1 + (-E_2) = \text{zero}$$

(c) At point to the right of the two parallel sheets

$$E = E_1 + E_2 = 2E$$

$$\therefore E = \frac{\sigma}{\varepsilon_o}$$

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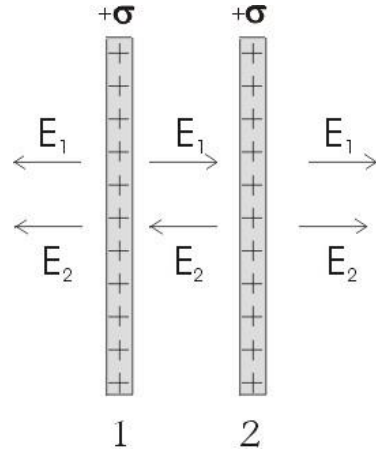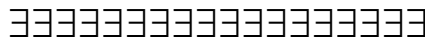
4.9 Problems

4.1) An electric field of intensity 3.5×10^3 N/C is applied the x-axis. Calculate the electric flux through a rectangular plane 0.35m wide and 0.70m long if (a) the plane is parallel to the yz plane, (b) the plane is parallel to the xy plane, and (c) the plane contains the y axis and its normal makes an angle of 40° with the x axis.

4.2) A point charge of $+5\mu C$ is located at the center of a sphere with a radius of 12cm. What is the electric flux through the surface of this sphere?

4.3) (a) Two charges of $8\mu C$ and $-5\mu C$ are inside a cube of sides 0.45m. What is the total electric flux through the cube? (b) Repeat (a) if the same two charges are inside a spherical shell of radius 0.45 m.

4.4) The electric field everywhere on the surface of a hollow sphere of radius 0.75m is measured to be equal to 8.90×10^2 N/C and points radially toward the center of the sphere. (a) What is the net charge within the surface? (b) What can you conclude about charge inside the nature and distribution of the charge inside the sphere?

4.5) Four closed surfaces, S_1, through S_4, together with the charges $-2Q$, $+Q$, and $-Q$ are sketched in figure 4.21. Find the electric flux through each surface.
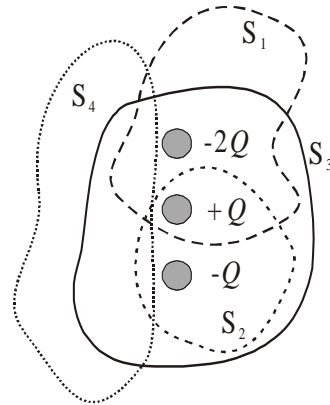
Figure 4.21

4.6) A conducting spherical shell of radius 15cm carries a net charge of $-6.4\mu C$ uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

4.7) A long, straight metal rod has a radius of 5cm and a charge per unit length of 30nC/m. Find the electric field at the following distances from the axis of the rod: (a) 3cm, (b) 10cm, (c) 100cm.

4.8) A square plate of copper of sides 50cm is placed in an extended electric field of 8×10^4N/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

4.9) A solid copper sphere 15cm in radius has a total charge of 40nC. Find the electric field at the following distances measured from the center of the sphere: (a) 12cm, (b) 17cm, (c) 75cm. (d) How would your answers change if the sphere were hollow?

4.10) A solid conducting sphere of radius 2cm has a positive charge of +8µC. A conducting spherical shell d inner radius 4cm and outer radius 5cm is concentric with the solid sphere and has a net charge of -4µC. (a) Find the electric field at the following distances from the center of this charge configuration: (a) r=1cm, (b) r=3cm, (c) r=4.5cm, and (d) r=7cm.

4.11) A non-conducting sphere of radius a is placed at the center of a spherical conducting shell of inner radius b and outer radius c, A charge +Q is distributed uniformly through the inner sphere (charge density ρC/m^3) as shown in figure 4.22. The outer shell carries -Q. Find $E(r)$ (i) within the sphere ($r<a$) (ii) between the sphere and the shell ($a<r<b$) (iii) inside the

shell ($b<r<c$) and (iv) out side the shell and (v) What is the charge appear on the inner and outer surfaces of the shell?
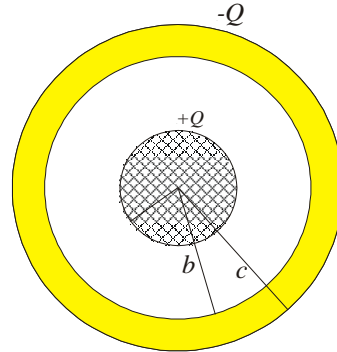
Figure 4.22

4.12) A solid sphere of radius 40cm has a total positive charge of 26µC uniformly distributed throughout its volume. Calculate the electric field intensity at the following distances from the center of the sphere: (a) 0 cm, (b) 10cm, (c) 40cm, (d) 60 cm.

4.13) An insulating sphere is 8cm in diameter, and carries a +5.7µC charge uniformly distributed throughout its interior volume. Calculate the charge enclosed by a concentric spherical surface with the following radii: (a) r=2cm and (b) r=6cm.

4.14) A long conducting cylinder (length l) carry a total charge +q is surrounded by a conducting cylindrical shell of total charge -$2q$

as shown in figure 4.23. Use Gauss's law to find (i) the electric field at points outside the conducting shell and inside the conducting shell, (ii) the distribution of the charge on the conducting shell, and (iii) the electric field in the region between the cylinder and the cylindrical shell?
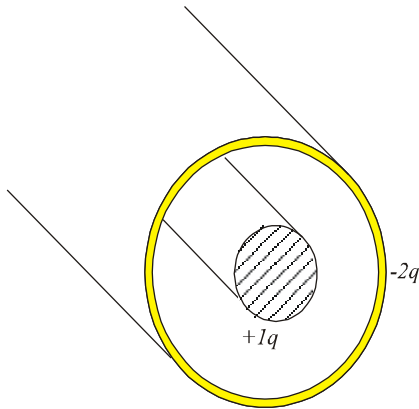
Figure 4.23

4.15) Consider a thin spherical shell of radius 14cm with a total charge of 32μC distributed uniformly on its surface. Find the electric field for the following distances from the center of the charge distribution: (a) r=10cm and (b) r =20cm.

4.16) A large plane sheet of charge has a charge per unit area of 9.0μC/m^2. Find the electric field intensity just above the surface of the sheet, measured from the sheet's midpoint.

4.17) Two large metal plates face each other and carry charges with surface density +σ and -σ respectively, on their inner surfaces as shown in figure 4.24. What is E at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?
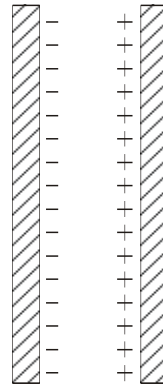
Figure 4.24

Chapter 5

Electric Potential
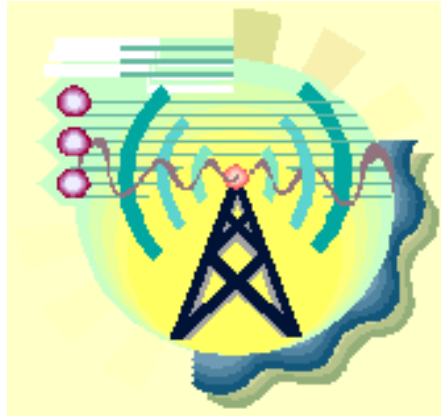
الجهد الكهربي

Electric Potential

The Electric Potential

تعلمنا في الفصول السابقة كيف يمكن التعبير عن القوى الكهربية أو التأثير الكهربي في الفراغ المحيط بشحنة أو أكثر باستخدام مفهوم المجال الكهربي. وكما نعلم أن المجال الكهربي هو كمية متجهة وقد استخدمنا لحسابه كلا من قانون كولوم وقانون جاوس. وقد سهل علينا قانون جاوس الكثير من التعقيدات الرياضية التي واجهتنا أثناء إيجاد المجال الكهربي لتوزيع متصل من الشحنة باستخدام قانون كولوم.

في هذه الفصل سوف نتعلم كيف يمكننا التعبير عن التأثير الكهربي في الفراغ المحيط بشحنة أو أكثر بواسطة كمية قياسية تسمى الجهد الكهربي The electric potential. وحيث أن الجهد الكهربي كمية قياسية وبالتالي فسيكون التعامل معه أسهل في التعبير عن التأثير الكهربي من المجال الكهربي.

في هذا الموضوع سندرس المواضيع التالية:-

(1)   تعريف الجهد الكهربي.

(2)   علاقة الجهد الكهربي بالمجال الكهربي.

(3)   حساب الجهد الكهربي لشحنة في الفراغ.

(4)   حساب المجال الكهربي من الجهد الكهربي.

(5)   أمثلة ومسائل محلولة.

قبل أن نبدأ بتعريف الجهد الكهربي أو بمعنى أصح فرق الجهد الكهربي بين نقطتين في مجال شحنة في الفراغ سوف نضرب بعض الأمثلة التوضيحية.

مثال توضيحي (1)

عند رفع جسم كتلته m إلى ارتفاع h فوق سطح الأرض فإننا نقول أن شغلا خارجيا (موجبا) تم بذله لتحريك الجسم ضد عجلة الجاذبية الأرضية، وهذا الشغل سوف يتحول إلى طاقة وضع مختزنة في المجموعة المكونة من الجسم m والأرض. وطاقة الوضع هذه تزداد بازدياد المسافة h لأنه بالطبع سيزداد الشغل المبذول. إذا زال تأثير الشغل المبذول على الجسم m فإنه سيتحرك من المناطق ذات طاقة الوضع المرتفعة إلى المناطق ذات طاقة الوضع المنخفضة حتى يصبح فرق طاقة الوضع مساوياً للصفر.

مثال توضيحي (2)
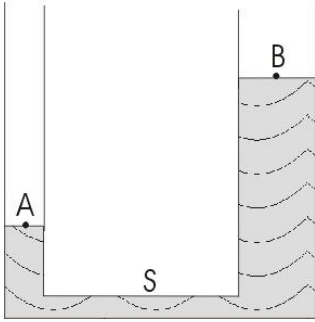
Figure 5.1

نفرض إناء على شكل حرف U به ماء كما في شكل 5.1 . تكون طاقة الوضع لجزئ الماء عند النقطة B أكبر من طاقة الوضع عند النقطة A ولذلك إذا فتح الصنبور S فإن الماء سوف يتدفق في اتجاه النقطة A إلى أن يصبح الفرق في طاقتي الوضع بين النقطتين A&B مساويا للصفر.

مثال توضيحي (3)

هناك حالة مشابهة تماما للحالتين السابقتين في الكهربية، حيث نفترض أن النقطتين A&B موجودتان في مجال كهربي ناتج من شحنة موجبة Q على سبيل المثال كما في شكل 5.2 . إذا كانت هناك شحنة اختبار $q_\circ$ (مناظرة للجسم m في مجال عجلة الجاذبية الأرضية وكذلك لجزئ الماء عند النقطة B في المثال السابق) موجودة بالقرب من الشحنة Q فإن الشحنة $q_\circ$ سوف تتحرك من نقطة قريبة من الشحنة إلى نقطة أكثر بعداً أي من B إلى A وفيزيائيا نقول أن الشحنة $q_\circ$ تحركت من مناطق ذات جهد كهربي مرتفع إلى مناطق ذات جهد كهربي منخفض. ولذلك يكون

تعريف فرق الجهد الكهربي بين نقطتين A&B واقعتين في مجال كهربي شدته E بحساب الشغل المبذول بواسطة قوة خارجية (F_{ex}) ضد القوى الكهربية (qE) لتحريك شحنة اختبار q_o من A إلى B بحيث تكون دائما في حالة اتزان (أي التحريك بدون عجلة).
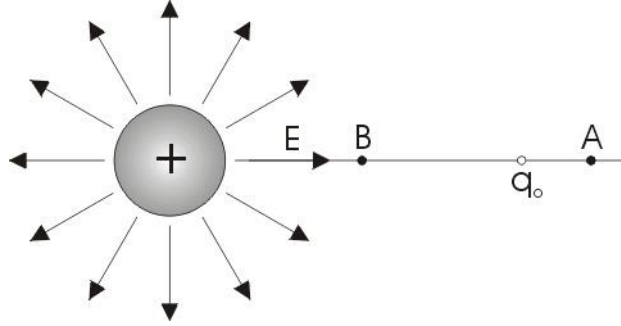
Figure 5.2

إذا كانت هنالك بطارية فرق الجهد بين قطبيها **1.5volt** فهذا يعنى إنها إذا ما وصلت في دائرة كهربية، فإن الشحنات الموجبة ستتحرك من الجهد المرتفع إلى الجهد المنخفض. كما حدث في حالة فتح الصنبور في الأنبوبة **U** وستستمر حركة الشحنات حتى يصبح فرق الجهد بين قطبي البطارية مساوياً للصفر.

5.1 Definition of electric potential difference

We define the potential difference between two points *A* and *B* as the work done by an external agent in moving a test charge q_o from *A* to *B* i.e.

$$V_B\text{-}V_A = W_{AB} / q_o \qquad\qquad (5.1)$$

The unit of the potential difference is (***Joule/Coulomb***) which is known as ***Volt (V)***

Notice

Since the work may be (a) positive *i.e* $V_B > V_A$

(b) negative *i.e* $V_B < V_A$

(c) zero *i.e* $V_B = V_A$

You should remember that the work equals

$$W = \vec{F}_{ex}.\vec{l} = F_{ex}\cos\theta\, l$$

- If $0 < \theta < 90 \Rightarrow \cos\theta$ is +ve and therefore the *W* is +ve
- If $90 < \theta < 180 \Rightarrow \cos\theta$ is -ve and therefore *W* is -ve
- If $\theta = 90$ between F_{ex} and l $\Rightarrow$ therefore *W* is zero

The potential difference is independent on the path between *A* and *B*. Since the work (W_{AB}) done to move a test charge q_o from *A* to *B* is independent on the path, otherwise the work is not a scalar quantity. (see example 5.2)

5.2 The Equipotential surfaces

As the electric field can be represented graphically by lines of force, the potential distribution in an electric field may be represented graphically by equipotential surfaces.

The equipotential surface is a surface such that the potential has the same value at all points on the surface. *i.e.* $V_B - V_A =$ zero for any two points on one surface.

The work is required to move a test charge between any two points on an equipotential surface is zero. (*Explain why*?)

In all cases the equipotential surfaces are at right angles to the lines of force and thus to *E*. (*Explain why*?)

Figure 5.3 (a)   *Figure 5.3 (b)*
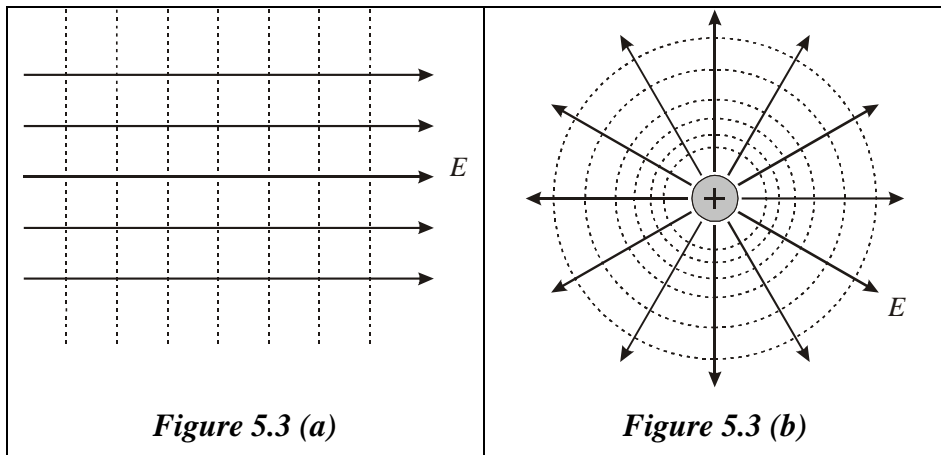
Figure 5.3 shows the equipotential surfaces (dashed lines) and the electric field lines (bold lines), (a) for uniform electric field and (b) for electric field due to a positive charge.

5.3 Electric Potential and Electric Field

Simple Case (Uniform electric field):

The potential difference between two points *A* and *B* in a Uniform electric field *E* can be found as follow,

Assume that a positive test charge q_o is moved by an external agent from *A* to *B* in uniform electric field as shown in figure 5.4.

The test charge q_o is affected by electric force of q_oE in the downward direction. To move the charge from *A* to *B* an external force *F* of the same magnitude to the electric force but in the opposite direction. The work *W* done by the external agent is:

$$W_{AB} = Fd = q_oEd \qquad (5.2)$$

The potential difference V_B-V_A is

$$V_B - V_A = \frac{W_{AB}}{q_o} = Ed \qquad (5.3)$$
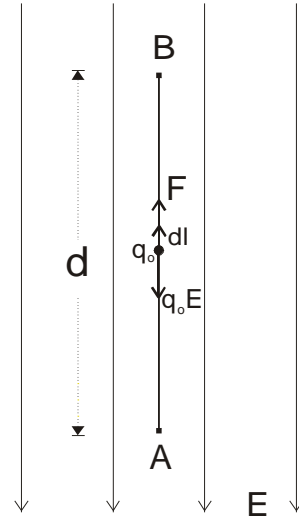
Figure 5.4

This equation shows the relation between the potential difference and the electric field for a special case (uniform electric field). Note that *E* has a new unit (V/m). hence,

$$\frac{Volt}{Meter} = \frac{Newton}{Coulomb}$$

The relation in general case (not uniform electric field):

If the test charge q_o is moved along a curved path from *A* to *B* as shown in figure 5.5. The electric field exerts a force q_oE on the charge. To keep the charge moving without accelerating, an external agent must apply a force *F* equal to $-q_oE$.

If the test charge moves distance *dl* along the path from *A* to *B*, the work done is *F.dl*. The total work is given by,

$$W_{AB} = \int_A^B \vec{F}.\vec{dl} = -q_o \int_A^B \vec{E}.\vec{dl} \quad (5.4)$$

The potential difference V_B-V_A is,

$$V_B - V_A = \frac{W_{AB}}{q_o} = -\int_A^B \vec{E}.\vec{dl} \quad (5.5)$$
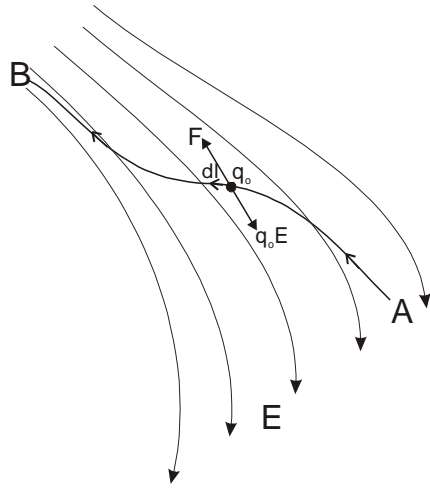
Figure 5.5

لاحظ هنا أن حدود التكامل من *A* إلى *B* هى التي تحدد المسار ومنه اتجاه متجه الإزاحة *dl* وتكون الزاوية θ هي الزاوية المحصورة بين منتجه الإزاحة ومتجه المجال الكهربي.

If the point *A* is taken to infinity then V_A=0 the potential *V* at point *B* is,

$$V_B = -\int_\infty^B \vec{E}.\vec{dl} \quad (5.6)$$

This equation gives the general relation between the potential and the electric field.

Example 5.1

Derive the potential difference between points A and B in uniform electric field using the general case.

Solution

$$V_B - V_A = -\int_A^B \vec{E}.d\vec{l} = -\int_A^B E\cos180^o\, dl = \int_A^B Edl \qquad (5.7)$$

E is uniform (constant) and the integration over the path A to B is d, therefore

$$V_B - V_A = E\int_A^B dl = Ed \qquad (5.8)$$

Example 5.2

In figure 5.6 the test charge moved from A to B along the path shown. Calculate the potential difference between A and B.
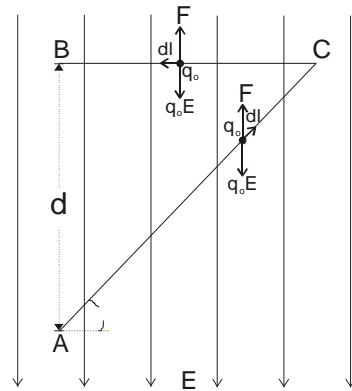
Figure 5.6

Solution

$$V_B - V_A = (V_B - V_C) + (V_C - V_A)$$

For the path *AC* the angle θ is 135º,

$$V_C - V_A = -\int_A^C \vec{E}.d\vec{l} = -\int_A^C E\cos135^o\,dl = \frac{E}{\sqrt{2}}\int_A^C dl$$

The length of the line *AC* is √2 *d*

$$V_C - V_A = \frac{E}{\sqrt{2}}(\sqrt{2}d) = Ed$$

For the path *CB* the work is zero and *E* is perpendicular to the path therefore, V_C-$V_A = 0$

$$V_B - V_A = V_C - V_A = Ed$$

The Electron Volt Unit

A widely used unit of energy in atomic physics is the electron volt (eV). **ELECTRON VOLT,** unit of energy, used by physicists to express the energy of ions and subatomic particles that have been accelerated in particle accelerators. One electron volt is equal to the amount of energy gained by an electron traveling through an electrical potential difference of 1 V; this is equivalent to 1.60207×10^{-19}J. Electron volts are commonly expressed as million electron volts (MeV) and billion electron volts (BeV or GeV).

5.4 Potential difference due to a point charge

Assume two points *A* and *B* near to a positive charge *q* as shown in figure 5.7. To calculate the potential difference V_B-V_A we assume a test charge q_o is moved without acceleration from *A* to *B*.
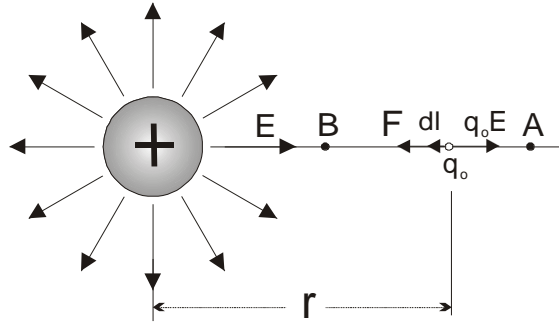
Figure 5.7

In the figure above the electric field *E* is directed to the right and *dl* to the left.

$$\vec{E}.\vec{dl} = E\cos180^o\, dl = -Edl \qquad (5.10)$$

However when we move a distance *dl* to the left, we are moving in a direction of decreasing *r*. Thus

$$\vec{dl} = -\vec{dr} \qquad (5.11)$$

Therefore

$$-Edl=Edr \qquad (5.12)$$

$$\therefore V_B - V_A = -\int_A^B \vec{E}.\vec{dl} = \int_{r_A}^{r_B} \vec{E}.\vec{dr} \qquad (5.13)$$

Substitute for *E*

$$\Theta\; E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad (5.14)$$

We get

$$\therefore V_B - V_A = -\frac{q}{4\pi\varepsilon_o} \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \quad (5.15)$$

لاحظ هنا أن هذا القانون يستخدم لإيجاد فرق الجهد الكهربي بين نقطتين في الفراغ المحيط بشحنة
q.

5.5 The potential due to a point charge

If we choose *A* at infinity then V_A=0 (*i.e.* $r_A \Rightarrow \infty$) this lead to the potential at distance r from a charge *q* is given by

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} \quad (5.16)$$

This equation shows that the equipotential surfaces for a charge are spheres concentric with the charge as shown in figure 5.8.
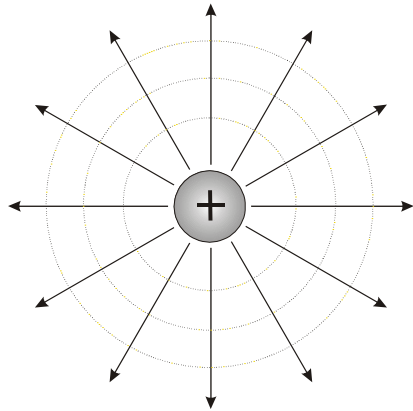
Figure 5.8

لاحظ أن المجال الكهربي لشحنة يتناسب عكسيا مع مربع المسافة، بينما الجهد الكهربي يتناسب
عكسيا مع المسافة.

5.6 The potential due to a point charge

يمكن باستخدام هذا القانون إيجاد الجهد الكهربي لنقطة تبعد عن شحنة أو أكثر عن طريق الجمع الجبري للجهد الكهربي الناشئ عن كل شحنة على حده عند النقطة المراد إيجاد الجهد الكلى عندها أي

$$V = V_1 + V_2 + V_3 + ...+ V_n \qquad (5.17)$$

$$\therefore V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o} \sum_n \frac{q_n}{r_n} \qquad (5.18)$$

عند التعويض عن قيمة الشحنة q تأخذ الإشارة في الحسبان، لأنك تجمع جمعاً جبرياً هنا وليس جمعاً اتجاهياً كما كنا نفعل في المجال الكهربي حيث تحدد الإشارة الاتجاه على الرسم.

Example 5.3

What must the magnitude of an isolated positive charge be for the electric potential at 10 cm from the charge to be +100V?

Solution

$$V = \frac{1}{4\pi\varepsilon_o} \frac{q}{r}$$

$$\therefore q = V 4\pi\varepsilon_o r^2 = 100 \times 4\pi \times 8.9 \times 10^{-12} \times 0.1 = 1.1 \times 10^{-9}\,C$$

Example 5.4

What is the potential at the center of the square shown in figure 5.9?
Assume that $q_1 = +1 \times 10^{-8}C$, $q_2 = -2 \times 10^{-8}C$, $q_3 = +3 \times 10^{-8}C$, $q_4 = +2 \times 10^{-8}C$,
and $a = 1m$.

Solution

$$\therefore V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o} \frac{q_1 + q_2 + q_3 + q_4}{r}$$

The distance r for each charge from P is 0.71m

$$\therefore V = \frac{9 \times 10^9 (1 - 2 + 3 + 2) \times 10^{-8}}{0.71} = 500V$$
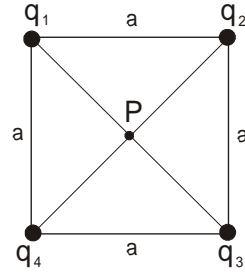
Figure 5.9

Example 5.5

Calculate the electric potential due to an electric dipole as shown in
figure 5.10.
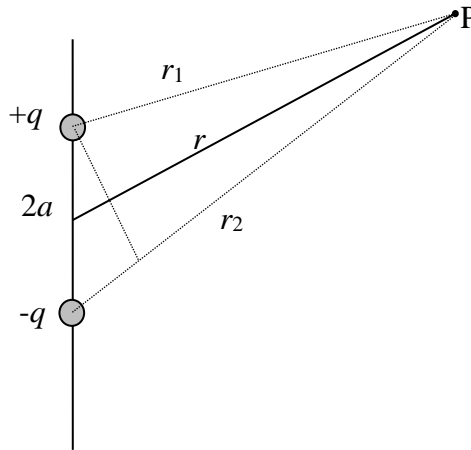
Figure 5.10

Solution

$V = \sum V_n = V_1 + V_2$

$$V = K\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = Kq\frac{r_2 - r_1}{r_2 r_1}$$

When $r >> 2a$,

$$r_2 - r_1 \cong 2a\cos\theta \qquad \text{and} \qquad r_1 r_2 \cong r^2,$$

$$V = Kq\frac{2a\cos\theta}{r^2} = K\frac{p\cos\theta}{r^2} \qquad\qquad (5.19)$$

where p is the dipole momentum

Note that $V = 0$ when $\theta = 90^\circ$ but V has the maximum positive value when $\theta = 0^\circ$ and V has the maximum negative value when $\theta = 180^\circ$.

5.7 Electric Potential Energy

The definition of the *electric potential energy* of a system of charges is the work required to bring them from infinity to that configuration.

To workout the electric potential energy for a system of charges, assume a charge q_2 at infinity and at rest as shown in figure 5.11. If q_2 is moved from infinity to a distance r from another charge q_1, then the work required is given by

$$W = V q_2$$

$$\Theta \; V = \frac{1}{4\pi\varepsilon_o} \frac{q_1}{r}$$

Substitute for V in the equation of work

$$U = W = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r_{12}} \qquad (5.20)$$

$$U = \frac{q_1 q_2}{4\pi\varepsilon_o r} \qquad (5.21)$$

Figure 5.11

To calculate the potential energy for systems containing more than two charges we compute the potential energy for every pair of charges separately and to add the results algebraically.

$$U = \sum \frac{q_i q_j}{4\pi\varepsilon_o r_{ij}} \qquad (5.22)$$

القانون الأول يطبق في حالة شحنتين فقط، ولكن إذا كانت المجموعة المراد إيجاد طاقة الوضع الكهربي لها أكثر من شحنتين نستخدم القانون الثاني حيث نوجد الطاقة المختزنة بين كل شحنتين على حده ثم نجمع جمعا جبريا، أي نعوض عن قيمة الشحنة ونأخذ الإشارة بالحسبان في كل مرة.

If the total electric potential energy of a system of charges is positive this correspond to a repulsive electric forces, but if the total electric potential energy is negative this correspond to attractive electric forces. (*explain why?*)
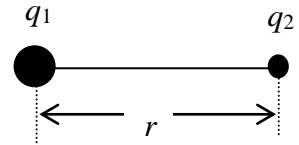
Example 5.6

Three charges are held fixed as shown in figure 5.12. What is the potential energy? Assume that $q=1 \times 10^{-7}$C and $a=10$cm.
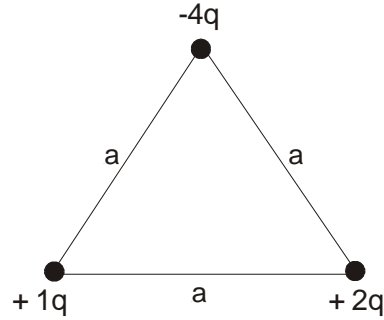
Solution

Figure 5.12

$U=U_{12}+U_{13}+U_{23}$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{(+q)(-q)}{a} + \frac{(+q)(+2q)}{a} + \frac{(-4q)(+2q)}{a}\right]$$

$$U = -\frac{10}{4\pi\varepsilon_o}\frac{q^2}{a}$$

$$\therefore U = -\frac{9\times 10^9 (10)(1\times 10^{-7})^2}{0.1} = -9\times 10^{-3}\, J$$

نلاحـــظ أن قيمـــة الطاقـــة الكليـــة ســـالبة، وهـــذا يعنـــي أن الشـــغل المبـــذول للحفـــاظ علـــى ثبـــات الشـــحنات ســـابقة الـــذكر ســـالب أيضـــاً. نستنتج مـــن ذلـــك أن القـــوة المتبادلـــة بـــين الشـــحنات هـــي قـــوة تجـــاذب، أمـــا فـــي حالـــة أن تكـــون الطاقـــة الكليـــة موجبـــة فـــإن هـــذا يعنـــي أن القـــوة المتبادلـــة بـــين الشـــحنات هـــي قوة تنافر.

5.8 Calculation of *E* from *V*

As we have learned that both the electric field and the electric potential can be used to evaluate the electric effects. Also we have showed how to calculate the electric potential from the electric field now we determine the electric field from the electric potential by the following relation.

$$\overset{\rho}{E} = -\frac{dV}{dl} \qquad (5.23)$$

New unit for the electric field is *volt/meter* (v/m)

لاحظ أن العلاقة الرياضية بين المجال الكهربي والجهد الكهربي هي علاقة تفاضل وتكامل وبالتالي إذا علمنا الجهد الكهربي يمكن بإجراء عملية التفاضل إيجاد المجال الكهربي. وتتذكر أن خطوط المجال الكهربي عمودية على أسطح متساوية الجهد equipotential surfaces.

Example 5.7

Calculate the electric field for a point charge *q*, using the equation $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$

Solution

$$E = -\frac{dV}{dl} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r}\right)$$

$$E = -\frac{q}{4\pi\varepsilon_o}\frac{d}{dr}\left(\frac{1}{r}\right) = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

5.9 Solution of some selected problems

في هذا الجزء سنعرض حلولاً لبعض المسائل التي تغطى موضوع
الجهد الكهربي والمجال الكهربي

Example 5.8

Two charges of 2µC and -6µC are located at positions (0,0) m and (0,3) m, respectively as shown in figure 5.13. (i) Find the total electric potential due to these charges at point (4,0) m.

(ii) How much work is required to bring a 3µC charge from ∞ to the point P?

(iii) What is the potential energy for the three charges?
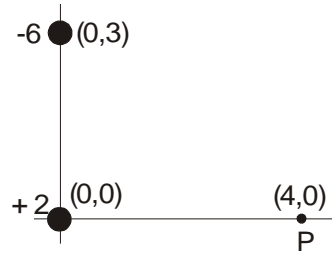
Figure 5.13

Solution

$$V_p = V_1 + V_2$$

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$$V = 9\times10^9\left[\frac{2\times10^{-6}}{4} - \frac{6\times10^{-6}}{5}\right] = -6.3\times10^3 \text{ volt}$$

(ii) the work required is given by

$$W = q_3\,V_p = 3\times10^{-6}\times\text{-}6.3\times10^3 = \text{-}18.9\times10^{-3}\text{ J}$$

The -ve sign means that work is done by the charge for the movement from ∞ to *P*.

(iii) The potential energy is given by

$$U = U_{12} + U_{13} + U_{23}$$

$$U = k\left[\frac{(2\times10^{-6})(-6\times10^{-6})}{3} + \frac{(2\times10^{-6})(3\times10^{-6})}{4} + \frac{(-6\times10^{-6})(3\times10^{-6})}{5}\right]$$

$$\therefore U = -5.5\times10^{-2}\,Joule$$

Example 5.9

A particle having a charge $q=3\times10^{-9}$C moves from point *a* to point *b* along a straight line, a total distance $d=0.5$m. The electric field is uniform along this line, in the direction from *a* to *b*, with magnitude $E=200$N/C. Determine the force on *q*, the work done on it by the electric field, and the potential difference V_a-V_b.

Solution

The force is in the same direction as the electric field since the charge is positive; the magnitude of the force is given by

$$F = qE = 3\times10^{-9} \times 200 = 600\times10^{-9}\text{N}$$

The work done by this force is

$$W = Fd = 600\times10^{-9} \times 0.5 = 300\times10^{-9}\text{J}$$

The potential difference is the work per unit charge, which is

$$V_a\text{-}V_b = W/q = 100\text{V}$$

Or

$$V_a\text{-}V_b = Ed = 200 \times 0.5 = 100\text{V}$$

Example 5.10

Point charge of $+12\times10^{-9}$C and -12×10^{-9}C are placed 10cm part as shown in figure 5.14. Compute the potential at point *a*, *b*, and *c*.

Compute the potential energy of a point charge $+4\times10^{-9}$C if it placed at points *a*, *b*, and *c*.
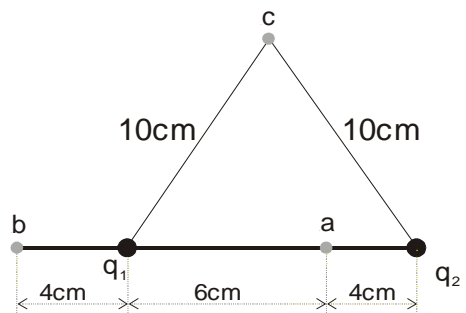
Figure 5.14

Solution

We need to use the following equation at each point to calculate the potential,

$$V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o} \sum \frac{q_i}{r_i}$$

At point *a*

$$V_a = 9 \times 10^9 \left(\frac{12 \times 10^{-9}}{0.06} + \frac{-12 \times 10^{-9}}{0.04} \right) = -900V$$

At point *b*

$$V_b = 9 \times 10^9 \left(\frac{12 \times 10^{-9}}{0.04} + \frac{-12 \times 10^{-9}}{0.14} \right) = -1930V$$

At point *c*

$$V_c = 9 \times 10^9 \left(\frac{12 \times 10^{-9}}{0.1} + \frac{-12 \times 10^{-9}}{0.14} \right) = 0V$$

We need to use the following equation at each point to calculate the potential energy,

$$U = qV$$

At point *a*

$$U_a = qV_a = 4 \times 10^{-9} \times (-900) = -36 \times 10^{-7} J$$

At point *b*

$$U_b = qV_b = 4 \times 10^{-9} \times 1930 = +77 \times 10^{-7} J$$

At point *c*

$$U_c = qV_c = 4 \times 10^{-9} \times 0 = 0$$

Example 5.11

A charge *q* is distributed throughout a nonconducting spherical volume of radius *R*. (a) Show that the potential at a distance *r* from the center where *r < R*, is given by

$$V = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_o R^3}$$

Solution

لايجاد الجهد داخل الكرة غير الموصلة عند نقطة *A*   مثلا فإننا سوف نحسب فرق الجهد بين موضع في مالانهاية والنقطة *A*.

$$V_B - V_\infty = -\int \vec{E}.d\vec{l}$$

وحيث أن للمجال قيمتين مختلفتين خارج الكرة وداخلها كما نعلم من مسألة سابقة من مسائل قانون جاوس.

$$E_{out} = \frac{q}{4\pi\varepsilon_o r^2} \qquad E_{in} = \frac{qr}{4\pi\varepsilon_o R^3}$$

$$V_A - V_\infty = (V_A - V_B) + (V_B - V_\infty)$$

$$V_A - V_\infty = -\int \vec{E}_{in}.d\vec{l} - \int \vec{E}_{out}.d\vec{l}$$

نلاحظ أن الزاوية بين *E & dl* هي 180° أي أن $\cos 180 = -1$ ولكن أيضا $dl = -dr$

$$V_A - V_\infty = -\int \frac{qr}{4\pi\varepsilon_o R^3} \ dr - \int \frac{q}{4\pi\varepsilon_o r^2} \ dr$$

$$= -\frac{q}{4\pi\varepsilon_o R^3}\left[\frac{r^2}{2}\right] + \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r}\right] \ = \ \frac{q(3R^2 - r^2)}{8\pi\varepsilon_o R^3}$$

وهذا هو الجهد عند النقطة *A* وهو المطلوب إثباته

إذا كانت *A* على سطح الكرة فإن الجهد في هذه الحالة $V = \frac{q}{4\pi\varepsilon_o R}$

أي كالجهد الكهربي لشحنة في الفراغ.

Example 5.12

For the charge configuration shown in figure 5.15, Show that $V(r)$ for the points on the vertical axis, assuming $r \gg a$, is given by

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{r} + \frac{2aq}{r^2}\right]$$

Solution

$V_p = V_1 + V_2 + V_3$

$$V = \frac{q}{4\pi\varepsilon_o(r-a)} + \frac{q}{4\pi\varepsilon_o r} - \frac{q}{4\pi\varepsilon_o(r+a)}$$

$$\frac{q(r+a) - q(r-a)}{4\pi\varepsilon_o(r^2 - a^2)} + \frac{q}{4\pi\varepsilon_o r}$$

$$\frac{2aq}{4\pi\varepsilon_o r^2(1 - a^2/r^2)} + \frac{q}{4\pi\varepsilon_o r}$$

when $r \gg a$ then $a^2/r^2 \ll 1$

$$V = \frac{2aq}{4\pi\varepsilon_o r^2}(1 - a^2/r^2)^{-1} + \frac{q}{4\pi\varepsilon_o r}$$
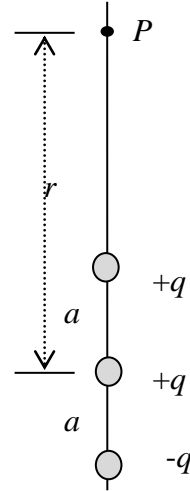
Figure 5.15

يمكن فك القوسين بنظرية ذات الحدين والاحتفاظ بأول حدين فقط كتقريب جيد

$(1 + x)^n = 1 + nx$ when $x \ll 1$

$$V = \frac{2aq}{4\pi\varepsilon_o r^2}(1 + a^2/r^2) + \frac{q}{4\pi\varepsilon_o r}$$

ويمكن إهمال a^2/r^2 بالنسبة لـ 1

$$\therefore V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{r} + \frac{2aq}{r^2}\right]$$

Example 5.13

Derive an expression for the work required to put the four charges together as indicated in figure 5.16.
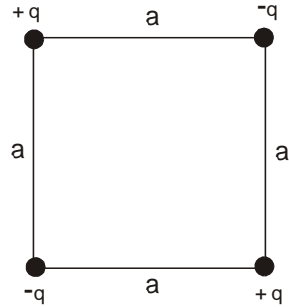
Solution

The work required to put these charges together is equal to the total electric potential energy.

Figure 5.16

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a}\right]$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-4q^2}{a} + \frac{2q^2}{\sqrt{2}a}\right]$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-\sqrt{2}4q^2 + 2q^2}{\sqrt{2}a}\right] = \frac{-0.2q^2}{\varepsilon_o a}$$

The minus sign indicates that there is attractive force between the charges

In Example 5.13 assume that if all the charges are positive, prove that the work required to put the four charges together is

$$U = \frac{1}{4\pi\varepsilon_o}\frac{5.41q^2}{\varepsilon_o a}$$

Example 5.14

In the rectangle shown in figure 5.17, q_1 = -5x10⁻⁶C and q_2 = 2x10⁻⁶C calculate the work required to move a charge q_3 = 3x10⁻⁶C from *B* to *A* along the diagonal of the rectangle.

q_1 15cm   A

q_3 5cm

B q_2
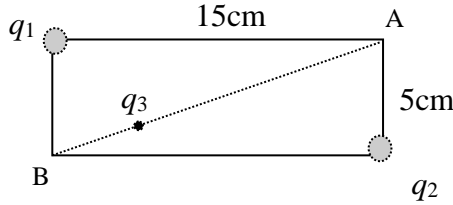
Figure 5.17

Solution

from the equation $V_B - V_A = W_{AB} / q_o$

$V_A = V_1 + V_2$ & $V_B = V_1 + V_2$

$$V_A = \frac{q}{4\pi\varepsilon_o}\left[\frac{-5\times10^{-6}}{0.15} + \frac{2\times10^{-6}}{0.05}\right] \quad = 6\times10^4 \text{ V}$$

$$V_B = \frac{q}{4\pi\varepsilon_o}\left[\frac{-5\times10^{-6}}{0.05} + \frac{2\times10^{-6}}{0.15}\right] \quad = -7.8\times10^4 \text{ V}$$

$$W_{BA} = (V_A - V_B)\, q_3$$

$$= (6\times10^4 + 7.8\times10^4)\, 3\times10^{-6} = 0.414 \text{ Joule}$$

Example 5.15

Two large parallel conducting plates are 10 cm a part and carry equal but opposite charges on their facing surfaces as shown in figure 5.18. An electron placed midway between the two plates experiences a force of 1.6×10^{-15} N.

What is the potential difference between the plates?

Solution

$$V_B - V_A = Ed$$

يمكن حساب المجال الكهربي عن طريق القوى الكهربية المؤثرة على الإلكترون

$$F = eE \Rightarrow E = F/e$$

$$V_B - V_A = 10000 \times 0.1 = 1000 \text{ volt}$$

Figure 5.18

ƎƎƎƎƎƎƎƎƎƎƎƎƎƎƎƎƎ
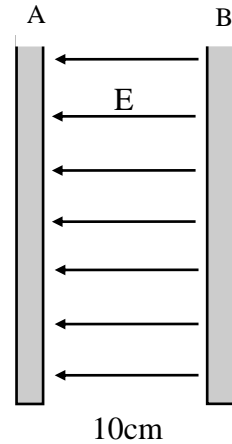
5.10 Problems

5.1) What potential difference is needed to stop an electron with an initial speed of 4.2×10^5m/s?

5.2) An ion accelerated through a potential difference of 115V experiences an increase in potential energy of 7.37×10^{-17}J. Calculate the charge on the ion.

5.3) How much energy is gained by a charge of 75 μC moving through a potential difference of 90V?

5.4) An infinite charged sheet has a surface charge density σ of 1.0×10^{-7} C/m^2. How far apart are the equipotential surfaces whose potentials differ by 5.0 V?

5.5) At what distance from a point charge of 8μC would the potential equal 3.6×10^4V?

5.6) At a distance r away from a point charge q, the electrical potential is $V=400$V and the magnitude of the electric field is $E=150$N/C. Determine the value of q and r.

5.7) Calculate the value of the electric potential at point P due to the charge configuration shown in Figure 5.19. Use the values

$q_1=5$μC, $q_2=-10$μC, $a=0.4$m, and $b=0.5$m.
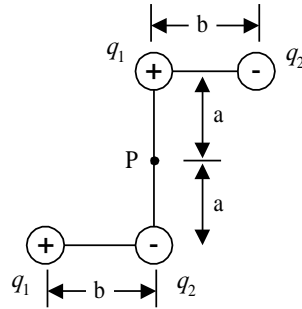
Figure 5.19

5.8) Two point charges are located as shown in Figure 5.20, where $q_1=+4$μC, $q_2=-2$μC, $a=0.30$m, and b $=0.90$m. Calculate the value of the electrical potential at points P_1, and P_2. Which point is at the higher potential?
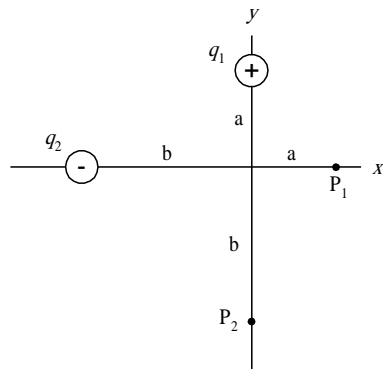
Figure 5.20

5.9) Consider a point charge with $q=1.5\times10^{-6}$C. What is the radius of an equipotential surface having a potential of 30V?

5.10) Two large parallel conducting plates are 10cm apart and carry equal and opposite charges on their facing surfaces. An electron placed midway between the two plates experiences a force of 1.6×10^{15}N. What is the potential difference between the plates?

5.11) A point charge has $q=1.0\times10^{-6}$C. Consider point A which is 2m distance and point B which is 1m distance as shown in the figure 5.21(a). (a) What is the potential difference V_A-V_B? (b) Repeat if points A and B are located differently as shown in figure 5.21(b).
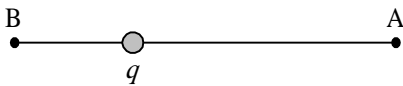
5.12) In figure 5.22 prove that the work required to put four charges together on the corner of a square of radius a is given by ($w=-0.21q^2/\varepsilon_o a$).
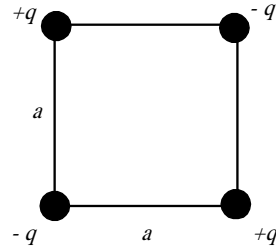
Figure 5.22

5.13) Two charges $q=+2\times10^{-6}$C are fixed in space a distance $d=2$cm) apart, as shown in figure 5.23 (a) What is the electric potential at point C? (b) You bring a third charge $q=2.0\times10^{-6}$C very slowly from infinity to C. How much work must you do? (c) What is the potential energy U of the configuration when the third charge is in place?
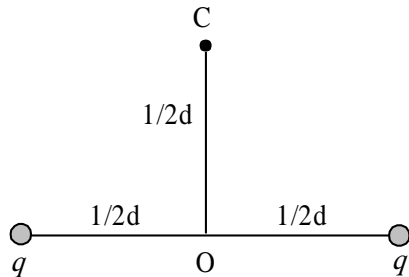
Figure 5.21(a)

Figure 5.21(b)

Figure 5.23

5.14) Four equal point charges of charge $q=+5\mu C$ are located at the corners of a 30cm by 40cm rectangle. Calculate the electric potential energy stored in this charge configuration.

5.15) Two point charges, $Q_1=+5nC$ and $Q_2=-3nC$, are separated by 35cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

Multiple Choice Questions

Part 1
Principles of Electrostatic

Coulomb's Law
Electric Field
Gauss's Law
Electric Potential Difference

Attempt the following question after the completion of part 1

[1] Two small beads having positive charges 3 and 1 are fixed on the opposite ends of a horizontal insulating rod, extending from the origin to the point x=d. As in Figure 1, a third small, charged bead is free to slide on the rod. At what position is the third bead in equilibrium?

$+3q$ $+q$

d
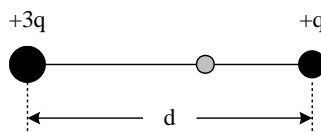
Figure 1

a.   x = 0.366d
b.   x = 0.634d
c.   x = 0.900d
d.   x = 2.37d

---

[2] Two identical conducting small spheres are placed with their centers 0.300m apart. One is given a charge of 12.0nC and the other one a charge of 18.0nC. (a) Find the electrostatic force exerted on one sphere by the other. (b) The spheres are connected by a conducting wire. After equilibrium has occurred, find the electrostatic force between the two.

a.   (a) 2.16×10^{-5} N attraction; (b) 0 N repulsion
b.   (a) 6.47×10^{-6} N repulsion; (b) 2.70×10^{-7} N attraction
c.   (a) 2.16×10^{-5} N attraction; (b) 8.99×10^{-7} N repulsion
d.   (a) 6.47×10^{-6} N attraction; (b) 2.25×10^{-5} N repulsion

---

[3] An electron is projected at an angle of 40.0º above the horizontal at a speed of 5.20×10^5 m/s in a region where the electric field is E = 3 50 j N/C. Neglect gravity and find (a) the time it takes the electron to return to its maximum height, (h) the maximum height it reaches and (c) its horizontal displacement when it reaches its maximum height.

a.   (a) 1.09×10^{-8} s; (b) 0.909 mm; (c) 2.17 m
b.   (a) 1.69×10^{-8} s; (b) 2.20 mm; (c) 4.40 m
c.   (a) 1.09×10^{-8} s; (b) 4.34 mm; (c) 0.909 m
d.   (a) 1.30×10^{-8} s; (b) 1.29 mm; (c) 2.17 m

[4] Two identical metal blocks resting on a frictionless horizontal surface are connected by a light metal spring for which the spring constant is k 175 N/m and the unscratched length is 0.350 m as in Figure 2a.
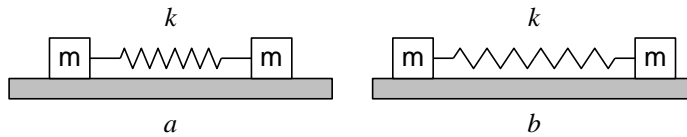
Figure 2

A charge Q is slowly placed on the system causing the spring to stretch to an equilibrium length of 0.460 m as in Figure 2b. Determine the value of Q, assuming that all the charge resides in the blocks and that the blocks can be treated as point charges.

a. 64.8 µC

b. 32.4 µC

c. 85.1 µ*C*

d. 42.6 µC

[5] A small plastic ball 1.00 g in mass is suspended by a 24.0 cm long string in a uniform electric field as shown in Figure P23.52.
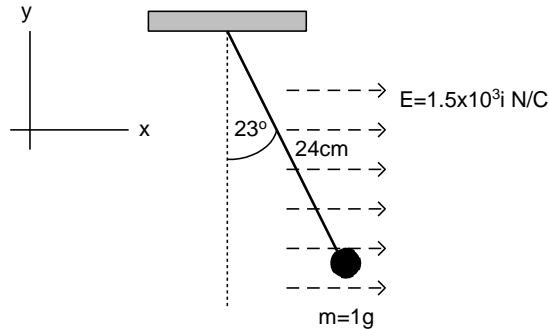
Figure 3

If the ball is in equilibrium when the string makes a 23.0^o angle with the vertical, what is the net charge on the ball?

a. 36.1 µC

b. 15.4 µC

c. 6.5 3 µC

d. 2.77 µC

[6] An object having a net charge of 24.0 μC is placed in a uniform electric field of 6 10 N/C directed vertically. What is the mass of the object if it "floats" in the field?

a. 0.386 g
b. 0.669 g
c. 2.59 g
d. 1.49 g

---

[7] Four identical point charges ($q = +14.0$ μC) are located on the corners of a rectangle as shown in Figure 4.
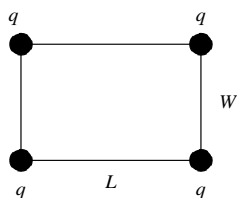
Figure 4

The dimensions of the rectangle are L = 55.0 cm and W= 13.0 cm. Calculate the magnitude and direction of the net electric force exerted on the charge at the lower left corner by the other three charges. (Call the lower left corner of the rectangle the origin.)

a.   106 mN @ 264º
b.   7.58 mN @ 13.3º
c.   7.58 mN @ 84.0º
d.   106 mN @ 193º

---

[8] An electron and proton are each placed at rest in an electric field of 720 N/C. Calculate the speed of each particle 44.0 ns after being released.

a. $v_e = 1.27 \times 10^6$ m/S, $v_p = 6.90 \times 10^3$ m/s
b. $v_e = 5.56 \times 10^6$ m/S, $v_p = 3.04 \times 10^3$ m/s
c. $v_e. = 1.27 \times 10^{14}$ m/S, $v_p = 6.90 \times 10^{10}$ m/s
d. $v_e = 3.04 \times 10^3$ m/S, $v_p = 5.56 \times 10^6$ m/s

[9] Three point charges are arranged as shown in Figure 5.
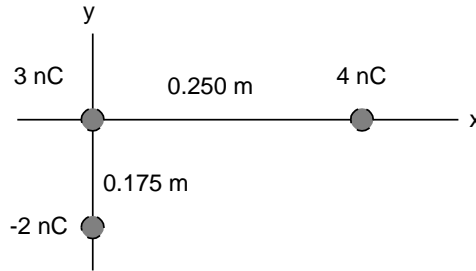
Figure 5

(a)   Find the vector electric field that the 4.00 nC and -2.00 nC charges together create at the origin. (b) Find the vector force on the 3.00 nC charge.

a. (a) (0.144i - 0.103 j) kN/C;   (b) (0.432i - 0.308j) μN
b. (a) (-0.575i - 0.587j) kN/C;   (b) (-1.73i- 1.76j) μN
c. (a) (-0.144i - 0.103j) kN/C;   (b) (-0.432i - 0.308j) μN
d. (a) (-0.575i + 0.587j) kN/C;   (b) (-1.73i + 1.76j) μN

[10] Two 1.00 μC point charges are located on the x axis. One is at x = 0.60 m, and the other is at x = -0.60 m. (a) Determine the electric field on the y axis at x = 0.90 m. (b) Calculate the electric force on a -5.00 $μC$ charge placed on the y axis at y = 0.90 m.

a. (a) $(8.52 \times 10^3 i + 1.28 \times 10^4 j)$N/C;   (b) $(-4.62 \times 10^{-2} i - 6.39 \times 10^{-2} j)$N
b. (a) $8.52 \times 10^3 j$ N/C;   (b) $-4.26 \times 10^{-2} j$ N
c. (a) $1.28 \times 10^4 j$ N/C;   (b) $-6.39 \times 10^{-2} j$ N
d. (a) -7.68×10^3 N/C;   (b) $3.84 \times 10^{-2} j$ N

[11] A 14.0μC charge located at the origin of a cartesian coordinate system is surrounded by a nonconducting hollow sphere of radius 6.00 cm. A drill with a radius of 0.800 mm is aligned along the z-axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.

a. 176 Nm^2/C
b.4.22 Nm^2/C
c. 0 Nm^2/C
d. 70.3 Nm^2/C

[12] An electric field of intensity 2.50 kN/C is applied along the x-axis. Calculate the electric flux through a rectangular plane 0.450 m wide and 0.800 m long if (a) the plane is parallel to the yz plane; (b) the plane is parallel to the *xy* plane; (c) the.plane contains the y-axis and its normal makes an angle of 30.0° with the x-axis.

a.   (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 779 Nm^2/C
b.   (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 779 Nm^2/C
c.   (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 450 Nm^2/C
d.   (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 450 Nm^2/C

[13] A conducting spherical shell of radius 13.0 cm carries a net charge of -7.40 μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

a.   (a) (-7.88 mN/C)r;   (b) (-7.88 mN/C)r
b.   (a) (7.88 mN/C)r;   (b) (0 mN/C)r
c.   (a) (-3.94 rnN/C)r;   (b) (0 mN/C)r
d.   (a) (3.94 mN/C)r;   (b) (3.94 mN/C)r

[14] A point charge of 0.0562 μC is inside a pyramid. Determine the total electric flux through the surface of the pyramid.

a.1.27 × 10^3 Nm^2/C^2
b.6.35 × 10^3 Nm^2/C^2
c. 0 Nm^2/C^2
d. 3.18 × 10^4 Nm^2/C^2

[15] A large flat sheet of charge has a charge per unit area of 7.00 μC/m^2. Find the electric field intensity just above the surface of the sheet, measured from its midpoint.

a.   7.91 × 10^5 N/C up
b.   1.98 × 10^5 N/C up
c.   3.95 × 10^5 N/C up
d.   1.58 × 10^6 N/C up

[16] The electric field on the surface of an irregularly shaped conductor varies from 60.0 kN/C to 24.0 kN/C. Calculate the local surface charge density at the point on the surface where the radius of curvature of the surface is (a) greatest and (b) smallest.

a.   0.531 μC/m^2;   (b) 0.2l2, μC/m^2
b.   l.06, μC/m^2;   (b) 0.425 μC/m^2
c.   0.425, μC/m^2 ; (b) 1.06μC/m^2
d.   0.212 μC/m^2;   (b) 0.531 μC/m^2

[17] A square plate of copper with 50.0 cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

a. (a) $\sigma = \pm 0.708$ μC/m^2;   (b) $Q = \pm 0.0885$ μC
b. (a) $\sigma = \pm 1.42$ μC/m^2;   (b) $Q = \pm 0.354$ μC
c. (a) $\sigma = \pm 0.708$ μC/m^2;   (b) $Q = \pm 0.177$ μC
d. (a) $\sigma = \pm 1.42$ μC/m^2;   (b) $Q = \pm 0.177$ μC

[18] The following charges are located inside a submarine: 5.00μC, -9.00μC, 27.0μC and -84.0μC. (a) Calculate the net electric flux through the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

a.   (a) 1.41×10^7 Nm^2/C;   (b) greater than
b.   (a) -6.89×10^6 Nm^2/C; (b) less than
c.   (a) -6.89×10^6 Nm^2/C; (b) equal to
d.   (a) 1.41×10^7 Nm^2/C;   (b) equal to

[19] A solid sphere of radius 40.0 cm has a total positive charge of 26.0μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field at 90.0 cm.

a. $(2.89 \times 10^5$ N/C)r
b. $(3.29 \times 10^6$ N/C)r
c. 0 N/C
d. $(1.46 \times 10^6$ N/C)r

[20] A charge of 190 μC is at the center of a cube of side 85.0 cm long. (a) Find the total flux through each face of the cube. (b) Find the flux through the whole surface of the cube.

a.   (a) 3.58×10^6 Nm^2/C;   (b) 2.15×10^7 Nm^2/C
b.   (a) 4.10×10^7 Nm^2/C;   (b) 4.10×10^7 Nm^2/C
c.   (a) 1.29×10^8 Nm^2/C;   (b) 2.15×10^7 Nm^2/C
d.   (a) 6.83×10^6 Nm^2/C;   (b) 4.10×10^7 Nm^2/C

---

[21] A 30.0 cm diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is found to be 3.20 x 105 Nm2/C. What is the electric field strength?

a. 3.40×10^5 N/C
b. 4.53×10^6 N/C
c. 1.13×10^6N/C
d. 1.70×10^5 N/C

---

[22] Consider a thin spherical shell of radius 22.0 cm with a total charge of 34.0μC distributed uniformly on its surface. Find the magnitude of the electric field (a) 15.0 cm and (b) 30.0 cm from the center of the charge distribution.

a.   (a) 6.32×10^6 N/C;   (b) 3.40×10^6 N/C
b.   (a) 0 N/C;   (b) 6.32×10^6 N/C
c.   (a) 1.36×10^7 N/C;   (b) 3.40×10^6 N/C
d.   (a) 0 N/C;   (b) 3.40×10^6 N/C

---

[23] A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field 100.0 cm from the axis of the rod, where distances area measured perpendicular to the rod.

a. $(1.08 \times 10^4$ N/C)r
b. $(2.70 \times 10^2$ N/C)r
c. $(5.39 \times 10^2$ N/C)r
d.   (0 N/C)r

[24] A solid conducting sphere of radius 2.00 cm has a charge of 8.00 μC. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge of -4.00 μC. Find the electric field at r = 7.00 cm from the center of this charge configuration.

a. $(2.20 \times 10^7 \text{ N/C})r$
b. $(4.32 \times 10^7 \text{ N/C})r$
c. $(7.34 \times 10^6 \text{ N/C})r$
d. $(1.44 \times 10^7 \text{ N/C})r$

[25] The electric field everywhere on the surface of a thin spherical shell of radius 0.650 m is measured to be equal to 790 N/C and points radially toward the center of the sphere. (a) What is the net charge within the sphere's surface? (b) What can you conclude about the nature and distribution of the charge inside the spherical shell?

a.   (a) 3.71×10^{-8}C; (b) The charge is negative, its distribution is spherically symmetric.
b.   (a) 3.71×10^{-8} C; (b) The charge is positive, its distribution is uncertain.
c.   (a) 1.93×10^{-4} C; (b) The charge is positive, its distribution is spherically symmetric.
d.   (a) 1.93×10^{-4} C; (b) The charge is negative, its distribution is uncertain.

[26] Four identical point charges (q = +16.0 μC) are located on the corners of a rectangle, as shown in Figure 6.
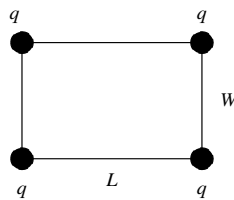
Figure 6

The dimensions of the rectangle are L 70.0 cm and W= 30.0 cm. Calculate the electric potential energy of the charge at the lower left corner due to the other three charges.

a. 14.9 J
b. 7.94 J
c. 14.0 J
d. 34.2 J

---

[27] The three charges in Figure 7 are at the vertices of an isosceles triangle.

q

4 cm

2 cm

-*q*   -*q*
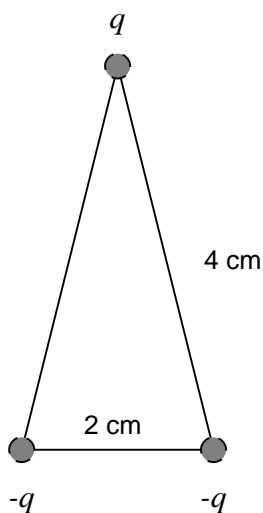
Figure 7

Calculate the electric potential at the midpoint of the base, taking *q*=7.00 μC.
a. -14.2 mV
b. 11.0 mV
c. 14.2 mV
d. -11.0mV

---

[28] An insulating rod having a linear charge density = 40.0 μC/m and linear mass density 0.100 kg/m is released from rest in a uniform electric field E=100 V/m directed perpendicular to the rod (Fig. 8).
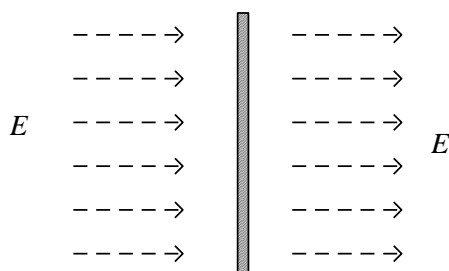
Figure 8

(a)   Determine the speed of the rod after it has traveled 2.00 m. (b) How does your answer to part (a) change if the electric field is not perpendicular to the rod?

a.   (a) 0.200 m/s; (b) decreases
b.   (a) 0.400 m/s; (b) the same
c.   (a) 0.400 m/s; (b) decreases
d.   (a) 0.200 m/s; (b) increases

[29] A spherical conductor has a radius of 14.0 cm and a charge of 26.0μC. Calculate the electric field and the electric potential at $r = 50.0$ cm from the center.

a. 9.35×10^5 N/C, 1.67 mV
b. 1.19×10^7 N/C, 0.468 mV
c. 9.35×10^5 N/C, 0.468 mV
d. 1.19×10^7 N/C, 1.67 mV

[30] How many electrons should be removed from an initially unchanged spherical conductor of radius 0.200 m to produce a potential of 6.50 kV at the surface?

a. 1.81×10^{11}
b. 2.38×10^{15}
c. 9.04×10^{11}
d. 1.06×10^{15}

[31] An ion accelerated through a potential difference of 125 V experiences an increase in kinetic energy of 9.37×10^{-17} J. Calculate the charge on the ion.

a. 1.33×10^{18} C
b. 7.50×10^{-19} C
c. 1.17×10^{-14} C
d. 1.60×10^{-19} C

[32] How much work is done (by a battery, generator, or some other source of electrical energy) in moving Avagadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is -5.00 V? (The potential in each case is measured relative to a common reference point.)

a. 0.482 MJ
b. 0.385 MJ
c. 1.35 MJ
d. 0.867 MJ

[33] At a certain distance from a point charge, the magnitude of the electric field is 600 V/m and the electric potential is -4.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

a.   (a) 0.150 m;   (b) 0.445 µC
b.   (a) 0.150 m;   (b) -1.50 µC
c.   (a) 6.67 m;   (b) 2.97 µC
d.   (a) 6.67 m;   (b) -2.97 µC

[34] An electron moving parallel to the x-axis has an initial speed of 3.70×10^6 m/s at the origin. Its speed is reduced to 1.40×10^5 m/s at the point x = 2.00 cm. Calculate the potential difference between the origin and that point. Which point is at the higher potential?

a.   -38.9 V, the origin
b.   19.5 V, x
c.   38.9 V, x
d.   -19.5 V, the origin

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Solution of the multiple choice questions

Q. No.	Answer	Q. No.	Answer
1	b	18	b
2	c	19	a
3	a	20	a
4	d	21	b
5	d	22	d
6	d	23	c
7	a	24	c
8	b	25	a
9	b	26	c
10	c	27	d
11	d	28	b
12	a	29	c
13	c	30	c
14	b	31	b
15	c	32	c
16	d	33	d
17	c	34	a

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Part 2

Applications of Electrostatic
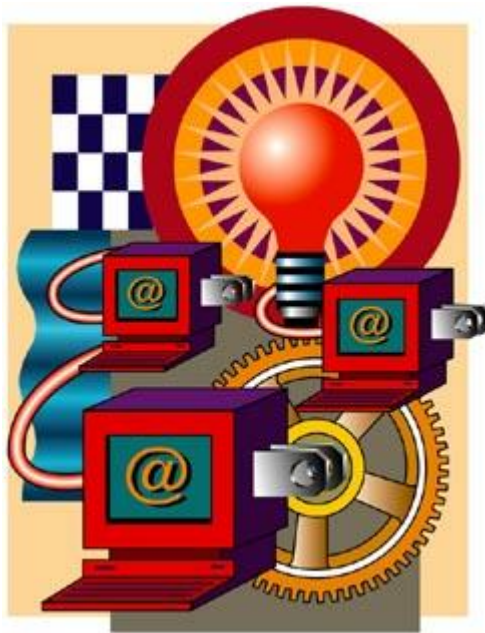
Chapter 6

Capacitors and Capacitance
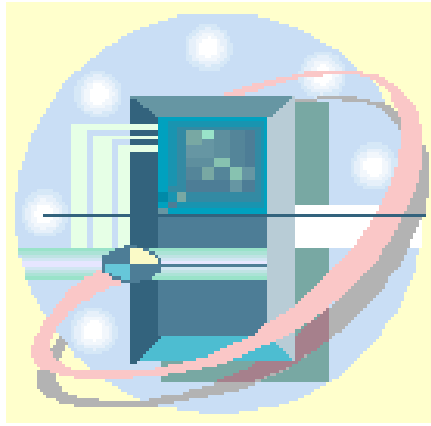
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Capacitors and Capacitance

Capacitors and Capacitance
المكثف الكهربي والسعة الكهربية

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يعتبر هذا الفصل تطبيقاً على المفاهيم الأساسية للكهربية الساكنة، حيث سنركز على التعرف على خصائص المكثفات Capacitors وهي من الأجهزة الكهربية التي لا تخلو منها أية دائرة كهربية. ويعد المكثف بمثابة مخزن للطاقة الكهربية. والمكثف عبارة عن موصلين يفصل بينهما مادة عازلة.

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6.1 Capacitor

A capacitor consists of two conductors separated by an insulator Figure 6.1. The capacitance of the capacitor depends on the geometry of the conductors and on the material separating the charged conductors, called dielectric that is an insulating material. The two conductors carry equal and opposite charge $+q$ and $-q$.
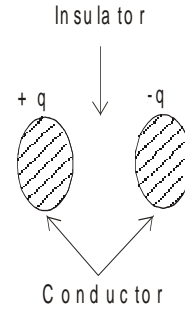
Figure 6.1

6.2 Definition of capacitance

The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them as shown in Figure 6.2.

$$C = \frac{q}{V} \qquad (6.1)$$

The capacitance C has a unit of C/v, which is called *farad F*

$$F = C/v$$

Figure 6.2

The farad is very big unit and hence we use submultiples of farad

$$1\mu F = 10^{-6}F$$

$$1nF = 10^{-9}F$$

$$1pF = 10^{-12}F$$

The capacitor in the circuit is represented by the symbol shown in Figure 6.3.
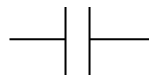
Figure 6.3
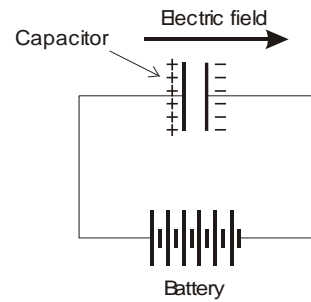
6.3 Calculation of capacitance

The most common type of capacitors are:-

- Parallel-plate capacitor
- Cylindrical capacitor
- Spherical capacitor

We are going to calculate the capacitance of parallel plate capacitor using the information we learned in the previous chapters and make use of the equation (6.1).

6.3.1 Parallel plate capacitor

Two parallel plates of equal area *A* are separated by distance *d* as shown in figure 6.4 bellow. One plate charged with +*q*, the other -*q*.
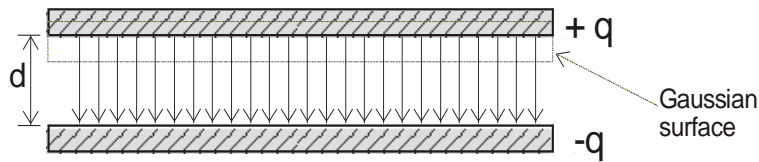
Figure 6.4

The capacitance is given by $C = \dfrac{q}{V}$

First we need to evaluate the electric field *E* to workout the potential *V*.

Using gauss law to find *E*, the charge per unit area on either plate is

$$\sigma = q/A. \tag{6.2}$$

$$\therefore E = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A} \tag{6.3), (4.9}$$

The potential difference between the plates is equal to *Ed*, therefore

$$V = Ed = \frac{qd}{\varepsilon_o A} \tag{6.4}$$

The capacitance is given by

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_o A} \qquad (6.5)$$

$$\therefore C = \frac{\varepsilon_o A}{d} \qquad (6.6)$$

Notice that the capacitance of the parallel plates capacitor is depends on the geometrical dimensions of the capacitor.

The capacitance is proportional to the area of the plates and inversely proportional to distance between the plates.

المعادلة (6.6) تمكننا من حساب سعة المكثف من خلال الأبعاد الهندسية له، حيث أن سعة المكثف تتناسب طردياً مع المساحة المشتركة بين اللوحين وعكسياً مع المسافة بين اللوحين.

Example 6.1

An air-filled capacitor consists of two plates, each with an area of 7.6cm², separated by a distance of 1.8mm. If a 20V potential difference is applied to these plates, calculate,

- **(a) the electric field between the plates,**
- **(b) the surface charge density,**
- **(c) the capacitance, and**
- **(d) the charge on each plate.**

Solution

(a) $E = \dfrac{V}{d} = \dfrac{20}{1.8\times10^{-3}} = 1.11\times10^4\, V/m$

(b) $\sigma = \varepsilon_o E = (8.85\times10^{-12})(1.11\times10^4) = 9.83\times10^{-8}\, C/m^2$

(c) $C = \dfrac{\varepsilon_o A}{d} = \dfrac{(8.85\times10^{-12})(7.6\times10^{-4})}{1.8\times10^{-3}} = 3.74\times10^{-12}\, F$

(d) $q = CV = (3.74\times10^{-12})(20) = 7.48\times10^{-11}\, C$

6.3.2 Cylindrical capacitor

In the same way we can calculate the capacitance of cylindrical capacitor, the result is as follow

$$C = \frac{2\pi\varepsilon_o l}{\ln(b/a)} \qquad (6.7)$$

Where l is the length of the cylinder, a is the radius of the inside cylinder, and b the radius of the outer shell cylinder.

6.3.3 Spherical Capacitor

In the same way we can calculate the capacitance of spherical capacitor, the result is as follow

$$C = \frac{4\pi\varepsilon_o ab}{b - a} \qquad (6.8)$$

Where a is the radius of the inside sphere, and b is the radius of the outer shell sphere.

Example 6.2

An air-filled spherical capacitor is constructed with inner and outer shell radii of 7 and 14cm, respectively. Calculate,
   (a) The capacitance of the device,
   (b) What potential difference between the spheres will result in a charge of 4µC on each conductor?

Solution

(a) $C = \dfrac{4\pi\varepsilon_o ab}{b - a} = \dfrac{(4\pi \times 8.85 \times 10^{-12})(0.07)(0.14)}{(0.14 - 0.07)} = 1.56 \times 10^{-11} F$

(b) $V = \dfrac{q}{C} = \dfrac{4 \times 10^{-6}}{1.56 \times 10^{-11}} = 2.56 \times 10^{5} V$

6.4 Combination of capacitors

Some times the electric circuit consist of more than two capacitors, which are, connected either in parallel or in series the equivalent capacitance is evaluated as follow

6.4.1 Capacitors in parallel:

In parallel connection the capacitors are connected as shown in figure 6.5 below where the above plates are connected together with the positive terminal of the battery, and the bottom plates are connected to the negative terminal of the battery.
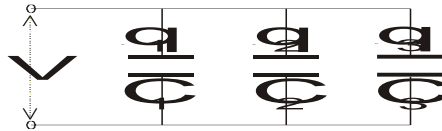
Figure 6.5

In this case the potential different across each capacitor is equal to the voltage of the battery V

$$i.e. \ V=V_1=V_2=V_3$$

The charge on each capacitor is

$$q_1 = C_1V_1; \qquad q_2 = C_2V_2; \qquad q_3 = C_3V_3$$

The total charge is

$$q = q_1 + q_2 + q_3$$
$$q = (C_1 + C_2 + C_3)V$$

$$\Theta \ C = \frac{q}{V}$$

في حالـة توصيـل المكثفـات علـى التـوازي يكـون فـرق الجهد على كل مكثف مساوياً لفـرق جهـد البطاريـة، أمـا الشحنة فتتوزع بنسبة سعة كل مكثف.

The Equivalent capacitance is

$$C = C_1 + C_2 + C_3 \qquad\qquad (6.9)$$

6.4.2 Capacitors in series:

In series connection the capacitors are connected as shown in figure 6.6 <mark>below where the above plates are connected together with the positive</mark>
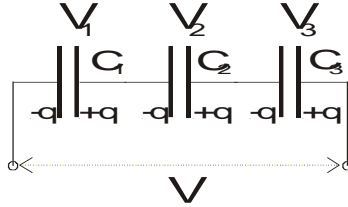
Figure 6.6

In this case the magnitude of the charge must be the same on each plate with opposite sign

$$i.e.\ q=q_1=q_2=q_3$$

The potential across each capacitor is

$$V_1 = q/C_1; \qquad V_2 = q/C_2; \qquad V_3 = q/C_3$$

The total potential V is equal the sum of the potential across each capacitor

$$V = V_1 + V_2 + V_3$$

$$V = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$C = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

في حالة توصيل المكثفات على التـوالي فـإن الشـحنة تتـوزع على كل مكثف بشكل متسـاو وتسـاوي الشـحنة الكليـة. أمـا مجموع فروق الجهد علـى كـل مكثـف يسـاوي فـرق جهـد البطارية.

The Equivalent capacitance is

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$(6.10)$$

Example 6.3

Find the equivalent capacitance between points a and b for the group of capacitors shown in figure 6.7 . $C_1=1\mu F$, $C_2=2\mu F$, $C_3=3\mu F$, $C_4=4\mu F$, $C_5=5\mu F$, and $C_6=6\mu F$.

(i)
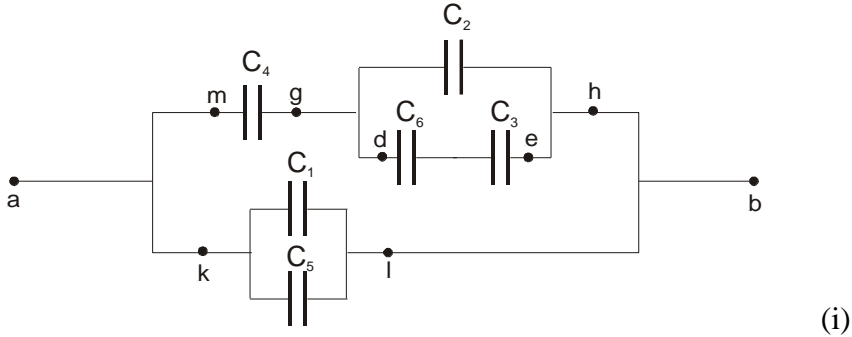
Figure 6.7

Solution

First the capacitor C_3 and C_6 are connected in series so that the equivalent capacitance C_{de} is

$$\frac{1}{C_{de}} = \frac{1}{6} + \frac{1}{3}; \Rightarrow C_{de} = 2\mu F$$

Second C_1 and C_5 are connected in parallel

$C_{kl}=1+5=6\mu F$

The circuit become as shown below

(ii)
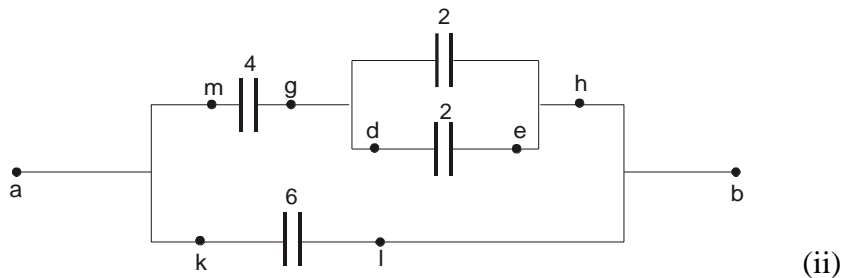
Continue with the same way to reduce the circuit for the capacitor C_2 and C_{de} to get $C_{gh}=4\mu F$

(iii)
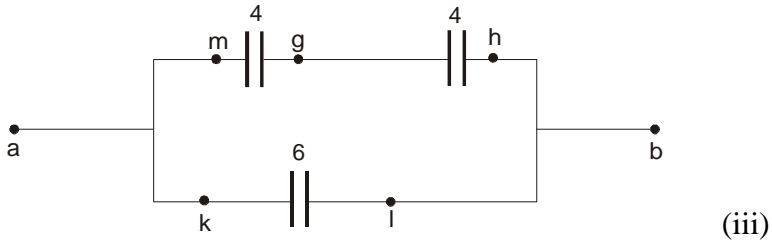
Capacitors C_{mg} and C_{gh} are connected in series the result is $C_{mh}=2\mu F$, The circuit become as shown below

(iv)
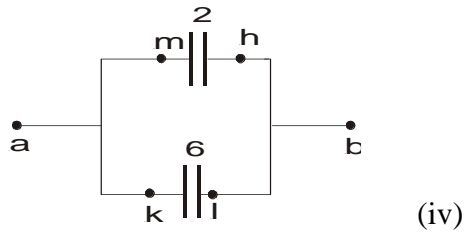
Capacitors C_{mh} and C_{kl} are connected in parallel the result is

(v)
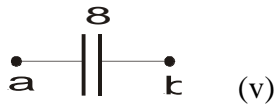
$C_{eq}=8\mu F$.

Example 6.4

In the above example 6.3 determine the potential difference across each capacitor and the charge on each capacitor if the total charge on all the six capacitors is 384µC.

Solution

First consider the equivalent capacitor C_{eq} to find the potential between points a and b (V_{ab})

$$V_{ab} = \frac{Q_{ab}}{C_{ab}} = \frac{384}{8} = 48V$$

Second notice that the potential $V_{kl}=V_{ab}$ since the two capacitors between k and l are in parallel, the potential across the capacitors C_1 and $C_5 = 48V$.

$V_1=48V$ and $Q_1=C_1V_1=48µC$

And for $C5$

$V_5=48V$ and $Q_5=C_5V_5=240µC$

For the circuit (iv) notice that $V_{mh}=V_{ab}=48V$, and

$Q_{mh}=C_{mh}V_{mh}=2\times28=96µC$

Since the two capacitors shown in the circuit (iii) between points m and h are in series, each will have the same charge as that of the equivalent capacitor, *i.e.*

$Q_{mh}=Q_{gh}=Q_{mh}=96µC$

$$V_{mg} = \frac{Q_{mg}}{C_{mg}} = \frac{96}{4} = 24V$$

$$V_{gh} = \frac{Q_{gh}}{C_{gh}} = \frac{96}{4} = 24V$$

Therefore for C_4, $V_4=24$ and $Q_4=96µC$

In the circuit (ii) the two capacitor between points g and h are in parallel so the potential difference across each is 24V.

Therefore for C_2, V_2=24V and Q_2=C_2V_2=48μC

Also in circuit (ii) the potential difference

V_{de}=V_{gh}=24V

And

Q_{de}=$C_{de}V_{de}$=2×24=48μC

The two capacitors shown in circuit (i) between points *d* and *a* are in series, and therefore the charge on each is equal to Q_{de}.

Therefore for C_6, Q_6=48μC

$$V_6 = \frac{Q_6}{C_6} = 8V$$

For C_3, Q_3=48μC and V_3=Q_3/C_3=16V

The results can be summarized as follow:

Capacitor	Potential Difference (V)	Charge (μC)
C_1	48	48
C_2	24	48
C_3	16	48
C_4	24	96
C_5	48	240
C_6	8	48
C_{eq}	48	384

4.5 Energy stored in a charged capacitor (in electric field)

If the capacitor is connected to a power supply such as battery, charge will be transferred from the battery to the plates of the capacitor. This is a charging process of the capacitor which mean that the battery perform a work to store energy between the plates of the capacitor.

Consider uncharged capacitor is connected to a battery as shown in figure 6.8, at start the potential across the plates is zero and the charge is zero as well.
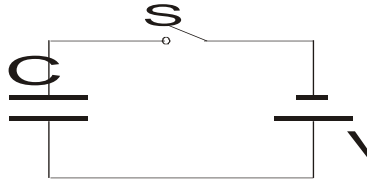
Figure 6.8

If the switch *S* is closed then the charging process will start and the potential across the capacitor will rise to reach the value equal the potential of the battery *V* in time *t* (called charging time).

بعد إغلاق المفتح S تستمر عملية شحن المكثف حتى يصبح فرق الجهد بين لوحي المكثف مساوياً لفرق جهد البطارية.

Suppose that at a time t a charge $q(t)$ has been transferred from the battery to capacitor. The potential difference $V(t)$ across the capacitor will be $q(t)/C$. For the battery to transferred another amount of charge dq it will perform a work dW

$$dW = Vdq = \frac{q}{C}dq \qquad (6.11)$$

The total work required to put a total charge Q on the capacitor is

$$W = \int dW = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C} \qquad (6.12)$$

Using the equation $q=CV$

$$W = U = \frac{Q^2}{2C} \tag{6.13}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \tag{6.14}$$

The energy per unit volume u (energy density) in parallel plate capacitor is the total energy stored U divided by the volume between the plates Ad

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} \tag{6.15}$$

For parallel plate capacitor $C = \frac{\varepsilon_o A}{d}$

$$u = \frac{\varepsilon_o}{2}\left(\frac{V}{d}\right)^2 \tag{6.16}$$

$$u = \frac{1}{2}\varepsilon_o E^2 \tag{6.17}$$

Therefore the electric energy density is proportional with square of the electric field.

لاحظ هنا أن الطاقة الكهربية المخزنة بين لوحي المكثف يمكن التعبير عنها باستخدام الطاقة الكلية U أو من خلال كثافة الطاقة u. الطاقة الكلية تساوي كثافة الطاقة في الحجم المحصور بين لوحي المكثف.

المعادلتان رقم (6.14)&(6.17) توضحان عنوان هذا الموضوع وهو الطاقة المخزنة في المكثف أو في المجال الكهربي.

Example 6.5

Three capacitors of 8µF, 10µF and 14µF are connected to a battery of 12V. How much energy does the battery supply if the capacitors are connected (a) in series and (b) in parallel?

Solution

(a) For series combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{8} + \frac{1}{10} + \frac{1}{14}$$

This gives

$$C = 3.37 \ \mu F$$

Then the energy U is

$$U = \frac{1}{2}CV^2$$

$$U = 1/2 \ (3.37 \times 10^{-6}) \ (12)^2 = 2.43 \times 10^{-4} J$$

(b) For parallel combination

$$C = C_1 + C_2 + C_3$$

$$C = 8+10+14 = 32\mu F$$

The energy U is

$$U = 1/2 \ (32 \times 10^{-6}) \ (12)^2 = 2.3 \times 10^{-3} J$$

Example 6.6

A capacitor C_1 is charged to a potential difference V_0. This charging battery is then removed and the capacitor is connected as shown in figure 6.9 to an uncharged capacitor C_2,
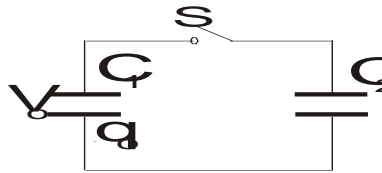
Figure 6.9

(a) What is the final potential difference V_f across the combination?

(b) What is the stored energy before and after the switch S is closed?

Solution

(a) The original charge q_o is shared between the two capacitors since they are connected in parallel. Thus

$$q_o = q_1 + q_2$$

$$q = CV$$

$$C_1 V_o = C_1 V_f + C_2 V_f$$

$$V_f = V_o \frac{C_1}{C_1 + C_2}$$

(b) The initial stored energy is U_o

$$U_o = \tfrac{1}{2} C_1 V_o^{\,2}$$

The final stored energy $U_f = U_1 + U_2$

$$U_f = \tfrac{1}{2} C_1 V_f^{\,2} + \tfrac{1}{2} C_2 V_f^{\,2} = \tfrac{1}{2}(C_1 + C_2)\left(\frac{V_o C_1}{C_1 + C_2} \right)^2$$

$$U_f = \left(\frac{C_1}{C_1 + C_2} \right) U_o$$

Notice that U_f is less than U_o (Explain why)

Example 6.7

Consider the circuit shown in figure 6.10 where
C_1=6μF, C_2=3μF, and V=20V. C_1 is first charged
by closing switch S_1. S_1 is then opened, and the
charged capacitor C_1 is connected to the
uncharged capacitor C_2 by closing the switch S_2.
Calculate the initial charge acquired by C_1 and
the final charge on each of the two capacitors.
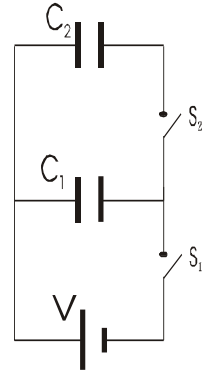
Figure 6.10

Solution

When S_1 is closed, the charge on C_1 will be

$$Q_1=C_1V_1=6μF \times 20V=120μC$$

When S_1 is opened and S_2 is closed, the total charge will remain constant and be distributed among the two capacitors,

$$Q_1=120μC-Q_2$$

The potential across the two capacitors will be equal,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{120\mu F - Q_2}{6\mu F} = \frac{Q_2}{3\mu F}$$

Therefore,

$$Q_2 = 40μC$$

$$Q_1=120μC-40μC=80μC$$

Example 6.8

Consider the circuit shown in figure 6.11 where $C_1=4\mu F$, $C_2=6\mu F$, $C_3=2\mu F$, and $V=35V$. C_1 is first charged by closing switch S to point 1. S is then connected to point 2 in the circuit.
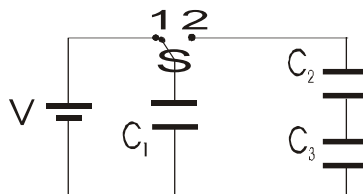
Figure 6.11

(a) Calculate the initial charge acquired by C_1,

(b) Calculate the final charge on each of the three capacitors.

(c) Calculate the potential difference across each capacitor after the switch is connected to point 2.

Solution

When switch S is connected to point 1, the potential difference on C_1 is 35V. Hence the charge Q_1 is given by

$$Q_1 = C_1 \text{x} V = 4 \text{x} 35 = 140\mu C$$

When switch S is connected to point 2, the charge on C_1 will be distributed among the three capacitors. Notice that C_2 and C_3 are connected in series, therefore

$$\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{6} + \frac{1}{2} = \frac{4}{6}$$

$$C' = 1.5\mu F$$

We know that the charges are distributed equally on capacitor connected in series, but the charges are distributed with respect to their capacitance when they are connected in parallel. Therefore,

$$Q_1 = \frac{140}{4+1.5} \times 4 = 101.8\mu C$$

But the charge Q' on the capacitor C' is

$$Q' = 140 - 101.8 = 38.2\mu C$$

Since C_1 and C_2 are connected in series then

$$Q_2=Q_3=Q'=38.2\mu C$$

To find the potential difference on each capacitor we use the relation $V=Q/C$

Then,

$$V_1=25.45V$$
$$V_2=6.37V$$
$$V_3=19.1V$$

Example 6.9

Consider the circuit shown in figure 6.12 where $C_1=6\mu F$, $C_2=4\mu F$, $C_3=12\mu F$, and $V=12V$.
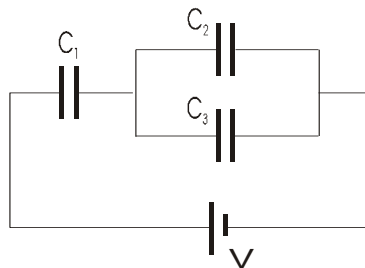
Figure 6.12

(a) **Calculate the equivalent capacitance,**

(b) **Calculate the potential difference across each capacitor.**

(c) **Calculate the charge on each of the three capacitors.**

Solution

C_2 and C_3 are connected in parallel, therefore

$$C' = C_2 + C_3 = 4 + 12 = 16\mu F$$

Now C' is connected in series with C_1, therefore the equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_1} = \frac{1}{6} + \frac{1}{16} = \frac{11}{48}$$

$$C = 4.36\mu F$$

The total charge $Q = CV = 4.36 \times 12 = 52.36\mu C$

The charge will be equally distributed on the capacitor C_1 and C'

$$Q_1 = Q' = Q = 52.36\mu C$$

But $Q' = C'V'$, therefore

$$V' = 52.36/16 = 3.27 \text{ volts}$$

The potential difference on C_1 is

$$V_1 = 12 - 3.27 = 8.73 \text{volts}$$

The potential difference on both C_2 and C_3 is equivalent to V' since they are connected in parallel.

$$V_2 = V_3 = 3.27 \text{volts}$$

$$Q_2 = C_2 V_2 = 13.08\mu C$$

$$Q_3 = C_3 V_3 = 39.24\mu C$$

Example 6.9

Four capacitors are connected as shown in Figure 6.13. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if $V_{ab}=15V$.
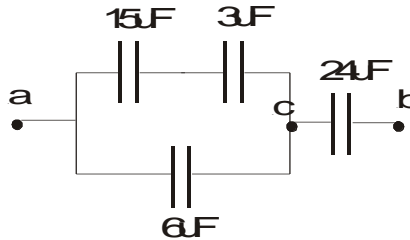
Figure 6.13

Solution

(a) We simplify the circuit as shown in the figure from (a) to (c).

(a)   (b)   (c)
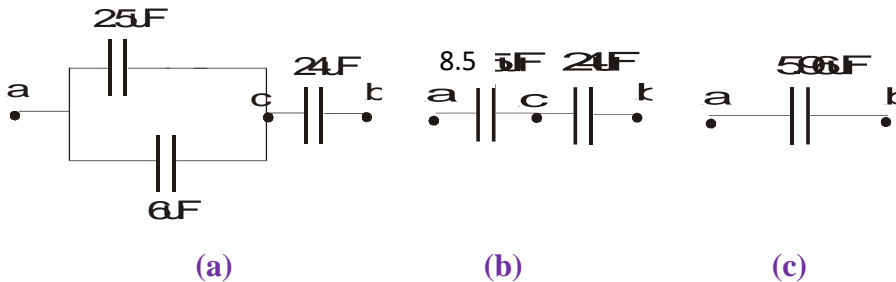
Firs the 15μF and 3μF in series are equivalent to

$$\frac{1}{(1/15)+(1/3)} = 2.5\,\mu F$$

Next 2.5μF combines in parallel with 6μF, creating an equivalent capacitance of 8.5μF.

The 8.5μF and 20μF are in series, equivalent to

$$\frac{1}{(1/8.5)+(1/20)} = 5.96\,\mu F$$

(b) We find the charge and the voltage across each capacitor by working backwards through solution figures (c) through (a).

For the 5.96μF capacitor we have

$$Q = CV = 5.96 \times 15 = 89.5 \mu C$$

In figure (b) we have, for the 8.5μF capacitor,

$$\Delta V_{ac} = \frac{Q}{C} = \frac{89.5}{8.5} = 10.5V$$

and for the 20μF in figure (b) and (a) $Q_{20} = 89.5 \mu C$

$$\Delta V_{cb} = \frac{Q}{C} = \frac{89.5}{20} = 4.47V$$

Next (a) is equivalent to (b), so $\Delta V_{cb} = 4.47V$ and $\Delta V_{ac} = 10.5V$

Thus for the 2.5μF and 6μF capacitors $\Delta V = 10.5V$

$$Q_{2.5} = CV = 2.5 \times 10.5 = 26.3 \mu C$$

$$Q_6 = CV = 6 \times 10.5 = 63.2 \mu C$$

Therefore

$$Q_{15} = 26.3 \mu C \qquad Q_3 = 26.3 \mu C$$

For the potential difference across the capacitors C_{15} and C_3 are

$$\Delta V_{15} = \frac{Q}{C} = \frac{26.3}{15} = 1.75V$$

$$\Delta V_3 = \frac{Q}{C} = \frac{26.3}{3} = 8.77V$$

6.6 Capacitor with dielectric

A dielectric is a non-conducting material, such as rubber, glass or paper. Experimentally it was found that the capacitance of a capacitor increased when a dielectric material was inserted in the space between the plates. The ratio of the capacitance with the dielectric to that without it called the dielectric constant κ of the material.

$$\kappa = \frac{C_d}{C_o} \qquad (6.18)$$

In figure 6.14 below two similar capacitors, one of them is filled with dielectric material, and both are connected in parallel to a battery of potential *V*. It was found that the charge on the capacitor with dielectric is larger than the on the air filled capacitor, therefore the $C_d > C_o$, since the potential *V* is the same on both capacitors.
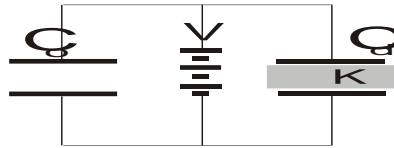
Figure 6.14

If the experiment repeated in different way by placing the same charge Q_o on both capacitors as shown in figure 6.15. Experimentally it was shown that $V_d < V_o$ by a factor of $1/\kappa$.
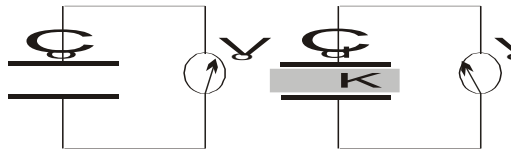
Figure 6.15

$$V_d = \frac{V_o}{\kappa} \qquad (6.19)$$

Since the charge Q_o on the capacitors does not change, then

$$C = \frac{Q_o}{V_d} = \frac{Q_o}{V_o/\kappa} = \kappa \frac{Q_o}{V_o} \qquad (6.20)$$

For a parallel plate capacitor with dielectric we can write the capacitance.

$$C = \kappa \frac{\varepsilon_o A}{d} \qquad (6.21)$$

Example 6.10

A parallel plate capacitor of area *A* and separation *d* is connected to a battery to charge the capacitor to potential difference V_o. Calculate the stored energy before and after introducing a dielectric material.

Solution

The energy stored before introducing the dielectric material,

$$U_o = \tfrac{1}{2} C_o V_o^{\,2}$$

The energy stored after introducing the dielectric material,

$$C = \kappa C_o \qquad \text{and} \qquad V_d = \frac{V_o}{\kappa}$$

$$U = \tfrac{1}{2} C V^2 = \tfrac{1}{2} \kappa C_o \left(\frac{V_o}{\kappa} \right)^2 = \frac{U_o}{\kappa}$$

Therefore, the energy is less by a factor of $1/\kappa$.

Example 6.11

A Parallel plate capacitor of area 0.64cm². When the plates are in vacuum, the capacitance of the capacitor is 4.9pF.

    (a) Calculate the value of the capacitance if the space between the plates is filled with nylon (κ=3.4).

    (b) What is the maximum potential difference that can be applied to the plates without causing discharge (E_{max}=14×10⁶V/m)?

Solution

(a) $C = \kappa C_o = 3.4 \times 4.9 = 16.7 \text{pF}$

(b) $V_{max} = E_{max} \times d$

To evaluate d we use the equation

$$d = \frac{\varepsilon_o A}{C_o} = \frac{8.85 \times 10^{-12} \times 6.4 \times 10^{-5}}{4.9 \times 10^{-12}} = 1.16 \times 10^{-4} m$$

$V_{max} = 1 \times 10^6 \times 1.16 \times 10^{-4} = 1.62 \times 10^3$ V

Example 6.12

A parallel-plate capacitor has a capacitance C_o in the absence of dielectric. A slab of dielectric material of dielectric constant κ and thickness $d/3$ is inserted between the plates as shown in Figure 6.16. What is the new capacitance when the dielectric is present?

1/3 d

2/3 d

d

K
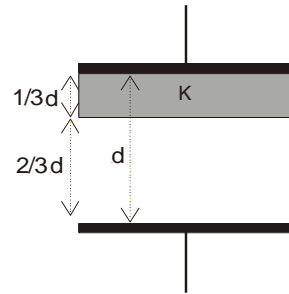
Figure 6.16

Solution

We can assume that two parallel plate capacitor are connected in series as shown in figure 6.17,

$$C_1 = \frac{\kappa \varepsilon_o A}{d/3} \qquad \text{and} \qquad C_2 = \frac{\varepsilon_o A}{2d/3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{\kappa \varepsilon_o A} + \frac{2d/3}{\varepsilon_o A}$$

Figure 6.17

$$\frac{1}{C} = \frac{d}{3\varepsilon_o A}\left(\frac{1}{\kappa} + 2\right) = \frac{d}{3\varepsilon_o A}\left(\frac{1+2\kappa}{\kappa}\right)$$

$$C = \left(\frac{3\kappa}{2\kappa+1}\right)\frac{\varepsilon_o A}{d} \qquad \Rightarrow \qquad C = \left(\frac{3\kappa}{2\kappa+1}\right)C_o$$

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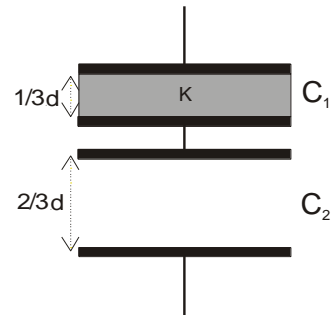
6.7 Problems

6.1) Two capacitors, $C_1 = 2\mu F$ and $C_2 = 16\mu F$, are connected in parallel. What is the value of the equivalent capacitance of the combination?

6.2) Calculate the equivalent capacitance of the two capacitors in the previous exercise if they are connected in series.

6.3) A 100pF capacitor is charged to a potential difference of 50V, the charging battery then being disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the measured potential difference drops to 35V, what is the capacitance of this second capacitor?

6.4) A parallel-plate capacitor has circular plates of 8.0cm radius and 1.0mm separation. What charge will appear on the plates if a potential difference of 100V is applied?

6.5) In figure 6.18 the battery supplies 12V. (a) Find the charge on each capacitor when switch S_1 is closed, and (b) when later switch S_2 is also closed. Assume $C_1 = 1\mu F$, $C_2 = 2\mu F$, $C_3 = 3\mu F$, and $C_4 = 4\mu F$.
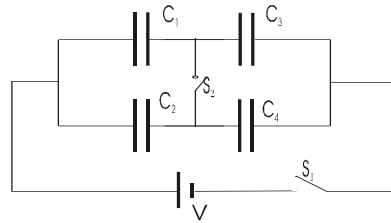
Figure 6.18

6.6) A parallel plate capacitor has a plate of area A and separation d, and is charged to a potential difference V. The charging battery is then disconnected and the plates are pulled apart until their separation is $2d$. Derive expression in term of A, d, and V for, the new potential difference, the initial and final stored energy, and the work required to separate the plates.

6.7) A 6.0μF capacitor is connected in series with a 4.0μF capacitor and a potential difference of 200 V is applied across the pair. (a) What is the charge on each capacitor? (b) What is the potential difference across each capacitor?

6.8) Repeat the previous problem for the same two capacitors connected in parallel.

6.9) Show that the plates of a parallel-plate capacitor attract each other with a force given by

$$F = \frac{q^2}{2\varepsilon_o A}$$

6.10) A parallel-plate air capacitor having area A (40cm^2) and spacing d (1.0 mm) is charged to a potential V (600V). Find (a) the capacitance, (b) the magnitude of the charge on each plate, (c) the stored energy, (d) the electric field between the plates and (e) the energy density between the plates.

6.11) How many 1µF capacitors would need to be connected in parallel in order to store a charge 1C with potential of 300V across the capacitors?

6.12) In figure 6.19 (a)&(b) find the equivalent capacitance of the combination. Assume that C_1=10µF, C_2=5µF, and C_3=4µF.
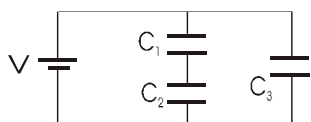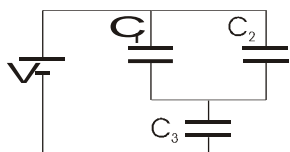
Figure 6.19(a)

Figure6.19(b)

6.13) Two capacitors (2.0µF and 4.0µF) are connected in parallel across a 300V potential difference.

Calculate the total stored energy in the system.

6.14) A 16pF parallel-plate capacitor is charged by a 10V battery. If each plate of the capacitor has an area of 5cm^2, what is the energy stored in the capacitor? What is the energy density (energy per unit volume) in the electric field of the capacitor if the plates are separated by air?

6.15) The energy density in a parallel-plate capacitor is given as 2.1×10^{-9} J/m^3. What is the value of the electric field in the region between the plates?

6.16) (a) Determine the equivalent capacitance for the capacitors shown in figure 6.20. (b) If they are connected to 12V battery, calculate the potential difference across each capacitor and the charge on each capacitor
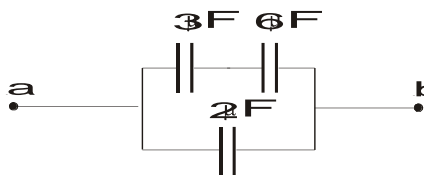
Figure 6.20

6.17) Evaluate the effective capacitance of the configuration shown in Figure 6.21. Each of the capacitors is identical and has capacitance C.

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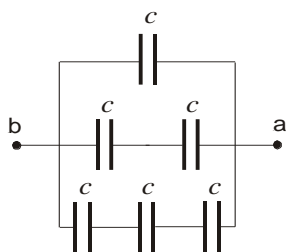
Figure 6.21

Figure 6.22

6.18) A parallel plate capacitor is constructed using a dielectric material whose dielectric constant is 3 an whose dielectric strength is 2×10^8V/m The desired capacitance is 0.25µF, and the capacitor must withstand a maximum potential difference of 4000V. Find the maximum area of the capacitor plate.

6.19) In figure 6.19(b) find (a) the charge, (b) the potential difference, (c) the stored energy for each capacitor. With V=100V.

6.20) (a) Figure 6.22 shows a network of capacitors between the terminals *a* and *b*. Reduce this network to a single equivalent capacitor. (b) Determine the charge on the 4µF and 8µF capacitors when the capacitors are fully charged by a 12V battery connected to the terminals. (c) Determine the potential difference across each capacitor.
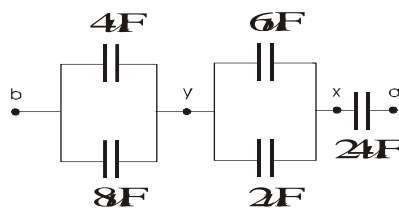
6.21) A uniform electric field $E=3000$V/m exists within a certain region. What volume of space would contain an energy equal to 10^{-7}J? Express your answer in cubic meters and in litters.

6.22) A capacitor is constructed from two square metal plates of side length *L* and separated by a distance *d* (Figure 6.23). One half of the space between the plates (top to bottom) is filled with polystyrene (κ=2.56), and the other half is filled with neoprene rubber (κ=6.7). Calculate the capacitance of the device, taking *L*=2cm and *d*=0.75mm. (Hint: The capacitor can be considered as two capacitors connected in parallel.)
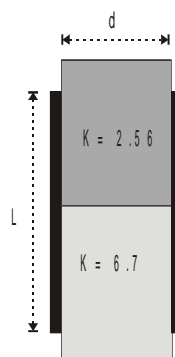
Figure 6.23

6.23) A parallel plate capacitor is constructed using three different dielectric materials, as shown in figure 6.24. (a) Find an expression for the capacitance in terms of the plate area A and κ_1, κ_2, and κ_3. (b) Calculate the capacitance using the value A=1cm^2, d=2mm, κ_1=4.9, κ_2=5.6, and κ_3=2.1.
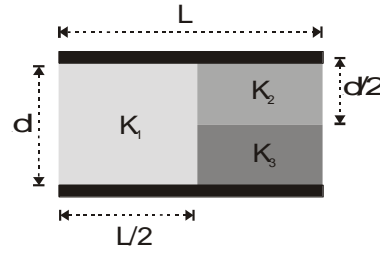
Figure 6.24

Chapter 7

Current and Resistance

التيار والمقاومـة

Current and Resistance

Current and Resistance

التيار والمقاومـة

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درسنا في الفصول السابقة بعض الظواهر الكهربية المتعلقة بالشحنة الساكنة، وفي هذا الفصل سنركز دراستنا على الشحنات الكهربية في حالة حركة أي "تيار كهربي". حيث نتعامل في حياتنا العملية مع العديد من الأجهزة الكهربية التي تعمل من خلال مرور شحنات كهربية فيها مثل البطارية والضوء وغيرها من الأمثلة الأخرى. ويجب أن نميز بين نوعين من التيار الكهربي وهما التيار الثابت والتيار المتردد، وفي هذا المقرر سنركز على التيار الثابت.

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7.1 Current and current density

درسنا في الفصل السابق تأثير فرق الجهد الصادر من بطارية على المكثف الكهربي حيث تتراكم الشحنات الموجبة على اللوح المتصل بالقطب الموجب والشحنات السالبة على اللوح المتصل بالقطب السالب للبطارية، وهذا أدى إلى تكون مجال كهربي في الفراغ بين لوحي المكثف. وعرفنا السعة C من خلال المعادلة

$$C = \frac{q}{V}$$

سنقوم هنا بتطبيق فرق جهد كهربي صادر من بطارية كهربية على طرفي موصل كهربي مثل سلك من النحاس مساحة مقطعة A. وسنتعرف على ظواهر فيزيائية جديدة مثل التيار الكهربي والمقاومة.
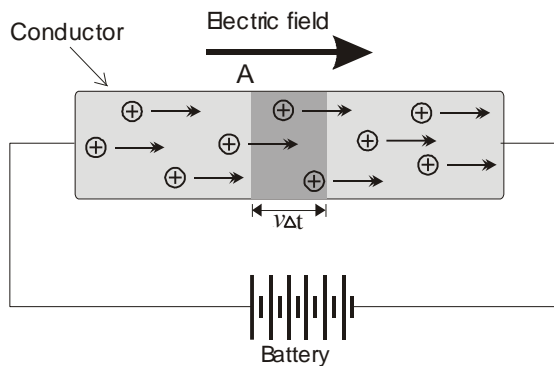
Figure 7.1

As shown in figure 7.1 above the electric field produces electric force (*F=qE*), this force leads the free charge in the conductor to move in one direction with an average velocity called ***drift velocity***.

The current is defined by the net charge flowing across the area *A* per unit time. Thus if a net charge ΔQ flow across a certain area in time interval Δt, the average current I_{av} across this area is

$$I_{av} = \frac{\Delta Q}{\Delta t} \qquad (7.1)$$

In general the current *I* is

$$I = \frac{dQ}{dt} \qquad (7.2)$$

Current is a *scalar* quantity and has a unit of *C/t*, which is called *ampere*.

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يحدد اتجاه التيار الكهربي في الدائرة الكهربية باتجاه التيار الاصطلاحي وهو اتجاه حركة الشحنات الموجبة في الدائرة والذي يكون من القطب الموجب إلى القطب السالب عبر الدائرة.

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7.2 Definition of current in terms of the drift velocity

Consider figure 7.1 shown above. Suppose there are *n* positive charge particle per unit volume moves in the direction of the field from the left to the right, all move in drift velocity *v*. In time Δt each particle moves distance $v\Delta t$ the shaded area in the figure, The volume of the shaded area in the figure is equal $Av\Delta t$, the charge ΔQ flowing across the end of the cylinder in time Δt is

$$\Delta Q = nqvA\Delta t \qquad (7.3)$$

where *q* is the charge of each particle.

Then the current *I* is

$$I = \frac{\Delta Q}{\Delta t} = nqvA \qquad (7.4)$$

7.3 Definition of the current density

The current per unit cross-section area is called the current density *J*.

$$\overset{\rho}{J} = \frac{I}{A} = nqv \qquad (7.5)$$

The current density is a vector quantity.

Example 7.1

A copper conductor of square cross section 1mm² on a side carries a constant current of 20A. The density of free electrons is 8×10^{28} electron per cubic meter. Find the current density and the drift velocity.

Solution

The current density is

$$J = \frac{I}{A} = 20 \times 10^6 \, A/m^2$$

The drift velocity is

$$v = \frac{J}{nq} = \frac{20 \times 10^6}{(8 \times 10^{28})(1.6 \times 10^{-9})} = 1.6 \times 10^{-3} \, m/s$$

This drift velocity is very small compare with the velocity of propagation of current pulse, which is $3 \times 10^8 m/s$. The smaller value of the drift velocity is due to the collisions with atoms in the conductor.

7.4 Resistance and resistivity (Ohm's Law)

The resistance R of a conductor is defined as the ratio V/I, where V is the potential difference across the conductor and I is the current flowing in it. Thus if the same potential difference V is applied to two conductors A and B, and a smaller current I flows in A, then the resistance of A is grater than B, therefore we write,

$$R = \frac{V}{I} \qquad \text{Ohm's law} \qquad (7.6)$$

This equation is known as Ohm's law, which show that a linear relationship between the potential difference and the current flowing in the conductor. Any conductor shows the lineal behavior its resistance is called *ohmic* resistance.

The resistance R has a unit of volt/ampere (*v/A*), which is called *Ohm* (Ω).

From the above equation, it also follows that

$$V = IR \qquad \text{and} \qquad I = \frac{V}{R}$$

The resistance in the circuit is drown using this symbol
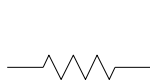
| Fixed resistor | Variable resistor | Potential divider |

Each material has different resistance; therefore it is better to use the resistivity ρ, it is defined from

$$\rho = \frac{E}{J} \qquad (7.8)$$

The resistivity has unit of $\Omega.m$

The inverse of resistivity is known as the conductivity σ,

$$\sigma = \frac{1}{\rho} \qquad (7.9)$$

7.5 Evaluation of the resistance of a conductor

Consider a cylindrical conductor as shown in figure 7.2, of cross-sectional area A and length λ, carrying a current I. If a potential difference V is connected to the ends the conductor, the electric field and the current density will have the values
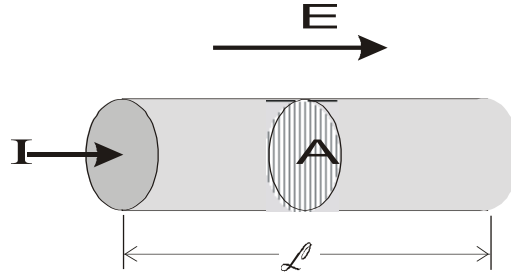
Figure 7.2

$$E = \frac{V}{\lambda} \tag{7.10}$$

and

$$J = \frac{I}{A}$$

The resistivity ρ is

$$\rho = \frac{E}{J} = \frac{V/I}{I/A} \tag{7.11}$$

But the *V/I* is the resistance R this leads to,

$$R = \rho \frac{\lambda}{A} \tag{7.12}$$

Therefore, the resistance R is proportional to the length λ of the conductor and inversely proportional the cross-sectional area A of it.

Notice that the resistance of a conductor depends on the geometry of the conductor, and the resistivity of the conductor depends only on the electronic structure of the material.

Example 7.2

Calculate the resistance of a piece of aluminum that is 20cm long and has a cross-sectional area of $10^{-4}m^2$. What is the resistance of a piece of glass with the same dimensions? $\rho_{Al}=2.82\times10^{-8}\Omega.m$, $\rho_{glass}=10^{10}\Omega.m$.

Solution

The resistance of aluminum

$$R_{Al} = \rho\frac{\lambda}{A} = 2.82\times10^{-8}\left(\frac{0.1}{10^{-4}}\right) = 2.82\times10^{-5}\Omega$$

The resistance of glass

$$R_{glass} = \rho\frac{\lambda}{A} = 10^{10}\left(\frac{0.1}{10^{-4}}\right) = 10^{13}\Omega$$

Notice that the resistance of aluminum is much smaller than glass.

Example 7.3

A 0.90V potential difference is maintained across a 1.5m length of tungsten wire that has a cross-sectional area of 0.60mm². What is the current in the wire?

Solution

From Ohm's law

$$I = \frac{V}{R} \qquad \text{where} \quad R = \rho\frac{\lambda}{A}$$

therefore,

$$I = \frac{VA}{\rho\lambda} = \frac{(0.90)(6.0\times10^{-7})}{(5.6\times10^{-8})(1.5)} = 6.43A$$

Example 7.4

(a) Calculate the resistance per unit length of a 22 nichrome wire of radius 0.321mm. (b) If a potential difference of 10V is maintained cross a1m length of nichrome wire, what is the current in the wire. $\rho_{nichromes} = 1.5 \times 10^{-6} \Omega.m$.

Solution

(a) The cross sectional area of the wire is

$$A = \pi r^2 = \pi (0.321 \times 10^{-3})^2 = 3.24 \times 10^{-7} m^2$$

The resistance per unit length is R/λ

$$\frac{R}{\lambda} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}}{3.24 \times 10^{-7}} = 4.6 \Omega/m$$

(b) The current in the wire is

$$I = \frac{V}{R} = \frac{10}{4.6} = 2.2A$$

Nichrome wire is often used for heating elements in electric heater, toaster and irons, since its resistance is 100 times higher than the copper wire.

	Material	Resistivity ($\Omega.m$)
1	Silver	1.59×10^{-8}
2	Copper	1.7×10^{-8}
3	Gold	2.44×10^{-8}
4	Aluminum	2.82×10^{-8}
5	Tungesten	5.6×10^{-8}
6	Iron	10×10^{-8}
7	Platinum	11×10^{-8}
8	Lead	20×10^{-8}
9	Nichrome	150×10^{-8}
10	Carbon	3.5×10^{-5}
11	Germanium	0.46
12	Silicon	640
13	Glass	$10^{10} - 10^{14}$

Table (7.1) Resistivity of various materials at 20ºC

7.6 Electrical Energy and Power

The current can flow in circuit when a battery is connected to an electrical device through conducting wire as shown in figure 7.3. If the positive terminal of the battery is connected to *a* and the negative terminal of the battery is connected to *b* of the device. A charge *dq* moves through the device from *a* to *b*. The battery perform a work $dW = dq\ V_{ab}$. This work is by the battery is energy *dU* transferred to the device in time *dt* therefore,
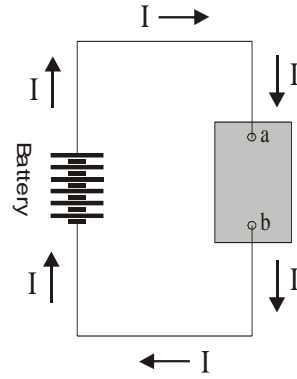
Figure 7.3

$$dU = dW = dq\ V_{ab} = I\ dt\ V_{ab} \qquad (7.13)$$

The rate of electric energy (*dU/dt*) is an electric power (*P*).

$$P = \frac{dU}{dt} = IV_{ab} \qquad (7.14)$$

Suppose a resistor replaces the electric device, the electric power is

$$P = I^2 R \qquad (7.15)$$

$$P = \frac{V^2}{R} \qquad (7.16)$$

The unit of power is (*Joule/sec*) which is known as *watt* (*W*).

Example 7.5

An electric heater is constructed by applying a potential difference of 110volt to a nichrome wire of total resistance 8Ω. Find the current carried by the wire and the power rating of the heater.

Solution

Since $V = IR$

$$\therefore I = \frac{V}{R} = \frac{110}{8} = 13.8A$$

The power P is

$$P = I^2R = (13.8)^2 \times 8 = 1520W$$

Example 7.6

A light bulb is rated at 120v/75W. The bulb is powered by a 120v. Find the current in the bulb and its resistance.

Solution

$$P = IV$$

$$\therefore I = \frac{P}{V} = \frac{75}{120} = 0.625A$$

The resistance is

$$R = \frac{V}{I} = \frac{120}{0.625} = 192\Omega$$

7.7 Combination of Resistors

Some times the electric circuit consist of more than two resistors, which are, connected either in parallel or in series the equivalent resistance is evaluated as follow:

7.7.1 Resistors in Series:

The figure 7.4 shows three resistor in series, carrying a current *I*.
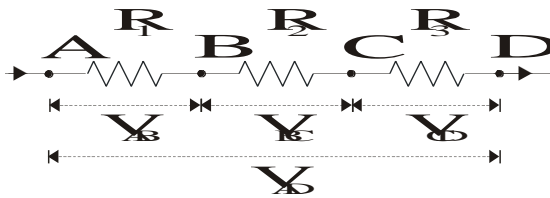
Figure 7.4

For a series connection of resistors, the current is the same in each resistor.

If V_{AD} is the potential deference across the whole resistors, the electric energy supplied to the system per second is IV_{AD}. This is equal to the electric energy dissipated per second in all the resistors.

$$IV_{AD} = IV_{AB} + IV_{BC} + IV_{CD} \qquad (7.17)$$

Hence

$$V_{AD} = V_{AB} + V_{BC} + V_{CD} \qquad (7.18)$$

The individual potential differences are

$$V_{AB} = IR_1, \qquad V_{BC} = IR_2, \qquad V_{CD} = IR_3$$

Therefore

$$V_{AD} = IR_1 + IR_2 + IR_3 \qquad (7.19)$$

$$V_{AD} = I\,(R_1 + R_2 + R_3) \qquad (7.20)$$

The equivalent resistor is

$$R = R_1 + R_2 + R_3 \qquad (7.21)$$

7.7.2 Resistors in Parallel:

The figure 7.5 shows three resistor in parallel, between the points A and B, A current *I* enter from point *A* and leave from point *B*, setting up a potential difference V_{AB}.
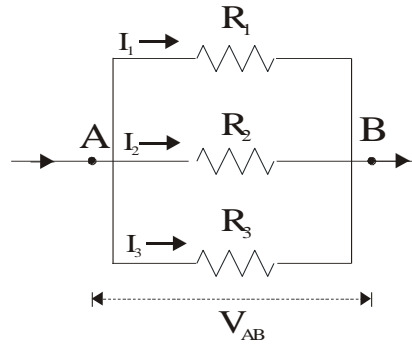
Figure 7.5

For a parallel connection of resistors, the potential difference is equal across each resistor.

The current branches into I_1, I_2, I_3, through the three resistors and,

$$I = I_1 + I_2 + I_3 \qquad (7.22)$$

The current in each branch is given by

$$I_1 = \frac{V_{AB}}{R_1}, \qquad I_2 = \frac{V_{AB}}{R_2}, \qquad I_3 = \frac{V_{AB}}{R_3}$$

$$\therefore I = V_{AB}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad (7.23)$$

The equivalent resistance is

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (7.24)$$

Physical facts for the series and parallel combination of resistors

No.	Series combination	Parallel combination
1	Current is the same through all resistors	Potential difference is the same through all resistors
2	Total potential difference = sum of the individual potential difference	Total Current = sum of the individual current
3	Individual potential difference directly proportional to the individual resistance	Individual current inversely proportional to the individual resistance
4	Total resistance is greater than greatest individual resistance	Total resistance is less than least individual resistance

Notice that parallel resistors combine in the same way that series capacitors combine, and vice versa.

Example 7.6

Find the equivalent resistance for the circuit shown in figure 7.6.
R_1=3Ω, R_2=6Ω, and R_3=4Ω.
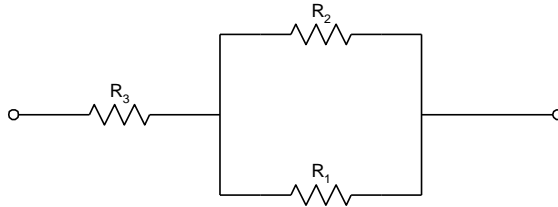
Figure 7.6

Solution

Resistance R_1 and R_2 are connected in parallel therefore the circuit is simplify as shown below
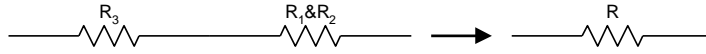
$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R'} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$$

$$R' = 2\Omega$$

Then the resultant resistance of R_1&R_2 (R') are connected in series with resistance R_3

$$R = R' + R_3 = 2 + 4 = 6\Omega$$

Example 7.7

Find the equivalent resistance for the circuit shown in figure 7.7. $R_1=4\Omega$, $R_2=3\Omega$, $R_3=3\Omega$, $R_4=5\Omega$, and $R_5=2.9\Omega$.
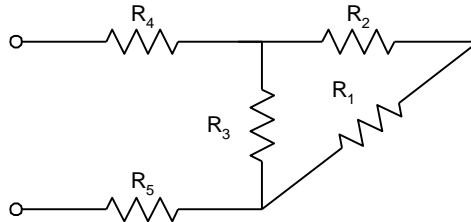
Figure 7.7

Solution

Resistance R_1 and R_2 are connected in series therefore the circuit is simplify as shown below
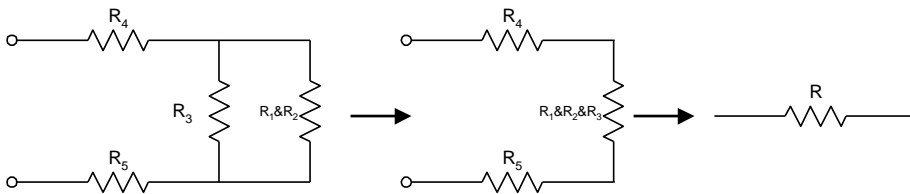
$$R' = R_1+R_2=4+3=7\Omega$$

Then the resultant resistance of R_1&R_2 (R') are connected in parallel with resistance R_3

$$\frac{1}{R''}=\frac{1}{R'}+\frac{1}{R_3}=\frac{1}{7}+\frac{1}{3}=\frac{10}{21}$$

$$R''=2.1\Omega$$

The resultant resistance R for R_5&R_4& R'' are connected in series.

$$R= R'' + R_5+R_4=2.1+5+2.9=10\Omega$$

Example 7.8

Three resistors are connected in parallel as in shown in figure 7.8. A potential difference of 18V is maintained between points *a* and *b*. (a) find the current in each resistor. (b) Calculate the power dissipated by each resistor and the total power dissipated by the three resistors. (c) Calculate the equivalent resistance, and the from this result find the total power dissipated.
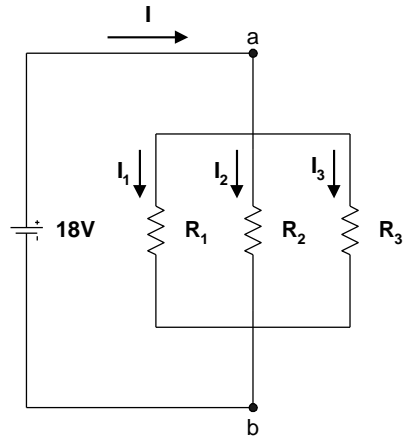
Figure 7.8

Solution

To find the current in each resistors, we make use of the fact that the potential difference across each of them is equal to 18v, since they are connected in parallel with the battery.

Applying *V=IR* to get the current flow in each resistor and then apply $P = I^2R$ to get the power dissipated in each resistor.

$$I_1 = \frac{V}{R_1} = \frac{18}{3} = 6A \quad \Rightarrow \quad P_1 = I_1^2 R_1 = 108W$$

$$I_2 = \frac{V}{R_2} = \frac{18}{6} = 3A \quad \Rightarrow \quad P_2 = I_2^2 R_2 = 54W$$

$$I_3 = \frac{V}{R_3} = \frac{18}{9} = 2A \quad \Rightarrow \quad P_3 = I_3^2 R_3 = 36W$$

The equivalent resistance R_{eq} is

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18}$$

$$R_{eq} = 1.6 \, \Omega$$

7.8 Solution of some selected problems

7.8 Solution of some selected problems

Example 7.9

Two wires A and B of circular cross section are made of the same metal and have equal length, but the resistance of wire A is three times greater than that of wire B. What is the ratio of their cross-sectional area? How do their radii compare?

Solution

Since $R=\rho L/A$, the ratio of the resistance $R_A/R_B=A_A/A_B$. Hence, the ratio is three times. That is, the area of wire B is three times that of B.

The radius of wire b is $\sqrt{3}$ times the radius of wire B.

Example 7.10

Two conductors of the same length and radius are connected across the same potential difference. One conductor has twice the resistance of the other. Which conductor will dissipate more power?

Solution

Since the power dissipated is given by $P=V^2/R$, the conductor with the lower resistance will dissipate more power.

Example 7.11

Two light bulbs both operate from 110v, but one has power rating 25W and the other of 100W. Which bulb has the higher resistance? Which bulb carries the greater current?

Solution

Since $P=V^2/R$, and V is the same for each bulb, the 25W bulb would have the higher resistance. Since $P=IV$, then the 100W bulb carries the greater current.

Example 7.12

The current I in a conductor depends on time as $I=2t_2-3t+7$, where t is in sec. What quantity of charge moves across a section through the conductor during time interval $t=2$sec to $t=4$sec?

Solution

$$I = \frac{dQ}{dt}; \qquad\qquad dQ = I\,dt$$

$$Q = \int I dt = \int_{2}^{4} (2t^2 - 3t + 7)\,dt$$

$$Q = \left[2t^3 - 3t^2 + 7t\right]_{2}^{4} = 33.3C$$

Example 7.13

A 2.4m length of wire that is 0.031cm² in cross section has a measured resistance of 0.24Ω. Calculate the conductivity of the material.

Solution

$$R = \rho \frac{L}{A} \qquad \text{and} \qquad \rho = \frac{1}{\sigma} \qquad \text{therefore}$$

$$\sigma = \frac{L}{RA} = \frac{2.4}{(0.24)(3.1 \times 10^{-6})} = 3.23 \times 10^6 \,/\Omega.m$$

Example 7.14

A 0.9V potential difference is maintained across a 1.5m length of tungsten wire that has cross-sectional area of 0.6mm². What is the current in the wire?

Solution

From Ohm's law,

$$I = \frac{V}{R} \qquad \text{where} \qquad R = \rho \frac{L}{A} \qquad \text{therefore}$$

$$I = \frac{VA}{\rho L} = \frac{0.9 \times 6 \times 10^{-7}}{5.6 \times 10^{-8} \times 1.5} = 6.43A$$

Example 7.15

A resistor is constructed by forming a material of resistivity ρ into the shape of a hollow cylinder of length L and inner and outer radii r_a and r_b respectively as shown in figure 7.9. In use, a potential difference is applied between the ends of the cylinder, producing a current parallel to the axis. **(a)** Find a general expression for the resistance of such a device in terms of L, ρ, r_a, and r_b. **(b)** Obtain a numerical value for R when L=4cm, r_a=0.5cm, r_b=1.2cm, and ρ=3.5×10⁵Ω.m.
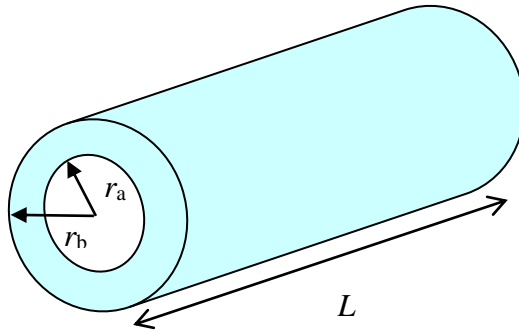
Figure 7.9

Solution

(a)
$$R = \rho \frac{L}{A} = \frac{\rho L}{\pi (r_b^2 - r_a^2)}$$

(b)
$$R = \frac{\rho L}{\pi (r_b^2 - r_a^2)} = \frac{(3.5 \times 10^5)(0.04)}{\pi \left[(0.012)^2 - (0.005)^2 \right]} = 3.74 \times 10^7 \, \Omega$$

Example 7.16

If a 55Ω resistor is rated at 125W, what is the maximum allowed voltage?

Solution

$$P = \frac{V^2}{R}$$

$$\therefore V = \sqrt{PR} = \sqrt{125 \times 55} = 82.9V$$

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7.9 Problems

7.1) A current of 5A exists in a 10 Ω resistor for 4min. (a) How many coulombs, and (b) how many electrons pass through any cross section of the resistor in this time?

7.2) A small but measurable current of 1.0×10^{-10} A exists in a copper wire whose diameter is 0.10in. Calculate the electron drift speed.

7.3) A square aluminum rod is 1.0m long and 5.0mm on edge. (a) What is the resistance between its ends? (b) What must be the diameter of a circular 1.0m copper rod if its resistance is to be the same?

7.4) A conductor of uniform radius 1.2cm carries a current of 3A produced by an electric field of 120V/m. What is the resistivity of the material?

7.5) If the current density in a copper wire is equal to $5.8 \times 10^6 A/m^2$, calculate the drift velocity of the free electrons in this wire.

7.6) A 2.4m length of wire that is $0.031 cm^2$ in cross section has a measured resistance of 0.24Ω. Calculate the conductivity of the material.

7.7) Aluminium and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?

7.8) What is the resistance of a device that operates with a current of 7A when the applied voltage is 110V?

7.9) A copper wire and an iron wire of the same length have the same potential difference applied to them. (a) What must be the ratio of their radii if the current is to be the same? (b) Can the current density be made the same by suitable choices of the radii?

7.10) A 0.9V potential difference is maintained across a 1.5m length of tungsten wire that has a cross-sectional area of $0.6mm^2$. What is the current in the wire?

7.11) A wire with a resistance of 6.0Ω is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire, assuming that the resistivity and density of the material are not changed during the drawing process.

7.12) A wire of Nichrome (a nickel-chromium alloy commonly used in

heating elements) is 1.0 m long and 1.0mm^2 in cross-sectional area. It carries a current of 4.0A when a 2.0V potential difference is applied between its ends. What is the conductivity σ, of Nichrome?

7.13) A copper wire and an iron wire of equal length l and diameter d are joined and a potential difference V is applied between the ends of the composite wire. Calculate (a) the potential difference across each wire. Assume that l=10m, d=2.0mm, and V=100V. (b) Also calculate the current density in each wire, and (c) the electric field in each wire.

7.14) Thermal energy is developed in a resistor at a rate of 100W when the current is 3.0A. What is the resistance in ohms?

7.15) How much current is being supplied by a 200V generator delivering 100kW of power?

7.16) An electric heater operating at full power draws a current of 8A from 110V circuit. (a) What is the resistance of the heater? (b) Assuming constant R, how much current should the heater draw in order to dissipate 750W?

7.17) A 500W heating unit is designed to operate from a 115V line. (a) By what percentage will its heat output drop if the line voltage drops to 110V? Assume no change in resistance. (b) Taking the variation of resistance with temperature into account, would the actual heat output drop be larger or smaller than that calculated in (a)?

7.18) A 1250W radiant heater is constructed to operate at 115V. *(a)* What will be the current in the heater? (b) What is the resistance of the heating coil?

Chapter 8

Direct Current Circuits

دوائر التيار المستمر

Direct Current Circuits

Direct Current Circuits

دوائر التيار المستمر

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سنتعامل في هذا الفصل مع الدوائر الكهربية التي تحتوي على بطارية ومقاومة ومكثف. سنقوم بتحليل هذه الدوائر الكهربية معتمدين على قاعدة كيرشوف *Kirchhoff's rule* لحساب التيار الكهربي المار في كل عنصر من عناصر الدائرة الكهربية. وبداية سنتعرف على مفهوم القوة الدافعة الكهربية *Electromotive force*.

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8.1 Electromotive Force

In any electrical circuit it must exist a device to provide energy to force the charge to move in the circuit, this device could be battery or generator; in general it is called electromotive force (*emf*) symbol (ξ). The electromotive force are able to maintain a potential difference between two points to which they are connected.

Then electromotive force (*emf*) (ξ) is defined as the work done per unit charge.

$$\xi = \frac{dW}{dq} \qquad\qquad (8.1)$$

The unit of ξ is *joule/coulomb*, which is *volt*.

The device acts as an *emf* source is drawn in the circuit as shown in the figure below, with an arrow points in the direction which the positive charge move in the external circuit. *i.e.* from the -ve terminal to the +ve terminal of the battery
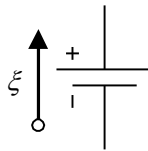
When we say that the battery is 1.5volts we mean that the *emf* of that battery is 1.5volts and if we measure the potential difference across the battery we must find it equal to 1.5volt.

A battery provide energy through a chemical reaction, this chemical reaction transfer to an electric energy which it can be used for mechanical work Also it is possible to transfer the mechanical energy to electrical energy and the electric energy can be used to charge the battery is chemical reaction. This mean that the energy can transfer in different forms in reversible process.

<div align="center">

Chemical $\Leftrightarrow$ *Electrical* $\Leftrightarrow$ *Mechanical*

</div>

<div align="center">

See appendix (A) for more information

</div>

8.2 Finding the current in a simple circuit

Consider the circuit shown in figure 8.1(a) where a battery is connected to a resistor *R* with connecting wires assuming the wires has no resistance.

In the real situation the battery itself has some internal resistance *r*, hence it is drawn as shown in the rectangle box in the diagram.

Figure 8.1(a)

Assume a +ve charge will move from point a along the loop *abcd*. In the graphical representation figure 8.1(b) it shows how the potential changes as the charge moves.

When the charge cross the *emf* from point a to b the potential increases to *a* the value of *emf* ξ, but when it cross the internal resistance *r* the potential decreases by value equal *Ir*. Between the point *b* to *c* the potential stay constant since the wire has no resistance. From point *c* to *d* the potential decreases by *IR* to the same value at point *a*.
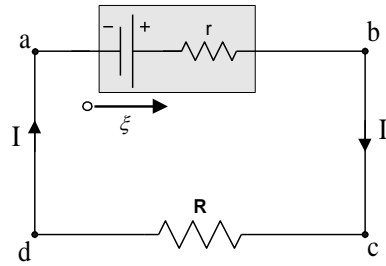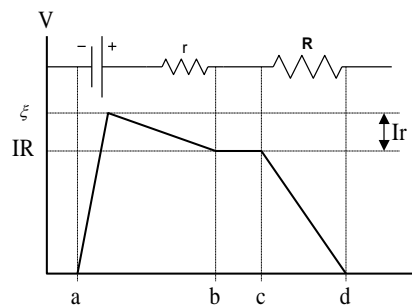
Figure 8.1(b)

The potential difference across the battery between point *a* and *b* is given by

$$V_b - V_a = \xi - Ir \tag{8.2}$$

Note that the potential difference across points a and b is equal to the potential difference between points *c* and *d i.e.*

$$V_b - V_a = V_d - V_c = IR \tag{8.3}$$

Combining the equations

$$IR = \xi - Ir \tag{8.4}$$

Or

$$\xi = IR + Ir \tag{8.5}$$

Therefore the current *I* is

$$I = \frac{\xi}{R+r} \tag{8.6}$$

This equation shows that the current in simple circuit depends on the resistors connected in series with the battery.

We can reach to the same answer using this rule

The algebraic sum of the changes in potential difference across each element of the circuit in a complete loop is equal to zero.

By applying the previous rule on the circuit above starting at point *a* and along the loop *abcda*

Here in the circuit we have three elements (one *emf* and two resistors *r&R*) applying the rule we get,

$$+\xi - Ir - IR = 0 \tag{8.7}$$

The +ve sine for ξ is because the change in potential from the left to the right across the battery the potential increases, the -ve sign for the change in potential across the resistors is due to the decrease of the potential as we move in the direction *abcda*.

$$\therefore I = \frac{\xi}{R+r} \tag{8.8}$$

Example 8.1

In figure 8.2 find the current flow in the branch if the potential difference V_b-V_a=12v. Assume ξ_1=10v, ξ_2=25v, R_1=3Ω, and R_2=5Ω.
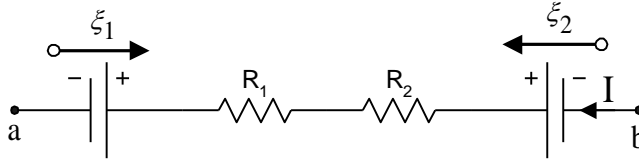
Figure 8.2

Solution

We must assume a direction of the current flow in the branch and suppose that is from point *b* to point *a*.

To find the current in the branch we need to add all the algebraic changes in the potential difference for the electrical element as we move from point *a* to point *b*.

$$V_b\text{-}V_a = +\xi_1 + IR_1 + IR_2 - \xi_2$$

لاحظ هنا أننا اخترنا المسار من النقطة *a* إلى النقطة *b*. وذلك معتمدين على أن فرق الجهد الكهربي *V*b-*V*a هو الشغل المبذول لتحريك شحنة من النقطة *a* إلى النقطة *b*.

Solving for *I*

$$I = \frac{(V_b - V_a) + (\xi_2 - \xi_1)}{R_1 + R_2} = \frac{(12) + (25 - 10)}{8} = 3.375A$$

Example 8.2

Find the potential difference V_a-V_b for the branches shown in figure 8.3 & figure 8.4.

Solution
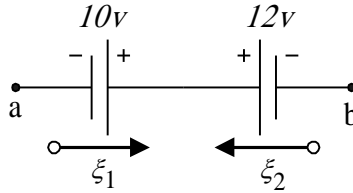
Figure 8.3

To find the potential difference V_a-V_b we should add the algebraic change in the potential difference for the two batteries as we move from point *b* to point *a*.

$$V_a\text{-}V_b = +\xi_2 - \xi_1 = 12 - 10 = 2v$$
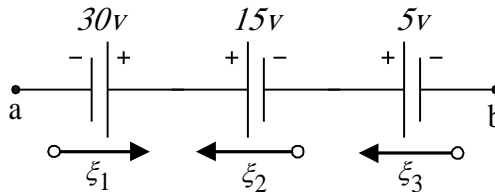
Figure 8.4

$$V_a\text{-}V_b = +\xi_3 + \xi_2 - \xi_1 = 5 + 15 - 30 = -10v$$

وهذا يعني أن النقطة *b* لها جهد أعلى من النقطة *a* وحيث أن التيار الكهربي يسري من الجهد المرتفع إلى الجهد المنخفض لذا فإن البطاريةξ_3 والبطارية ξ_2 تكونان في حالة شحن بواسطة البطارية ξ_1.

8.3 Kirchhoff's Rules

A practical electrical circuit is usually complicated system of many electrical elements. Kirchhoff extended Ohm's law to such systems, and gave two rules, which together enabled the current in any part of the circuit to be calculated.

Statements of Kirchhoff's Rules

(1) The algebraic sum of the currents entering any junction must equal the sum of the currents leaving that junction.

$$\sum_i I_i = 0 \qquad \text{at the junction} \qquad (8.9)$$

(2) The algebraic sum of the changes in potential difference across all of the elements around any closed circuit loop must be zero.

$$\sum_i \Delta V_i = 0 \qquad \text{for the loop circuit} \qquad (8.10)$$

Note that the first Kirchhoff's rule is for the current and the second for the potential difference.

Applying the first rule on the junction shown below (figure 8.5)
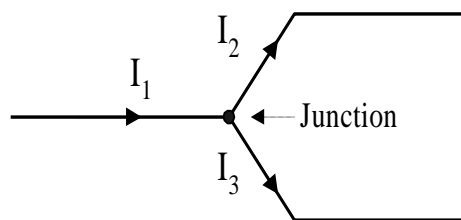
Figure 8.5

$$I_1 = I_2 + I_3$$

Applying the second rule on the following cases

(1) If a resistor is traversed in the direction of the current, the change in potential difference across the resistor is *-IR*.

$$\overset{I}{\longrightarrow}$$

a $\text{---}\Lambda\Lambda\Lambda\text{---}$ b

$$V_b\text{-}V_a\text{=-}IR$$
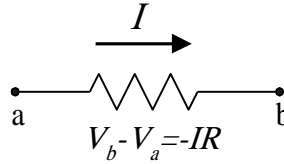
(2) If a resistor is traversed in the direction opposite the current, the change in potential difference across the resistor is *+IR*.

$$\overset{I}{\longleftarrow}$$

a $\text{---}\Lambda\Lambda\Lambda\text{---}$ b

$$V_b\text{-}V_a\text{=+}IR$$
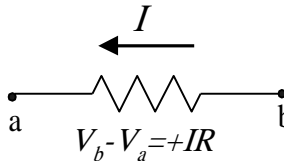
(3) If a source of *emf* is traversed in the direction of the *emf* (from - to + on the terminal), the change in potential difference is $+\xi$.

$$\overset{\xi}{\longrightarrow}$$

a $\quad -\|\,\|+\quad$ b

$$V_b\text{-}V_a\text{=+ }\xi$$
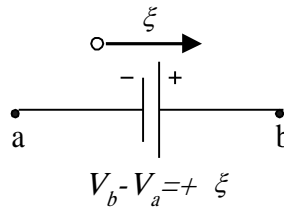
(4) If a source of *emf* is traversed in the direction opposite the *emf* (from + to - on the terminal), the change in potential difference is $-\xi$.

$$\overset{\xi}{\longleftarrow}$$

a $\quad -\|\,\|+\quad$ b

$$V_b\text{-}V_a\text{=- }\xi$$
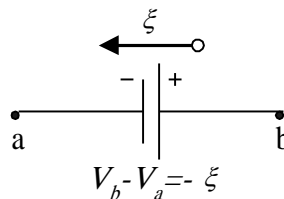
Hints for solution of problems using Kirchhoff's rules

لاستخدام قاعدة كيرشوف يجب اتباع الخطوات التالية:

(1) حدد اتجاه القوة الدافعة الكهربية *emf* لكل بطارية في الدائرة الكهربية بسهم متجه من القطب السالب إلى القطب الموجب للبطارية.

(2) حدد اتجاه التيار الكهربائي لكل عنصر من عناصر الدائرة الكهربية مثل المقاومة بافتراض اتجاه محدد للتيار حتى تتمكن من تطبيق قاعدتي كيرشوف. فإذا كان الحل النهائي يظهر إشارة موجبة للتيار يكون الاتجاه المفترض صحيحاً، أما إذا ظهرت إشارة التيار سالبة فإن قيمة التيار صحيحة، ولكن اتجاه التيار في الاتجاه المعاكس للاتجاه المفترض.

(3) نطبق القاعدة الأولى لكيرشوف عند العقدة الموجودة في الدائرة الكهربية بحيث تكون إشارة التيارات الداخلة على العقدة موجبة والخارجة من العقدة سالبة.

(4) نطبق القاعدة الثانية لكيرشوف على مسار مغلق محدد لكل فرع من أفرع الدائرة الكهربية ونراعي التغير في فرق الجهد على كل عنصر من عناصر الدائرة الكهربية إذا كان سالباً أو موجباً.

(5) نحل المعادلات الرياضية التي نتجت من تطبيق الخطوتين (3) و(4) حلاً جبرياً.

8.4 Single-Loop Circuit

In a single-loop circuit there is no junctions and the current is the same in all elements of the circuit, therefore we use only the second Kirchhoff rule.

Example 8.3

Two battery are connected in opposite in a circuit contains two resistors as shown in figure 8.6 the *emf* and the resistance are ξ_1=6v, ξ_2=12v, R_1= 8Ω, and R_2=10 Ω. (a) Find the current in the circuit. (b) What is the power dissipated in each resistor?
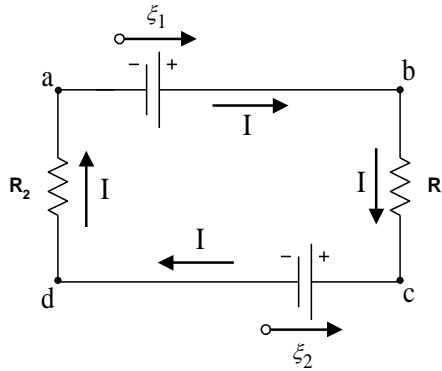
Figure 8.6

Solution

From figure 8.6 the circuit is a single-loop circuit. We draw an arrow for each *emf* in the circuit directed from the -ve to +ve terminal of the *emf*. If we assume the current in the circuit is in the clockwise direction (*abcda*).

Applying the second Kirchhoff's rule along arbitrary loop (*abcda*) we get

$$\sum_i \Delta V_i = 0$$

Starting at point (*a*) to point (*b*) we find the direction of the loop is the same as the direction of the emf therefore ξ_1 is +ve, and the direction of the loop from point (*b*) to point (*c*) is the same with the direction of the current then the change in potential difference is -ve and has the value -IR_1. Complete the loop with the same principle as discussed before we get;

$$+\xi_1 - IR_1 - \xi_2 + IR_2 = 0$$

Solving for the current we get

$$I = \frac{\xi_1 - \xi_2}{R_1 + R_2} = \frac{6 - 12}{8 + 10} = -\frac{1}{3}A$$

The -ve sign of the current indicates that the correct direction of the current is opposite the assumed direction *i.e.* along the loop (*adba*)

The power dissipated in R_1 and R_2 is

$$P_1 = I^2 R_1 = 8/9\text{W}$$

$$P_2 = I^2 R_2 = 10/9\text{W}$$

In this example the battery ξ_2 is being charged by the battery ξ_1.

Example 8.4

Three resistors are connected in series with battery as shown in figure 8.7, apply second Kirchhoff's rule to (a) Find the equivalent resistance and (b) find the potential difference between the points *a* and *b*.
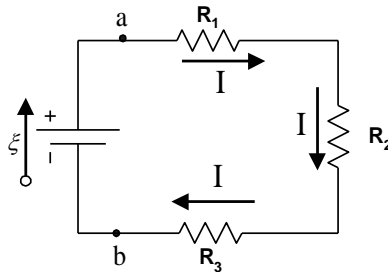
Figure 8.7

Solution

Applying second Kirchhoff's rule in clockwise direction we get

$$- IR_1 - IR_2 - IR_3 + \xi = 0$$

or

$$I = \frac{\xi}{R_1 + R_2 + R_3}$$

$$\Theta\ I = \frac{\zeta}{R}$$

therefore,

$$R = R_1 + R_2 + R_3$$

This is the same result obtained in section 7.1.1

To find the potential difference between points a and b V_{ab} $(=V_a\text{-}V_b)$ we use the second Kirchhoff's rule along a direction starting from point (b) and finish at point (a) through the resistors. We get

$$V_b + IR = V_a$$

Where R is the equivalent resistance for R_1, R_2 and R_3

$$V_{ab} = V_a - V_b = + IR$$

The +ve sign for the answer means that $V_a > V_b$

Substitute for the current *I* using the equation

$$I = \frac{\zeta}{R}$$

we get

$$V_{ab} = \xi$$

This means that the potential difference between points *a* and *b* is equal to the *emf* in the circuit (when the internal resistance of the battery is neglected).

Example 8.5

In the circuit shown in figure 8.8 let ξ_1 and ξ_2 be 2v and 4v, respectively; r_1, r_2 and R be 1Ω, 2Ω, and 5Ω, respectively. (a) What is the current in the circuit? (b) What is the potential difference V_a-V_b and V_a-V_c?
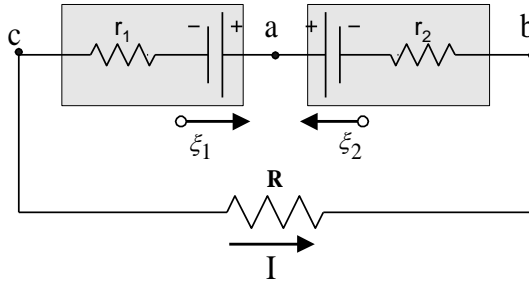
Figure 8.8

Solution

Since the emf ξ_2 is larger than ξ_1 then ξ_2 will control the direction of the current in the circuit. Hence we assume the current direction is counterclockwise as shown in figure 8.8. Applying the second Kirchhoff's rule in a loop clockwise starting at point *a* we get

$$-\xi_2 + Ir_2 + IR + Ir_1 + \xi_1 = 0$$

Solving the equation for the current we get

$$I = \frac{\zeta_2 - \xi_1}{R + r_1 + r_2} = \frac{4 - 2}{5 + 1 + 2} = +0.25A$$

The +ve sign for the current indicates that the current direction is correct. If we choose the opposite direction for the current we would get as a result -0.25A.

The potential difference V_a-V_b we apply second Kirchhoff's rule starting at point *b* to finishing at point *a*.

$$V_a\text{-}V_b = -Ir_2 + \xi_2 = (-0.25 \times 2) + 4 = +3.5\text{v}$$

Note that same result you would obtain if you apply the second Kirchhoff's rule to the other direction (the direction goes through R, r_1, and ξ_1)

The potential difference $V_a\text{-}V_c$ we apply second Kirchhoff's rule starting at point c to finishing at point a.

$$V_a\text{-}V_c = +\xi_1 + Ir_1 = +2 + (-0.25 \times 1) = +2.25\text{v}$$

Note that same result you would obtain if you apply the second Kirchhoff's rule to the other direction (the direction goes through R, r_2, and ξ_2)

8.5 Multi-Loop Circuit

Some circuits involving more than one current loop, such as the one shown in figure 8.9. Here we have a circuit with three loops: a left inside loop, a right inside loop, and an outside loop. There is no way to reduce this multi-loop circuit into one involving a single battery and resistor.
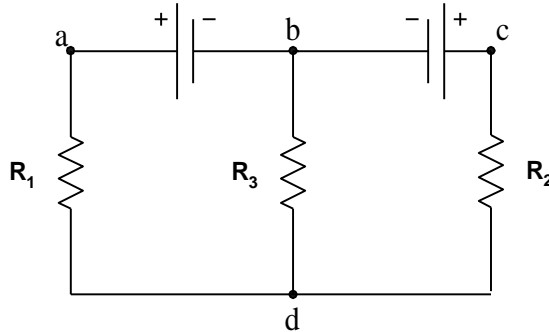
Figure 8.9

In the circuit shown above there are two junctions *b* and *d* and three branches connecting these junctions. These branches are bad, *bcd*, and *bd*. The problem here is to find the currents in each branch.

A general method for solving multi-loop circuit problem is to apply Kirchhoff*'s* rules.

You should always follow these steps:

(1) Assign the direction for the *emf* from the -ve to the +ve terminal of the battery.
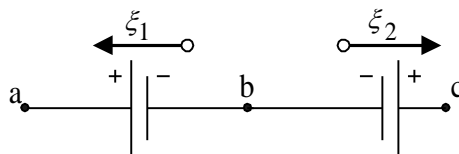
Figure 8.10

(2) Assign the direction of the currents in each branch assuming arbitrary direction.
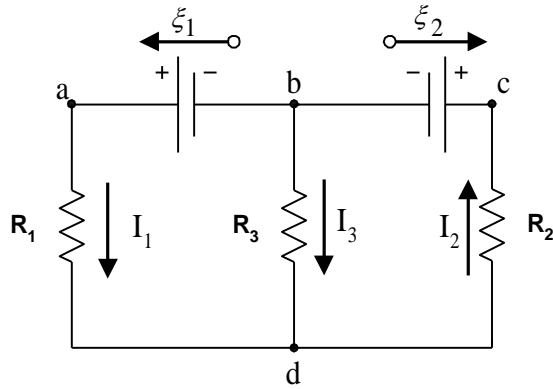
Figure 8.11

After solving the equations the +ve sign of the current means that the assumed direction is correct, and the -ve sign for the current means that the opposite direction is the correct one.

(3) Chose one junction to apply the first Kirchhoff's rule.

$$\sum_i I_i = 0$$

At junction *d* current I_1 and I_3 is approaching the junction and I_2 leaving the junction therefore we get this equation

$$I_1 + I_3 - I_2 = 0 \qquad\qquad (1)$$

(4) For the three branches circuit assume there are two single-loop circuits and apply the second Kirchhoff's rule on each loop.
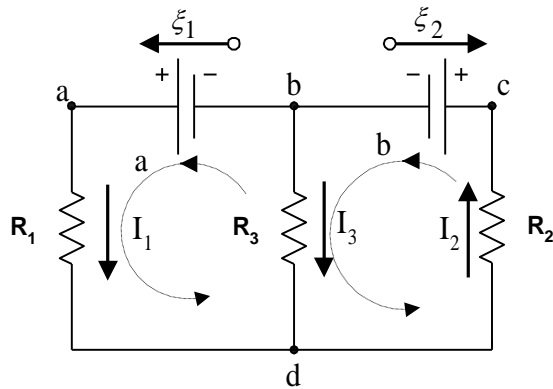
Figure 8.12

For loop a on the left side starting at point b we get

$$+\xi_1 - I_1R_1 + I_3R_3 = 0 \qquad (2)$$

For loop b on the left side starting at point b we get

$$- I_3R_3 - I_2R_2 - \xi_2 = 0 \qquad (3)$$

Equations (1), (2), and (3) can be solved to find the unknowns currents I_1, I_2, and I_3.

The current can be either positive or negative, depending on the relative sizes of the *emf* and of the resistances.

Example 8.6

In the circuit shown in figure 8.13, find the unknown current *I*, resistance *R*, and *emf* ξ.
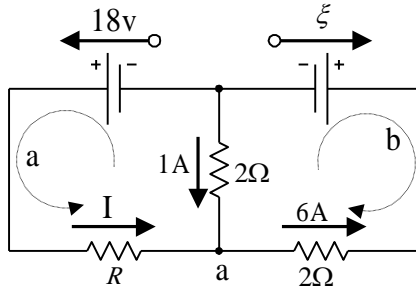
Figure 8.13

Solution

At junction a we get this equation

$$I + 1 - 6 = 0$$

Therefore the current

$$I = 5A$$

To determine *R* we apply the second Kirchhoff's rule on the loop (a), we get

$$18 - 5R + 1 \times 2 = 0$$

$$R = 4\Omega$$

To determine ξ we apply the second Kirchhoff's rule on the loop (a), we get

$$\xi + 6 \times 2 + 1 \times 2 = 0$$

$$\xi = -14v$$

Example 8.7

In the circuit shown in figure 8.14, (a) find the current in the 2Ω resistor, (b) the potential difference between points *a* and *b*.

(Use the current as labeled in the figure 8.14).
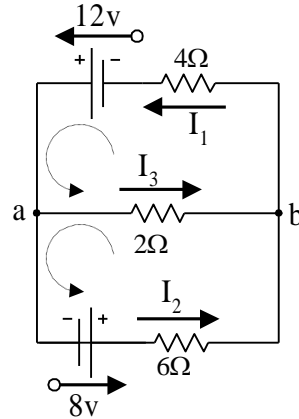
Figure 8.14

Solution

At junction *a* we get

$$I_1 = I_2 + I_3 \qquad (1)$$

For the top loop

$$12 - 2 \times I_3 - 4 \times I_1 = 0 \qquad (2)$$

For the bottom loop

$$8 - 6 \times I_2 + 2 \times I_3 = 0 \qquad (3)$$

From equation (2)

$$I_1 = 3 - 1/2 \ I_3$$

From equation (2)

$$I_2 = 4/3 - 1/3 \ I_3$$

Substituting these values in equation (1), we get

$$I_3 = 0.909\text{A the current in the resistor } 2\Omega$$

The potential difference between points *a* and *b*

$$V_a - V_b = I_3 \times R = 0.909 \times 2 = 1.82\text{v} \qquad V_a > V_b$$

Example 8.8

In the circuit shown in figure 8.15, (a) find the current I_1, I_2, and I_3, (b) the potential difference between points *a* and *b*. Use these values, ξ_1=10v, ξ_2=6v, ξ_3=4v, R_1=6Ω, R_2=2Ω, R_3=1Ω, and R_4=4Ω.

(Use the current as labeled in the figure below).
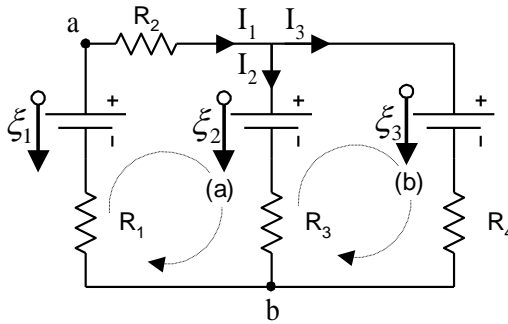
Figure 8.15

Solution

For the junction at the top we get

$$I_1 + I_2 - I_3 = 0 \tag{1}$$

For loop a on the left side we get

$$+\xi_1 - I_1R_2 - \xi_2 + I_2R_3 - I_1R_1 = 0$$
$$+10 - 2I_1 - 6 + I_2 - 6I_1 = 0$$
$$+4 - 8I_1 + I_2 = 0 \tag{2}$$

For loop b on the right side we get

$$- I_2R_3 + \xi_2 - \xi_2 + I_3R_4 = 0$$
$$- I_2 + 6 - 4 - 4I_3 = 0$$
$$+2 - I_2 - 4I_3 = 0 \tag{3}$$

From equation (2)

$$I_1 = \frac{4 + I_2}{8} \qquad (4)$$

From equation (3)

$$I_3 = \frac{2 - I_2}{4} \qquad (5)$$

Substitute in equation (1) from equations (4)&(5) we get

$$\frac{4 + I_2}{8} + I_2 - \frac{2 - I_2}{4} = 0$$

$I_2 = 0$

From equation (4)

$I_1 = 0.5A$

From equation (4)

$I_3 = 0.5A$

The potential difference between points a and b we use the loop (a)

$$V_b - V_a = -\xi_1 + I_1 R_2$$

$$V_b - V_a = -\xi_1 + I_1 R_2$$

$$V_b - V_a = 10 - 0.5 \times 6 = -7v \qquad (V_b < V_a)$$

Example 8.9

Consider the circuit shown in Figure 8.16. Find (a) the current in the 20.0 Ω resistor and (b) the potential difference between points a and b.
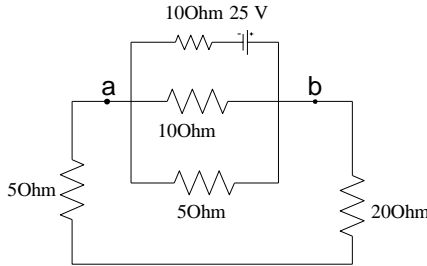
Figure 8.16

Solution

Turn the diagram ion figure 8.16 on its side, we find that the 20Ω and 5Ω resistors are in series, so the first reduction is as shown in (b). In addition, since the 10Ω, 5Ω, and 25Ω resistors are then in parallel, we can solve for their equivalent resistance as

$$R_{eq} = \frac{1}{\left(\dfrac{1}{10} + \dfrac{1}{5} + \dfrac{1}{25}\right)} = 2.94\Omega$$

This is shown in figure (c), which in turn reduces to the circuit shown in (d).

Next we work backwards through the diagrams, applying $I=V/R$ and $V=IR$. The 12.94Ω resistor is connected across 25V, so the current through the voltage source in every diagram is

$$I = \frac{V}{R} = \frac{25}{12.94} = 1.93A$$

(a)

(b)

(c)

(d)
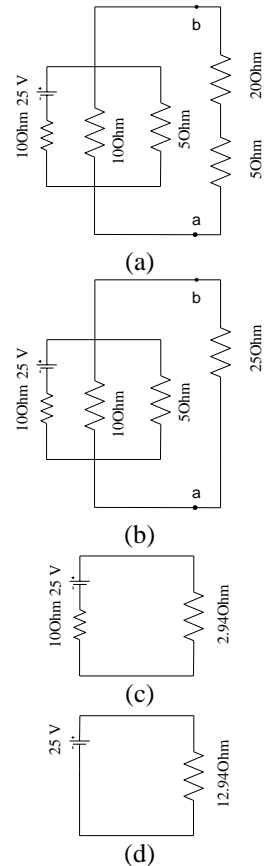
In figure (c), the current 1.93A goes through the 2.94Ω equivalent resistor to give a voltage drop of:

$$V = IR = (1.93)(2.94) = 5.68V$$

From figure (b), we see that this voltage drop is the same across V_{ab}, the 10Ω resistor, and the 5Ω resistor.

Therfore

$$V_{ab} = 5.68V$$

Since the current through the 20Ω resistor is also the current through the 25Ω

$$I = V_{ab}/R_{ab} = 5.68/25 = 0.227A$$

Example 8.10

Determine the current in each of the branches of the circuit shown in figure 8.17.
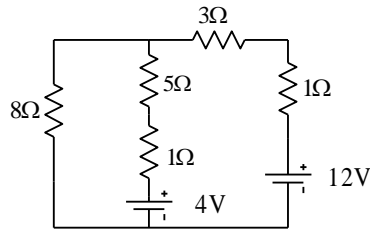
Figure 8.17

Solution

First we should define an arbitrary direction for the current as shown in the figure below.

$$I_3 = I_1 + I_2 \qquad (1)$$
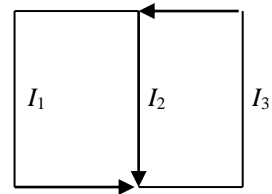
By the voltage rule the left-hand loop

$$+I_1(8Ω)-I_2(5Ω)-I_2(1Ω)-4V=0 \qquad (2)$$

For the right-hand loop

$$4V + I_2(5\Omega + 1\Omega) + I_3(4\Omega) - 12V = 0 \qquad (3)$$

Substitute for I_3 from eqn. (1) into eqns. (2)&(3)

$$8I_1 - 6I_2 - 4 = 0 \qquad\qquad (4)$$

$$4 + 6I_2 + 4(I_1 + I_2) - 12 = 0 \qquad (5)$$

Solving eqn. (4) for I_2

$$I_2 = \frac{8I_1 - 4}{6}$$

Rearranging eqn. (5) we get

$$I_1 + \frac{10}{4}I_2 = \frac{8}{4}$$

Substitute for I_2 we get

$$I_1 + 3.33I_1 - 1.67 = 2$$

Then,

$$I_1 = 0.846A$$

$$I_2 = 0.462A$$

$$I_3 = 1.31A$$

8.6 RC Circuit

In the previous section we studied either circuits with resistors only or with capacitors only, now we will deal with circuits contains both the resistors and capacitors together, these circuits are time dependent circuit where the current in the is varying with time.

In the circuit shown in figure 8.18 we have connected an *emf* with resistor *R* and uncharged capacitor *C* using a switch *S*.
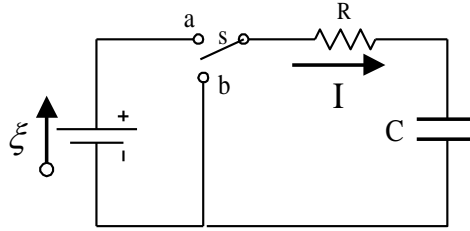
Figure 8.18

8.6.1 Charging a capacitor

When the switch *S* is connected to point (a) the battery will force charges to move to the capacitor this called *charging* process of the capacitor. Note that the current will not flow through the capacitor since there is no way for the charge to jump from one plate to the other. However a positive charge will accumulate on the plate connected with the positive terminal of the battery. The same number of a negative charge will accumulate on the other plate.

The current must stop after the capacitor will become fully charged and its potential difference equals the *emf*.

نلاحظ هنا في هذه الحالة أن التيار يكون ذا قيمة عظمى عند غلق المفتاح وبمرور الزمن يتناقص التيار إلي أن يصل إلى الصفر. وعندها يكون شحن المكثف قد اكتمل.

To analyze this circuit let's assume that in time *dt* a charge *dq* moves through the resistor and the capacitor. Apply the first Kirchhoff's rule in a direction from the battery to the resistor to the capacitor we get,

$$\xi - IR - \frac{q}{C} = 0 \qquad\qquad (8.11)$$

Where *IR* is the potential difference across the resistor and *q/C* is the potential difference across the capacitor.

The current *I* and the charge *q* are varying with time. Substitute for

$$I = \frac{dq}{dt} \tag{8.12}$$

$$\zeta - R\frac{dq}{dt} - \frac{q}{C} = 0 \tag{8.13}$$

By solving the differential equation to find the *q* as a function of time we get,

$$q = C\zeta(1 - e^{-t/RC}) \tag{8.14}$$

The quantity $C\xi$ is the maximum charge *Q* in the capacitor.

$$q = Q(1 - e^{-t/RC}) \tag{8.15}$$

The current *I* is

$$I = \frac{dq}{dt} = \frac{\zeta}{R}e^{-t/RC} \tag{8.16}$$
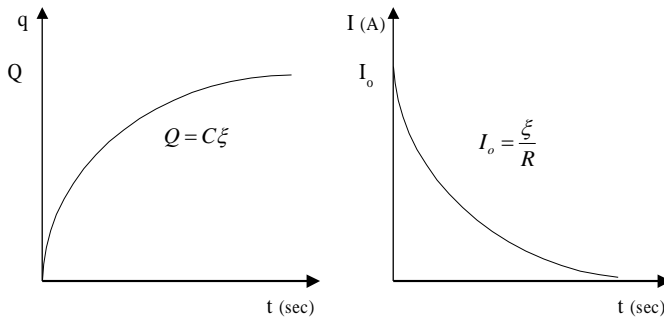
Figure 8.19

Plots of the charge Q and the current I as a function of

*time in the **charging process***

Note that the quantity *RC* in the equation has a unit of time (sec). Therefore it is called the *time constant* of the circuit.

Unit of *RC* is *Ohm . Farad=Sec*

$$Ohm.Farad = Ohm.\frac{Coulomb}{Volt} = Ohm.\frac{Amp.Sec}{Volt} = \frac{Volt.Sec}{Volt} = Sec$$

8.6.2 Discharging a capacitor

When the switch *S* is connected to point (b) the battery is disconnected and now the charged capacitor plays the role of *emf*. Therefore the capacitor will force a charge *q* to move through the resistor *R* this called *discharging* process of the capacitor.

Apply the first Kirchhoff's rule in a direction from the resistor to the capacitor we get,

$$- IR - \frac{q}{C} = 0 \qquad\qquad (8.17)$$

The current *I* and the charge *q* are varying with time. Substitute for

$$I = \frac{dq}{dt} \qquad\qquad (8.18)$$

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad\qquad (8.19)$$

By solving the differential equation to find the *q* as a function of time we get,

$$q = Qe^{-t/RC} \qquad\qquad (8.20)$$

The current *I* during the discharging process is

$$I = \frac{dq}{dt} = -\frac{Q}{RC}e^{-t/RC} \qquad\qquad (8.21)$$

The -ve sign indicates that the direction of the current in the discharging process is in the opposite direction of the charging process.

The quantity Q/RC is equal to the initial current I_o (*i.e.* when t=0)

$$I = I_o e^{-t/RC} \qquad\qquad (8.22)$$
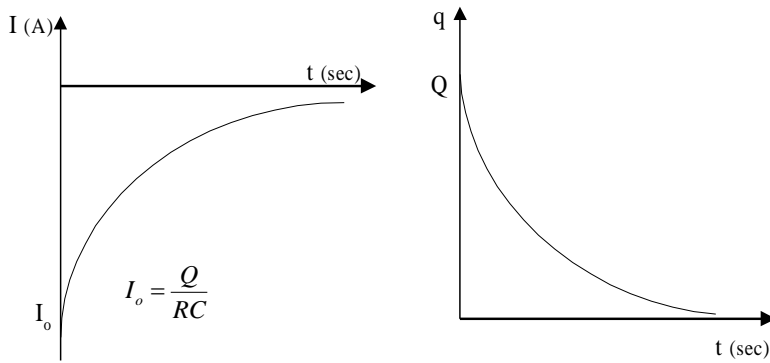
Figure 8.20

Plots of the charge Q and the current I as a function of

*time in the **discharging process***

In the end we found that charging and discharging process of the capacitor is exponentially depends on the time constant (*RC*).

Example 8.11

Consider an *RC* circuit in which the capacitor is being charged by a battery connected in the circuit. In five time constants, what percentage of the final charge is on the capacitor?

Solution

From equation (8.20)

$$q = Q(1 - e^{-t/RC})$$

$$t = 5RC$$

$$\frac{q}{Q} = 1 - e^{-t/RC}$$

$$\frac{q}{Q} = 1 - e^{-5RC/RC} = 1 - e^{-5} = 99.3\%$$

Example 8.12

In figure 8.21 (a) find the time constant of the circuit and the charge on the capacitor after the switch is closed. (b) find the current in the resistor *R* at a time 10sec after the switch is closed. Assume $R=1\times10^6\Omega$, *emf*=30Vand $C=5\times10^{-6}\mu F$.
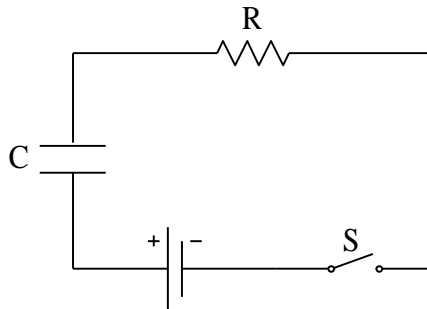
Figure 8.21

Solution

(a) The time constant $= RC = (1 \times 10^{-6})(5 \times 10^{-6}) = 5$ sec

The charge on the capacitor $= Q = C\xi = (5 \times 10^{-6})(30) = 150$ μC

(b) The current in charging of the capacitor is given by

$$I = \frac{\xi}{R} e^{-t/RC}$$

$$I = \frac{30}{1 \times 10^6} e^{-\left(\frac{10}{(1 \times 10^6)(5 \times 10^{-6})}\right)} = 4.06 \times 10^{-6} A$$

Example 8.13

Determine the potential difference V_b-V_a for the circuit shown in figure 8.22

Solution

The current is zero in the middle branch since there is discontinuity at the points *a* and *b*. Applying the second Kirchhoff's rule for the outside loop we get,

Figure 8.22

$+12 - 10I - 5I - 8 - 1/2I - 1/2I = 0$

$+4 - 16I = 0$

$I = 4$A

The potential difference V_b-V_a is found by applying the second Kirchhoff's rule at point *a* and move across the upper branch to reach point *b* we get,

V_b-$V_a = +10I - 12 + 1/2I + 4$

V_b-$V_a = -8 + 10.5I = -8 + (10.5 \times 4) = 34$volt
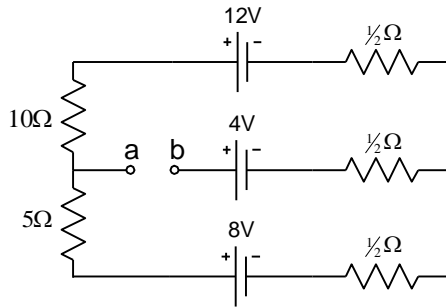
Example 8.14

The circuit has been connected as shown in figure 8.23 for a long time. (a) What is the voltage across the capacitor? (b) If the battery is disconnected, how long does it take for the capacitor to discharge to 1/10 of its initial voltage? The capacitance $C=1\mu F$.
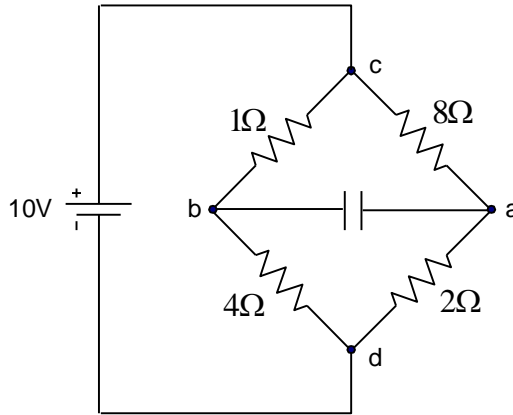
Figure 8.23

Solution

After long time the capacitor would be fully charged and the current in the branch *ab* equal zero.

The resistors in the left hand (1Ω, and 4Ω) are connected in series and assume the current in this branch *cbd* is I_1. The resistors in the right hand (8Ω, and 2Ω) are connected in series and assume the current in this branch *cad* is I_2.

The potential difference across the points *c* and *d* is the same as the *emf* = 10volt. Therefore,

$$I_1 = \frac{10}{1+4} = 2A \qquad\qquad I_2 = \frac{10}{2+8} = 1A$$

The total current,

$$I = I_1 + I_2 = 3A$$

The potential difference across the capacitor V_b-V_a is

$$V_b\text{-}V_a = 8I_2 - 1I_1 = 8 \times 1 - 1 \times 2 = 6\text{volt}$$

To find the answer of (b) we need to find the equivalent resistance,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

Where R_1 is the equivalent resistance for (1Ω, and 4Ω), and R_2 is the equivalent resistance for (2Ω, and 8Ω)

R=3.3Ω

From equation 8.17

$$q = Qe^{-t/RC}$$

Divide by the capacitance C, therefore

$$\frac{q}{C} = \frac{Q}{C}e^{-t/RC}$$

$$v = Ve^{-t/RC}$$

$$\therefore \frac{v}{V} = e^{-t/RC}$$

The time for the capacitor to discharge to 1/10 of its initial voltage

$$\frac{1}{10} = e^{-t/RC}$$

$$Ln1 - Ln10 = -t/RC$$

$$t = Ln10 \times R \times C$$

$$t = 7.7\mu s$$

8.7 Electrical Instruments

8.7.1 Ammeter and Voltmeter

A device called *ammeter* is used to measure the current flow in a circuit, the ammeter must connected in *series* in the circuit so that the current to be measured actually passes through the meter. In order that the ammeter will not affect the current in the circuit it must has *very small* resistance.

A device called *voltmeter* is used to measure the potential difference between two points, and its terminal must be connected to these points in *parallel*.
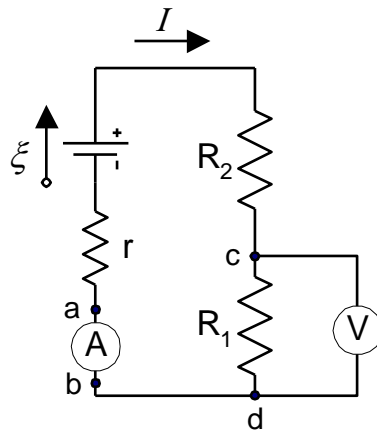
Figure 8.24

In figure 8.24 shows an ammeter (A) measure the current *I* in the circuit. Voltmeter (V), measure the potential difference across the resistor R_1, (V_c-V_d). In order that the voltmeter will not affect the current in the circuit it must has *very large* resistance.

Note that the ammeter is connected in series is the circuit and the voltmeter is connected in parallel with the points to measure the potential difference across them.

8.7.2 The Wheatstone Bridge

This is a circuit consist of four resistors, emf, and galvanometer. The Wheatstone bridge circuit is used to measure unknown resistance. In figure 8.25 show three resistors R_1, R_2, and R_3 are known with R_1 is a variable resistance and resistor R_x is the unknown one.
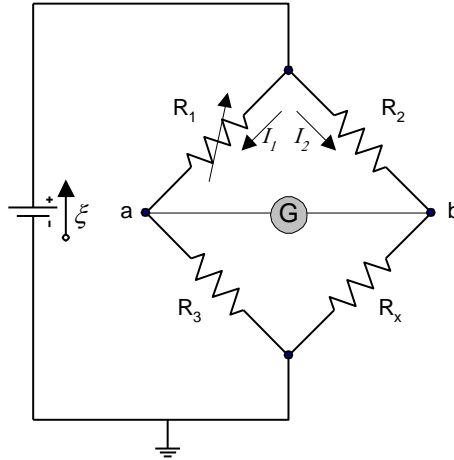
Figure 8.25

To find the resistance R_x the bridge is balanced by adjusting the variable resistance R_1 until the current between *a* and *b* is zero and the galvanometer reads zero. At this condition the voltage across R_1 is equal the voltage across R_2 and the same for R_3 and R_x. Therefore,

$$I_1R_1 = I_2R_2$$

$$I_1R_2 = I_2R_x$$

Dividing the to equation and solving for R_x we get,

$$R_x = \frac{R_2 R_3}{R_1} \tag{8.23}$$

This shows how unknown resistor can be determined using the Wheatstone bridge.

8.7.3 The potentiometer

This circuit is used to measure potential differences by comparison with a slandered voltage source. The circuit is shown in figure 8.26 where the working *emf* is ξ_w and the unknown *emf* is ξ_x. The current flow in the circuit is I in the left branch and I_x is the current in the right branch and $I\text{-}I_x$ is the current flow in the variable resistor. Apply second Kirchhoff's rule on the right branch *abcd* we get,
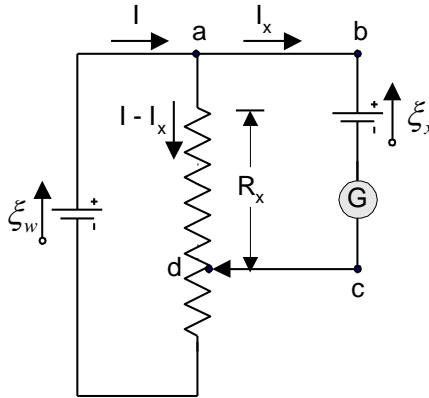
Figure 8.26

$$-(I\text{-}I_x)R_x + \xi_x = 0$$

When the variable resistance adjusted until the galvanometer reads zero, this mean that $I_x=0$.

$$\xi_x = IR_x$$

In the next step the emf ξ_x is replaced with standard *emf* ξ_s, and adjusted the resistance until the galvanometer reads zero, therefore,

$$\xi_s = IR_x$$

Where the current I remains the same, divide the two equations we get,

$$\zeta_x = \frac{R_x}{R_s}\zeta_s \qquad\qquad (8.24)$$

This shows how unknown *emf* can be determined using known *emf*.

8.8 Problems

8.1) A battery with an *emf* of 12V and internal resistance of 0.9Ω is connected across a load resistor R. If the current in the circuit is 1.4A, what is the value of R?

8.2) What power is dissipated in the internal resistance of the battery in the circuit described in Problem 8.1?

8.3) (a) What is the current in a 5.6Ω resistor connected to a battery with an 0.2Ω internal resistance if the terminal voltage of the battery is 10V? (b) What is the *emf* of the battery?

8.4) If the emf of a battery is 15V and a current of 60A is measured when the battery is shorted, what is the internal resistance of the battery?

8.5) The current in a loop circuit that has a resistance of R_1 is 2A. The current is reduced to 1.6A when an additional resistor $R_2=3\Omega$ is added in series with R_1. What is the value of R_1?

8.6) A battery has an emf of 15V. The terminal voltage of the battery is 11.6V when it is delivering 20W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance of the battery?

8.7) A certain battery has an open-circuit voltage of 42V. A load resistance of 12Ω reduces the terminal voltage to 35V. What is the value of the internal resistance of the battery?

8.8) Two circuit elements with fixed resistances R_1 and R_2 are connected in *series* with a 6V battery and a switch. The battery has an internal resistance of 5Ω, $R_1= 32\Omega$, and $R_2=56\Omega$. (a) What is the current through R_1 when the switch is closed? (b) What is the voltage across R_2 when the switch is closed?

8.9) The current in a simple series circuit is 5.0A. When an additional resistance of 2.0Ω is inserted, the current drops to 4.0 A. What was the resistance of the original circuit?

8.10) Three resistors (10Ω, 20Ω, and 30Ω) are connected in parallel. The total current through this network is 5A. (a) What is the voltage drop across the network (b) What is the current in each resistor?

8.11) (a) Find the equivalent resistance between points a and b in Figure 8.27. (b) A potential difference of 34V is applied between points a and b in Figure 28.28. Calculate the current in each resistor.
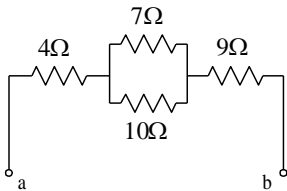
Figure 8.27

8.12) Evaluate the effective resistance of the network of identical resistors, each having resistance R, shown in figure 8.28.
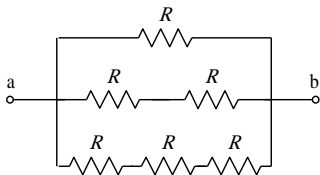
Figure 8.28

8.13) Calculate the power dissipated in each resistor in the circuit of figure 8.29.
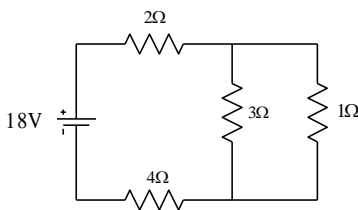
Figure 8.29

8.14) Consider the circuit shown in Figure 8.30. Find (a) the current in the 20Ω resistor and (b) the potential difference between points a and b.
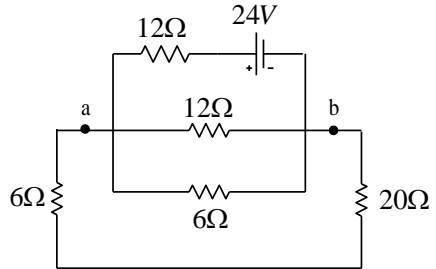
Figure 8.30

8.15) (*a*) In Figure 8.31 what value must R have if the current in the circuit is to be 0.0010A? Take & $\xi_1=2.0V$, $\xi_2=3.0V$, and $r_1=r_2=3.0\Omega$. (b) What is the rate of thermal energy transfer in R?
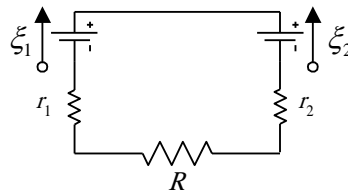
Figure 8.31

8.16) In Figure 8.32 (a) calculate the potential difference between *a* and *c* by considering a path that contains R and ξ_2.
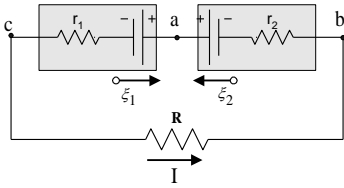
Figure 8.32

8.17) In Figure 8.33 find the current in each resistor and the potential difference between *a* and *b*. Put $\xi_1=6.0V$, $\xi_2=5.0V$, $\xi_3=4.0V$, $R_1=100\Omega$ and $R_2=50\Omega$.
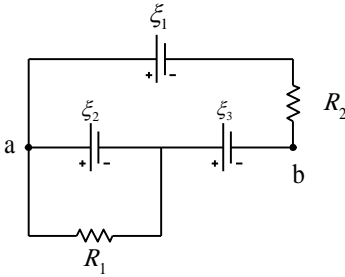
Figure 8.33

8.18) (a) Find the three currents in Figure 8.34. (b) Find V_{ab}. Assume that $R_1=1.0\Omega$, $R_2=2.0\Omega$, $\xi_1=2.0$ V, and $\xi_2=\xi_3=4.0V$.
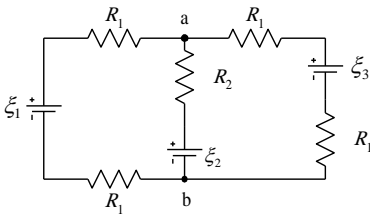
Figure 8.34

8.19) (a) Find the potential difference between points a and b in the circuit in Figure 8.35. (b) Find the currents I_1, I_2, and I_3 in the circuit.
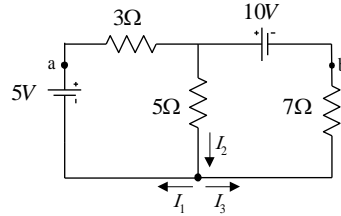
Figure 8.35

8.20) Determine the current in each of the branches of the circuit shown in figure 8.36.
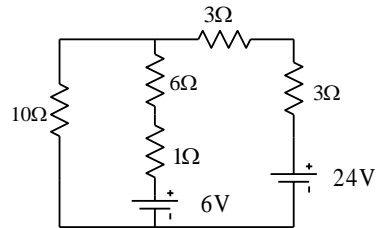
Figure 8.36

8.21) Calculate the power dissipated in each resistor in the circuit shown in figure 8.37.
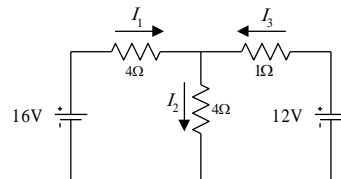
Figure 8.37

8.22) Consider a series RC circuit for which $R=1M\Omega$, $C=5\mu F$, and $\xi=30V$. Find (a) the time constant of the circuit and (b) the maximum charge on the capacitor after the switch is closed. (c) If the switch in the RC circuit is closed at $t=0$. Find the current in the resistor R at a time 10s after the switch is closed.

8.23) At $t=0$, an unchanged capacitor of capacitance C is connected through a resistance R to a battery of constant emf (Figure 8.38). (a) How long does it take for the capacitor to reach one half of its final charge? (b) How long does it

take for the capacitor to become fully charged?

Close at $t=0$

Figure 8.38

8.24) A 4MΩ resistor and a 3μF capacitor are connected in series with a 12V power supply. (a) What is the time constant for the circuit? (b) Express the current in the circuit and the charge on the capacitor as a function of time.

Multiple Choice Questions

Part 2
Applications of Electrostatic

Capacitors and Capacitance
Current and Resistance
Direct Current Circuits

Attempt the following question after the completion of part 2

[1] (a) How much charge is on a plate of a 4.00μF capacitor when it is connected to a 12.0 V battery? (b) If this same capacitor is connected to a 1.50 V battery, what charge is stored?

a. (a) 3.00 μC; (b) 2.67 μC
b. (a) 3.00 μC; (b) 0.375 μC
c. (a) 0.333 μC; (b) 2.67 μC
d. (a) 48.00 μC; (b) 6.00 μC

---

[2] Calculate the equivalent capacitance between points a and b in Figure 1.
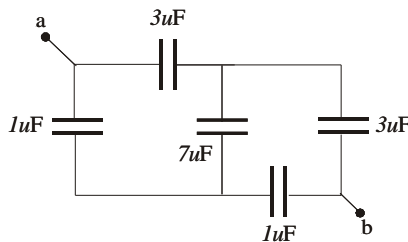
Figure 1

Note that this is not a simple series or parallel combination. (Hint: Assume a potential difference ΔV between points a and b. Write expressions for ΔV_{ab} in terms of the charges and capacitances for the various possible pathways from a to b, and require conservation of charge for those capacitor plates that are connected to each other.)

a. 4.68 μF
b. 15.0 μF
c. 0.356 μF
d. 200 μF

---

[3] Two capacitors $C_1 = 27.0$ μF and $C_2 = 7.00$ μF, are connected in parallel and charged with a 90.0 V power supply. (a) Calculate the total energy stored in the two capacitors. (b) What potential difference would be required across the same two capacitors connected in series in order that the combinations store the same energy as in (a)?

a. (a) 0.0225 J;   (b) 25.7 V
b. (a) 0.0225 J;   (b) 36.4 V
c. (a) 0.138 J; (b) 223 V
d. (a) 0.13 8 J;   (b) 157 V

---

[4] Each capacitor in the combination shown in Figure 2 has a breakdown voltage of 19.0V.

2μF   2μF

2μF   2μF
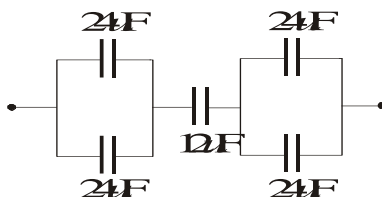
Figure 2

What is the breakdown voltage of the combination?

a. 57.0 V
b. 28.5 V
c. 95.0 V
d. 19.0 V

---

[5] When a potential difference of 190 V is applied to the plates of a parallel plate capacitor, the plates carry a surface charge density of 20.0 nC/cm^2. What is the spacing between the plates?

a. 8.41 μm
b. 0.119 μm
c. 0.429 μm
d. 2.3 3 μm

---

[6] A parallel plate capacitor is constructed using a dielectric material whose dielectric constant is 4.00 and whose dielectric strength is 2.50×10^8 V/m. The desired capacitance is 0.450μF, and the capacitor must withstand a maximum potential difference of 4000V. Find the minimum area of the capacitor plates.

a. 3.25 m^2
b. 0.203 m^2
c. 0.795 m^2
d. 0.814 m^2

[7] Find the equivalent capacitance between points a and b in the combination of capacitors shown in Figure 3.
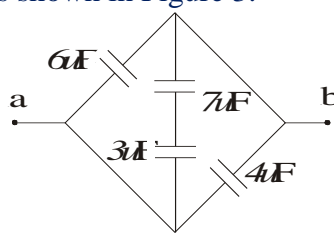
Figure 3

a. 1.12 µF
b. 1.94 µF
c. 1.12 µF
d. 20.0 µF

---

[8] The inner conductor of a coaxial cable has a radius of 0.500 mm, and the outer conductor's inside radius is 4.00 mm. The space between the conductors is filled with polyethylene, which has a dielectric constant of 2.30 and a dielectric strength of 20.0×10^6 V/m. What is the maximum potential difference that this cable can withstand?

a. 30.4 kV
b. 70.0 kV
c. 20.8 kV
d. 166 kV

---

[9] Two capacitors when connected in parallel give an equivalent capacitance of 27.0 pF and give an equivalent capacitance of 4.00 pF when connected in series. What is the capacitance of each capacitor?

a. 10.4 pF, 16.6 pF
b. 9.76 pF, 44.2 pF
c. 9.77 pF, 17.23 pF
d. 4.88 pF, 22.12 pF

---

[10] An isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the charged capacitor is then connected in parallel to an unchanged 10.0 µF capacitor, the voltage across the combination is 30.0 V. Calculate the unknown capacitance.

---

a. 7.00 µF
b. 2.31 µF
c. 4.29 µF
d. 13.0 µF

---

[11] Four capacitors are connected as shown in Figure 4.
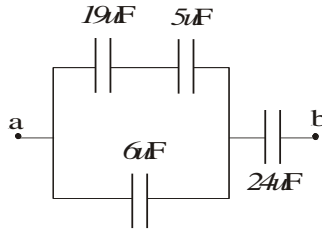
Figure 4

(a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if ΔV_{ab} = 11.0 V.

a. (a) 28.8 µF; (b) q_{19} = 1254, q_5 = 330, q_6 = 396, q_{24} = 317
b. (a) 7.04 µF; (b) q_{19} = 30.8, q_5 = 30.8, q_6 = 46.7, q_{24} = 77.4
c. (a) 28.8 µF; (b) q_{19} = 1584, q_5 = 1584, q_6 = 396, q_{24} = 317
d. (a) 7.04 µF; (b) q_{19} = 148, q_5 = 38.9, q_6 = 46.7, q_{24} = 77.4

---

[12] A 40.0µF spherical capacitor is composed of two metal spheres one having a radius four times as large as the other. If the region between the spheres is a vacuum, determine the volume of this region.

a. 5.18×10^{18} m^3
b. 1.46×10^{13} m^3
c. 1.37×10^{13} m^3
d. 5.26×10^{18} m^3

---

[13] A 18.0 m metal wire is cut into five equal pieces that are then connected side by side to form a new wire the length of which is equal to one-fifth the original length. What is the resistance of this new wire?

a. 90.0 Ω
b. 3.60 Ω
c. 0.720 Ω
d. 19.0 Ω

---

[14] A small sphere that carries a charge of 8.00 nC is whirled in a circle at the end of an insulating string. The angular frequency of rotation is 100 πrad/s. What average current does the rotating rod represent?

a. 251 nA
b. 400 nA
c. 127 nA
d. 160 nA

[15] An aluminun wire with a cross sectional area of 4.00×10^{-6} m^2 carries a current of 5.00A. Find the drift speeds of the electrons in the wire. The density of aluminum is 2.70 g/cm^3. (Assume that one electron is supplied by each atom.)

a. 9.45×10^{-4} m/s
b. 1.30×10^{-4} m/s
c. 1.78×10^{-7} m/s
d. 7.71×10^{-3} m/s

[16] A 16.0 V battery is connected to a 100 Ω resistor. Neglecting the internal resistance of the battery, calculate the power dissipated in the resistor.

a. 0.0625 W
b. 1.60 W
c. 16.0 W
d. 0.391 W

[17] An electric current is given by $I(t) = 120.0 \sin(1.10\pi t)$, where I is in amperes and t is in seconds. What is the total charge carried by the current from $t = 0$ to $t = 1/220$ s?

a. 0.347 C
b. 120 C
c. 415 C
d. 1.09 C

[18] A resistor is constructed of a carbon rod that has a uniform cross sectional area of 3.00mm When a potential difference of 19.0 V is applied across the ends of the rod, there is a current of 2.00×10^{-3} A in the rod. Find (a) the resistance of the rod and (b) the rod's length.

a. (a) 1.05 kΩ; (b) 998 mm
b. (a) 9.50 kΩ; (b) 111 mm
c. (a) 9.50 kΩ; (b) 814 mm
d. (a) 1.05 kΩ; (b) 902 mm

[19] A copper cable is designed to carry a current of 500 A with a power loss of 1.00 W/m. What is the required radius of this cable?

a. 3.68 cm
b. 6.76 cm
c. 13.5 cm
d. 7.36 cm

[20] In a certain stereo system, each speaker has a resistance of 6.00 Q. The system is rated at 50.0 W in each channel, and each speaker circuit includes a fuse rated at 2.00 A. Is the system adequately protected against overload?

a. yes
b. no
c. n/a
d. n/a

[21] Compute the cost per day of operating a lamp that draws 1.70 A from a 110 V line if the cost of electrical energy is $0.06kWh.

a.   0.458 cents/day
b.   45.8 cents/day
c.   1.12 cents/day
d.   26.9 cents/day

[22] Calculate the power dissipated in each resistor of the circuit of Figure 5.
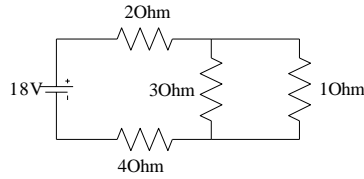
Figure 5

a. $P_2 = 14.2$ W, $P_4 = 28.4$ W, $P_3 = 1.33$ W, $P_1 = 4.00$ W
b. $P_2 = 162$ W, $P_4 = 81\ 0$ W, $P_3 = 60.8$ W, $P_1 = 20.3$ W
c. $P_2 = 14.2$ W, $P_4 = 28.4$ W, $P_3 = 12.0$ W, $P_1 = 0.444$ W
d. $P_2 = 162$ W, $P_4 = 81.0$ W, $P_3 = 20.3$ W, $P_1 = 60.8$ W

[23] When two unknown resistors are connected in series with a battery, 225 W is dissipated with a total current of 7.00 A. For the same total current, 45.0 W is dissipated when the resistors are connected in parallel. Determine the values of the two resistors.

a. $3.32\ \Omega$, $2.40\ \Omega$
b. $3.32\ \Omega$, $7.92\ \Omega$
c. $1.27\ \Omega$, $5.86\ \Omega$
d. $1.27\ \Omega$, $3.32\ \Omega$

[24] A fully charged capacitor stores 14.0 J of energy. How much energy remains when its charge has decreased to half its original value?

a. 3.50 J
b. 7.00 J
c. 56.0 J
d. 14.0 J

[25] For the current shown in Figure 6, calculate (a) the current in the 1.00 Ω resistor and (b) the potential difference between the points a and b.
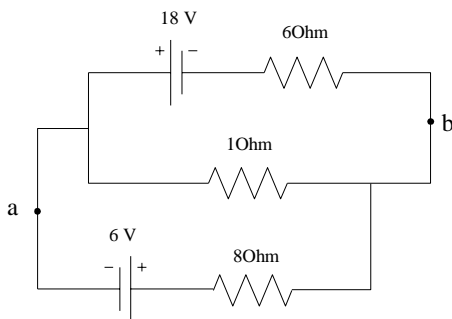
Figure 6

a.   (a) 2.90 A; (b) -2.90 V
b.   (a) 1.74 A; (b) -1.74 V
c.   (a) 1.74 A; (b) 1.74 V
d.   (a) 2.90 A; (b) 2.90 V

---

[26] Three 4.00 Ω resistors are connected as in Figure 7.
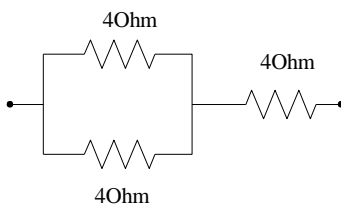
Figure 7

Each can dissipate a maximum power of 34.0 W without being excessively heated. Determine the maximum power the network can dissipate.

a. 434 W
b. 22.7 W
c. 51.0 W
d. 102 W

---

[27] Consider the circuit shown in Figure 8.
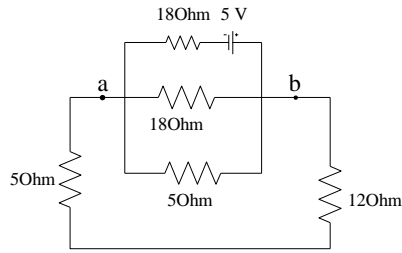
Figure 8

Find (a) the current in the 12.0 Ω resistor and (b) the potential difference between points a and b.

a.   (a) 0.0554 A; (b) 0.665 V
b.   (a) 0.294 A; (b) 5.00 V
c.   (a) 0.250 A; (b) 4.25 V
d.   (a) 0.0442 A; (b) 0.751 V

---

[28] Two resistors connected in series have an equivalent resistance of 590 Ω. When they are connected in parallel, their equivalent resistance is 125 Ω. Find the resistance of each resistor.

a.   327 Ω, 263 Ω
b.   327 Ω, 202 Ω
c.   180 Ω, 410 Ω
d.   180 Ω, 54.8 Ω

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Solution of the multiple choice questions

Q. No.	Answer	Q. No.	Answer
1	d	15	b
2	d	16	c
3	c	17	a
4	b	18	c
5	a	19	a
6	b	20	a
7	c	21	d
8	c	22	a
9	d	23	d
10	c	24	a
11	b	25	b
12	a	26	c
13	c	27	d
14	b	28	c

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APPENDICES
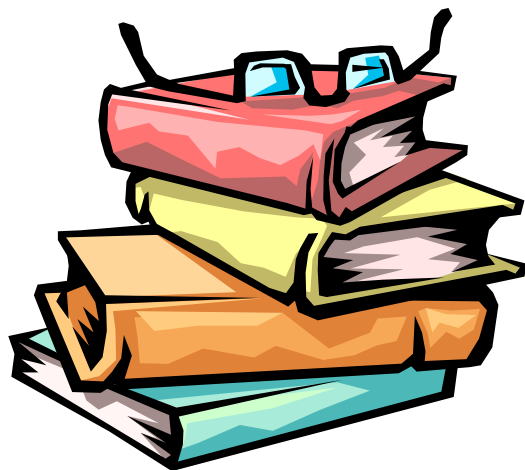
APPENDIX (A)

Some practical application of electrostatic

Understanding the Van de Graaff generator

Now that you understand something about electrostatics and static electricity, it is easy to understand the purpose of the Van de Graaff generator. A Van de Graaff generator is a device designed to create static electricity and make it available for experimentation.

The American physicist Robert Jemison Van de Graaff invented the Van de Graaff Generator in 1931. The device that bears his name has ability to produce extremely high voltages - as high as 20 million volts. Van de Graaff invented the Van de Graaff Generator to supply the high energy needed for early particle accelerators. These accelerators were known as *atom smashers* because they accelerated sub-atomic particles to very high speeds and then "smashed" them into the target atoms. The resulting collisions create other subatomic particles and high-energy radiation such as X-rays. The ability to create these high-energy collisions is the foundation of particle and nuclear physics.

Van de Graaff Generators are described as "constant current" electrostatic devices. When you put a load on a Van de Graaff generator, the current (amperage) remains the same. It's the voltage that varies with the load. In the case of the Van de Graaff generator, as you approach the output terminal (sphere) with a grounded object, the voltage will decrease, but as stated above, the current will remain the same. Conversely, batteries are known as "constant voltage" devices because when you put a load on them, the voltage remains the same. A good example is your car battery. A fully charged car battery will produce about 12.75 volts. If you turn on your headlights and then check your battery voltage, you will see that it remains relatively unchanged (providing your battery is healthy). At the same time, the current will vary with the load. For example, your headlights may require 10 amps, but your windshield wipers may only require 4 amps. Regardless of which one you turn on, the voltage will remain the same.

There are two types of Van de Graaff generators, one that uses a high voltage power supply for charging and one that uses belts and rollers for charging. Here we will discuss the belts and rollers type.

Van de Graaff generators are made up of a motor, two rollers, a belt, two brush assemblies, and an output terminal (usually a metal or aluminum sphere), as shown in the figure:

When the motor is turned on, the lower roller (charger) begins turning the belt. Since the belt is made of rubber and the lower roller is covered in silicon tape, the lower roller begins to build a negative charge and the belt builds a positive charge. By now, you should understand why this charge imbalance occurs, but you may want to look at the Triboelectric Series again (see chapter 1). Silicon is more negative than rubber; therefore, the lower roller is capturing electrons from the belt as it passes over the roller. It is important to realize that the charge on the roller is much more concentrated than the charge on the belt. Because of this concentration of charge, the roller's electric field is much stronger than the belt's at the location of the roller and lower brush assembly. The strong negative charge from the roller now begins to do two things:

1. It repels the electrons near the tips of the lower brush assembly. Metals are good conductors because they are basically positive atoms surrounded by easily movable electrons. The brush assembly now has wire tips that are positively charged because the electrons have moved away from the tips, towards the connection at the motor housing.
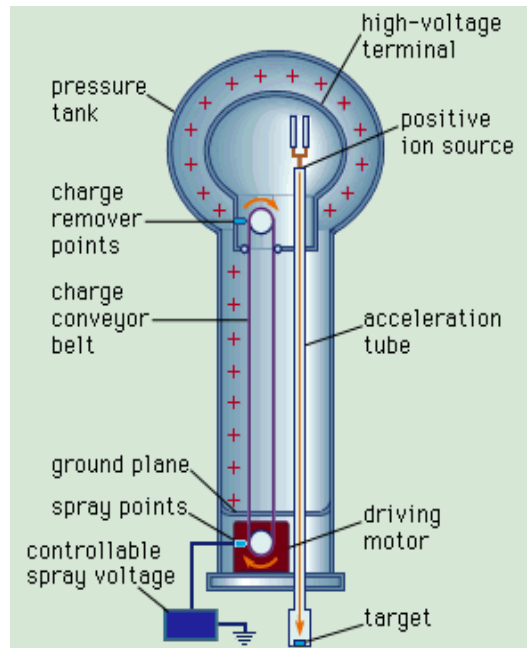
2. It begins to strip nearby air molecules of their electrons. When an atom is stripped of its electrons, it is said to be plasma, the fourth state of matter. So we have free electrons and positively charged atoms of air existing between the roller and the brush. The electrons repel from the roller and attract to the electron-less brush tips while the positive atoms attract to the negatively charged roller.

The positively charged atomic nuclei from the air molecules try to move toward the negatively charged roller, but the belt is in the way. So now the belt gets "coated" with the positive charge, which it then carries away from the roller.

You should now understand that as long as there is air between the lower roller and brush assembly, the Van de Graaff generator will continue to charge the belt. Theoretically, the Van de Graaff generator can continue to charge forever. Unfortunately, dirt and other impurities in the surroundings will limit the actual charge that develops on the sphere.

Let's return to the belt. The belt, as we left it, is positively charged and rolling toward the upper roller and upper brush assembly. Since I used nylon for my upper roller, it wants to repel the charge on the belt. The upper brush assembly is connected to the inside of the sphere and hangs near the upper roller and belt location. The electrons in the brush move to the tips of the wires, because they are attracted to the positively charged belt. Once the air breaks down as before, the positive atomic nuclei of air are attracted to the brush. At the same time, the free electrons in the air move to the belt. When a charged object touches the inside of a metal container, the container will take all of the charge, leaving the object neutral. The excess charge then shows up on the outside surface of the container. Here our container is the sphere. It is through this effect that the de Graaff generator is able to achieve its huge voltages (over one million volts). For the Van de Graaff generator, the belt is the charged object delivering a continuous positive charge to the sphere.

One last note is a neutral material is used for the upper roller; thus the belt becomes neutral after the sphere sucks its excess charge away.

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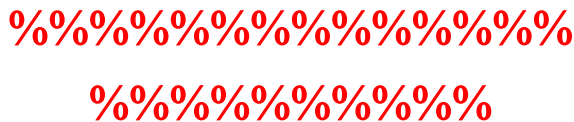
Cathode Ray Oscilloscope
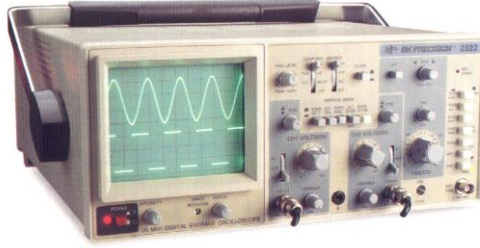
جهاز راسم الذبذبات الكهربائية

OSCILLOSCOPE, electronic instrument that records changes in the voltage of an electric or electronic circuit by a trace of light on the face of a cathode-ray tube (CRT). Oscilloscopes are widely used throughout industry and in scientific laboratories to test and adjust electronic equipment and to follow extremely rapid oscillations in electric voltages; the oscilloscope is capable of following changes that occur within several billionths of a second. Special converters attached to an oscilloscope can convert mechanical vibrations, sound waves, and other forms of oscillatory motion into electrical impulses that can be observed on the face of the CRT.

Oscilloscopes measure 2 things: voltage and time (and with time, often, frequency). An electron beam is swept across a phosphorescent screen horizontally (X direction) at a known rate (say 1 sweep in a millisecond). An input signal is used to change the position of the beam in the Y direction. The trace left behind can be used to measure the voltage of the input signal (off the Y axis) and the duration or frequency can be read off the X axis.

تنبعث الإلكترونات من الفتيلة بواسطة التسخين. ثم تعجل هذه الإلكترونات في اتجاه الانود بواسطة فرق جهد بناء على العلاقة التالية

$$\frac{1}{2}mv^2 = qV$$

حيث أن q شحنة الإلكترون و V فرق الجهد بين الكاثود والانود.
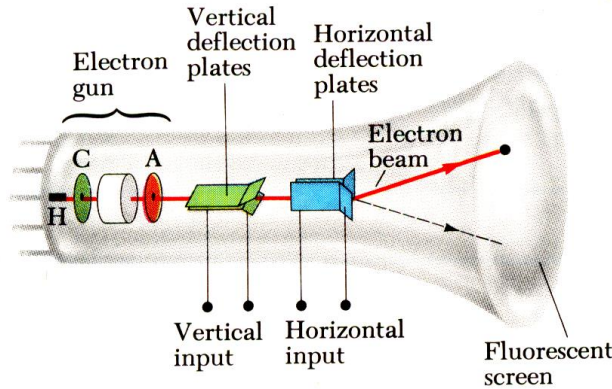
يمر شعاع الإلكترونات المعجلة بين زوجين من الألواح. الزوج الأول في وضع أفقي X-plate والزوج الثاني في وضع رأسي Y-plate. وبتطبيق فرق جهد بين اللوحين X-plate واللوحين -Y plate يمكن التحكم في مسار الشعاع الإلكتروني أفقيا ورأسيا. غالبا يكون X-plate موصلا في مصدر داخلي يعطى فرق جهد متغير له شكل سن المنشار sawtooth waveform وهو يتحكم في الثابت الزمني (Time base)

أما الإشارة المراد تتبعها تطبق على اللوحين Y-plate من خلال القناة Channel وفى اغلب أجهزة CRT الحديثة يكون هنالك قناتين وبالتالي شعاعين إلكترونيين. في النهاية يصطدم الشعاع الإلكتروني بشاشة فوسفورية كما هو الحال في جهاز التلفزيون وترسم شكل الموجة الداخلة إلى تلك القناة على الشاشة.

Speed of response is the cathode-ray oscilloscope's chief advantage over other plotting devices. General-purpose oscilloscopes have plotting frequencies of up to 100 megahertz (MHz). Response times as rapid as 2,000 MHz are achievable with special-purpose, high-speed oscilloscopes.
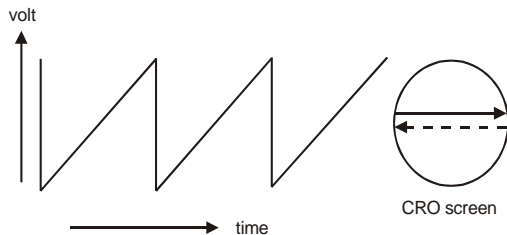
XEROGRAPHY (Photocopier)

XEROGRAPHY, electrostatic dry-printing process for the reproduction of images or documents, widely employed in commerce and industry in copying machines

The process was invented by the American printer Chester F. Carlson (1906–68) in 1937 and first commercially developed in 1950. It makes use of the principle of photoconductivity, that is, that certain substances resist passage of an electric current except when struck by light. Silicon, germanium, and selenium are poor conductors of electricity, but when light energy is absorbed by some of their electrons, the electrons are able to pass from one atom to another, thus allowing a current to flow when a voltage is applied. When the light is removed, their conductivity again becomes low. Xerography employs a photoconductive insulating layer, such as selenium, on an aluminum or other conductive metal support.

The layer is charged electrostatically, either with positive or negative ions, the polarity of the charge depending on the type of photoconductive insulating layer selected. When the plate is exposed, in a camera or photographic machine, those areas of the coating subjected to light lose a varying portion of the charge, depending upon the intensity of the illumination. Thus, the variation of the amount of charge retained on the coated metal plate is established as an electrical or electrostatic pattern of the image. The image is rendered visible by sprinkling over the exposed plate a special, charged powder, which carries an opposite charge to the initial charge applied to the plate and insulating layer. The powder adheres to those areas that have retained their charge. The print is obtained by covering the plate with paper, then applying a charge over the back of the paper of the same polarity as the initial charge applied to the photoconductive insulating layer. In this way the opposite charged powders are transferred to the paper surface. The powder image is then fused onto the paper by exposure to solvent vapors or heat to make the image permanent.

The entire xerographic process can be carried out, in high-speed mechanized equipment, in less than 5 seconds, and it is comparatively inexpensive to make these images because the photoconductive insulating layer can be

recycled many thousand times. The process has found its primary usefulness in copying office documents and in low-volume duplication of data.

(a) The photoconductive surface is positively charged. (b) Using a light source and lens, an image is formed in the surface in the form of hidden positive charges. (c) The surface containing the image with a charged powder, which adheres only to the image area. (d) A piece of paper is placed over the surface. This transfer the visible image to the paper, which is heated to ix the powder to the paper.

The xerographic method also permits the making, quickly and cheaply; of paper offset masterplates for low-to-medium-volume runs on office offset-printing presses. The method has also been applied to the production of X-ray images in a technique that is known as xeroradiography and is used in mammography for early detection of breast cancer. Xerography is applied in industrial nondestructive testing.

A variety of automatic xerographic machines are available today that make office copies on the push of a button and that are almost unable to fail. Xerographic machines can reproduce half-tone photographs and can enlarge or reduce copy. They can be set to print on either one or both sides of paper, to produce automatically a large or small number of copies without further intervention by the operator, and to sort and collate. A development of the late 1970s is color xerography. The economy, versatility, and flexibility of the xerographic process have created legal problems relating to copyright and helped stimulate numerous changes in the copyright laws in the U.S. and elsewhere.

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Battery

Battery, is a devices that convert chemical energy directly into electrical energy. Although the term battery, in strict usage, designates an assembly of two or more voltaic cells capable of such energy conversion, it is commonly applied to a single cell of this kind.

The mechanism by which a battery generates an electric current involves the arrangement of constituent chemicals in such a manner that electrons are released from one part of the battery and made to flow through an external circuit to another part. The part of the battery at which the electrons are released to the circuit is called the anode, or the negative electrode; the part that receives the electrons from the circuit is known as the cathode, or the positive electrode. (In a device that consumes current--*e.g.*, electroplating cell, electron tube, etc.--the term anode is often applied to the positive electrode, while the negative electrode is called the cathode.)
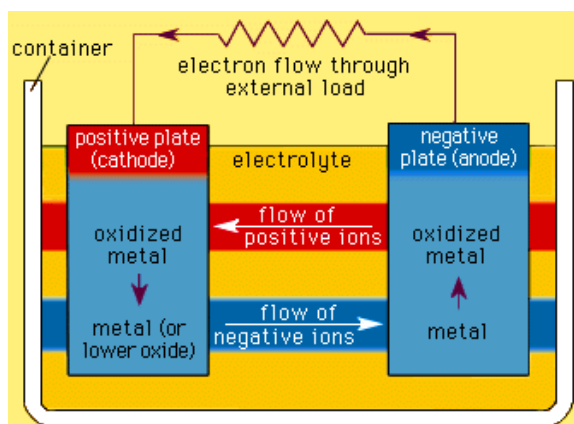
The first battery appears to have been constructed about 1800 by Alessandro Volta, a professor of natural philosophy at the University of Pavia in Italy. This device, later known as the voltaic pile, was composed of a series of silver and zinc disks in pairs, each of which was separated with a sheet of pasteboard saturated in salt water. A current was produced when the uppermost disk of silver was connected by a wire to the bottom disk of zinc. In 1836 the English chemist John Daniell developed what is considered the classic form of the voltaic cell.

A voltaic cell is composed of two chemicals with different electron-attracting capabilities that are immersed in an electrolyte and connected to each other through an external circuit. These two chemicals are called an

electrochemical couple. In a zinc-acid cell, for example, the electrochemical couple is a zinc-hydrogen ion couple. The reaction that occurs between an electrochemical couple in a voltaic cell is an oxidation-reduction reaction.
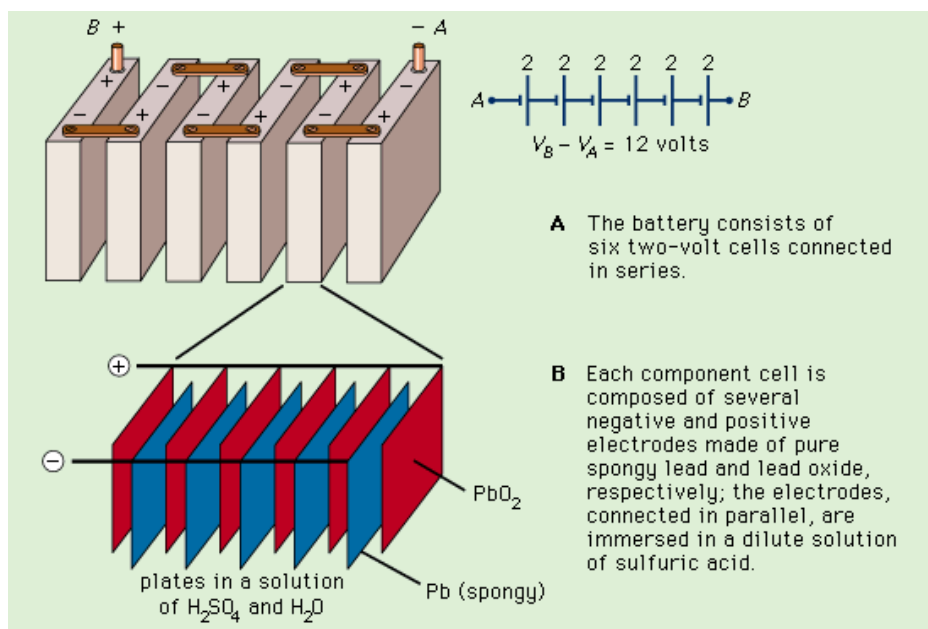
At rest, a voltaic cell exhibits a potential difference (voltage) between its two electrodes that is determined by the amount of chemical energy available when an electron is transferred from one electrode to the other and is thus subject to the chemical nature of the materials used in the electrode. The current that flows from a cell is determined by the resistance of the total circuit, including that of the cell itself. A low-resistance cell is required if extremely large currents are desired. This can be achieved by the utilization of electrodes with large areas. In short, the maximum current that can be drawn from a cell is dependent on the area of the electrodes. Whenever a current flows, the voltage of a cell decreases because of internal resistance of the cell and the slowness of the chemical process at the electrodes.

A voltaic cell has a limited energy content, or capacity, which is generally given in ampere-hours and determined by the quantity of electrons that can be released at the anode and accepted at the cathode. When all of the chemical energy of the cell has been consumed--usually because one of the electrodes has been completely exhausted--the voltage falls to zero and will not recover. The capacity of the cell is determined by the quantity of active ingredients in the electrode.

There are two major types of voltaic cells: primary batteries and secondary, or storage, batteries. (The latter are sometimes also called accumulators.) Primary cells are constructed in such a way that only one continuous or intermittent discharge can be obtained. Secondary devices, on the other hand, are constructed so that they can be discharged and then recharged to approximately their original state. The charging process is the reverse of the discharge process; therefore, the electrode reactions in these batteries must be reversible.

Primary batteries.

Several varieties of primary cells are available. These include dry, wet, and solid electrolyte.

Dry cells are not actually dry but contain an aqueous electrolyte that is unspillable or immobilized. Many of these cells are sealed to prevent seepage of the electrolyte or reaction products. Common examples of such primary batteries are acidic dry cells (*e.g.*, carbon-zinc cells), used in flashlights, toys, and certain transistorized portable radios; alkaline dry cells, employed in cameras, tape recorders, and electric razors; and mercury cells, utilized in hearing aids and photographic flash guns.

Wet cells contain a free and mobile electrolyte. They are used when service requirements include large capacity and moderately high currents, as, for example, in telephone and telegraph circuits and in signal systems for marine, mine, highway, and railway use. The Lalande cell of zinc-copper oxide--sodium hydroxide and the zinc-air-sodium hydroxide cell have been widely employed for such applications. Often included among wet primary cells is the so-called reserve battery, in which either magnesium-silver chloride or magnesium-cuprous chloride serves as the electrode material. Because these compounds are readily attacked by electrolytes (either seawater or liquid ammonia), the battery is assembled with the electrolyte stored in a separate container. The electrolyte is added immediately before use. The reserve battery has been utilized primarily for military applications.

Solid electrolyte cells contain electrolytes of crystalline salts such as silver iodide or lead chloride that have predominantly ionic conductivity. They are suitable for long-term operations that require very low drain or for stand-by services. Miniature versions of solid salt electrolyte batteries have been developed for use in various kinds of electronic devices.

Secondary batteries.

Such batteries consist of an assemblage of several identical voltaic cells. Of the various types of storage batteries available, the lead-acid type is the most widely used. It serves as the power source for the electrical systems of many kinds of motor vehicles, particularly automobiles and trucks. It is also commonly used to provide electricity for emergency lighting and communication circuits. The active constituents of a lead-acid battery are sulfuric acid and two sets of plates (electrodes)--one containing pure lead and the other lead dioxide. Each component cell consists of several of these plate pairs connected in parallel and is capable of delivering approximately two volts. Three or six cells are connected in series to make a 6- or 12-volt battery, respectively. The sulfuric-acid electrolyte in each of the cells is stored separately in its own compartment. During discharge, the plate materials are converted into lead sulfate and the sulfuric acid is depleted. Discharge stops before all component chemicals are exhausted, usually when the acid can no longer reach the active materials. Charging the battery by passing a direct current through it reverses the chemical changes, displacing the sulfate from the plates and causing a rise in the specific gravity of the sulfuric acid.

Another important type of storage battery is the nickel-cadmium battery, which operates much like the lead-acid variety but with different chemical ingredients. It consists of a nickel hydroxide cathode and a cadmium anode immersed in an electrolyte of potassium hydroxide solution. The nickel-cadmium battery is lighter in weight than are other storage batteries and can be hermetically sealed. Because of these features, nickel-cadmium cells are utilized in many kinds of cordless appliances and other portable equipment.

A third storage battery of significance is the silver-zinc type. The anode of this device is composed of a porous plate of zinc, the cathode of a silver screen pasted with silver oxide, and the electrolyte of a solution of potassium hydroxide saturated with zinc hydroxide. Silver-zinc batteries can be charged and discharged only a limited number of times because of the structure used to separate the electrodes. They do have, however, a high energy-to-weight ratio and are suitable for applications in which light weight is important.

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APPENDIX (B)

Answer of Some Selected Problems
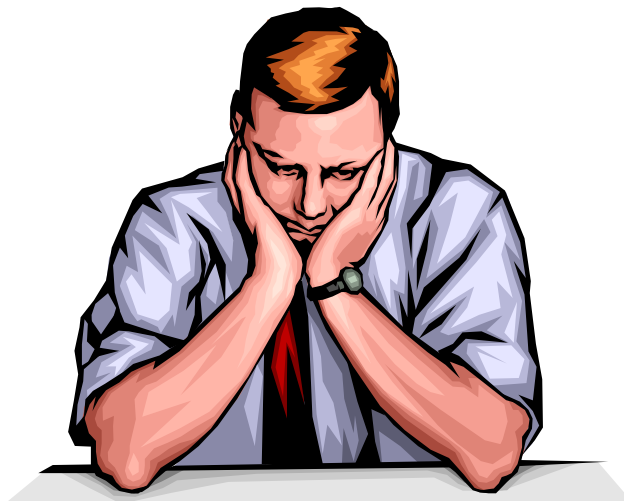
APPENDIX (B)

Answer of Some Selected Problems

Chapter 2

2.1) 1.6×10^{-9}N repelling each other

2.4) 1.2×10^{-5}C and 3.8×10^{-5}C

2.5) q=1/2Q

2.6) 8.5×10^{-2}N in the positive x-axis

2.8) 0.873N at 330°

2.11) (0,0.853m)

2.13) (b) $\pm2.4\times10^{-8}$C

Chapter 3

3.2) (a) $(-5.58\times10^{-11}$N/C)j, (b) $(1.02\times10^{-7}$N/C)j

3.3) (a) $(-5.20\times10^{3}$N/C)j, (b) $(2.93\times10^{3}$N/C)j and (c) 5.85×10^{3}N/C at 225°.

3.4) 5.6×10^{-11}C

3.7) 1×10^{3}N/C in the direction of the beam

3.8) (a) 1.5×10^{3}N/C, (b) 2.4×10^{-16}N up, (c) 1.6×10^{-26}N and (d) 1.5×10^{10}.

3.15) $E = \dfrac{q}{\pi\varepsilon_o a^2}$ along the bisector, away from the triangle

3.17) (a) 2.7×10^{6}m/s, (b) 1×10^{3}N/C

Chapter 4

4.2) 5.56×10^{5}N.m^2/C

4.7) (a) zero, (b) 5.4×10^{3}N/C and (c) 540N/C both radially outward

4.9) (a) zero, (b) 125×10^{4}N/C radially outward, (c) 639N/C radially outward and (d) no charge

4.10) (a) zero, (b) 8×10^{7}N/C radially outward (c) zero and (d) 7.35×10^{6}N/C radially outward

4.11) (i) $E = \dfrac{Qr}{4\,\pi\varepsilon_o a^3}$,

(ii) $E = \dfrac{Q}{4\,\pi\varepsilon_o r^2}$,

(iii) zero (iv) zero and (v) inner Q; outer zero.

4.12) (a) zero, (b) 3.66×10^{5}N/C, (c) 1.46×10^{6}N/C and (d) 6.5×10^{5}N/C radially outward

4.13) (a) 0.713μC, (b) 5.7μC

4.14) (i) $E = \dfrac{q}{2\,\pi\varepsilon_o l\,r}$ radially outward,

(ii)-q on both inner and outer surfaces,

(iii) $E = \dfrac{q}{2\,\pi\varepsilon_o l\,r}$ radially outward

4.15) (a) zero, (b) 7.2×10^{6}N/C away from the center

4.16) 5.08×10^{5}N/C up

4.17) (i) zero, (ii) $E = \dfrac{\sigma}{\varepsilon_o}$, and (iii) zero

Chapter 5

5.2) 6.41×10^{-19}C

5.4) 0.89m

5.5) 2m

5.6) 119nC, 2.67m

5.11) (a) –4500V, (b) same as (a) because potential is scalar quantity

5.13) (a) 2.5×10^6V, (b) 5.1J, and (c) 6.9J

5.15) (a) –386nJ positive binding energy would have to be put in to separate them. (b) 103V

Chapter 6

6.1) 18μC

6.3) 4.3pF

6.5) (a) q_1=9μC; q_2=16μC; q_3=9μC; q_4=16μC.

    (b) q_1=8.4μC; q_2=17μC; q_3=11μC; q_4=14μC.

6.6) (a) V_f=2V; (b) $U_i = \dfrac{1}{2}\dfrac{\varepsilon_o A V^2}{d}$, U_f=2U_i; (c) $W = \dfrac{1}{2}\dfrac{\varepsilon_o A V^2}{d}$

6.7) (a) 4.8×10^{-4}C, (b) V_4=120V; V_6=80V

6.11) 3300

6.12) (b) 3.2μF

6.14) 800pJ, 5.79mJ/m^3

6.16) (a) 4μF, (b) 8V; 24μC, 4V; 24μC, 12V, 24μC

6.18) 0.188m^2

6.19) (a) q_1=2.1×10^{-4}C; q_2=1.1×10^{-4}C; q_3=3.2×10^{-4}C. (b) V_1=V_2=21V; V_3=79V. (c) U_1=2.2×10^{-3}J; U_2=1.1×10^{-3}J; U_3=1.3×10^{-3}J;

Chapter 7

7.1) (a) 1.2×10^3C, (b) 7.5×10^{21} electron

7.2) 1.5×10^{-15}m/s

7.6) 3.23×10^6/Ω.m

7.9) (a) 2.4 iron being larger, (b) no

7.10) 6.43A

7.12) 2×10^6s

7.13) (a) 15V copper; 85V iron, (b) 8.5×10^7A/m^2, (c) 1.5V/m copper; 8.5V/m iron

7.18) (a) 11A, (b) 11Ω and (c) 1100kcal

Chapter 8

8.1) 7.67Ω

8.3) (a) 1.79A, (b) 10.4V

8.5) 12Ω

8.7) 2.4Ω

8.9) 8Ω

8.10) (a) 27.3V, (b) 2.73A; 1.37A; 0.91A

8.11) (b) 1.99A; 1.17A; 0.819

8.12) (6/11)R

8.13) 14W; 28.5W; 1.33W; 4w

8.17) i_1=50mA to the right; i_2=60mA down; V_{ab}=9V

8.18) (a) the left branch, 0.67A down; center branch, 0.33A up; the right branch, 0.33A up

8.19) (a) -10.4V

APPENDIX (C)

The international system of

units (SI)

APPENDIX (C)

The international system of units (SI)

SI Units http://tcaep.co.uk/science/siunits/index.htm

SI Units and Definitions

The Fundamental SI Units

Quantity	Unit	Abbreviation
Mass	kilogram	kg
Length	meter	m
Time	second	s
Temperature	kelvin	K
Electrical current	ampere	A
Luminous intensity	candela	cd
Amount of substance	mole	mol
Plane angle	radian	rad
Solid angle	steradian	sr

SI Prefixes

Prefix	Symbol	Factor	Prefix	Symbol	Factor
yatta	Y	10^{24}	deci	d	10^{-1}
zetta	Z	10^{21}	centi	c	10^{-2}
exa	E	10^{18}	milli	m	10^{-9}
peta	P	10^{15}	micro	μ	10^{-6}
tera	T	10^{12}	nano	n	10^{-9}
giga	G	10^{9}	pico	p	10^{-12}
mega	M	10^{6}	femto	f	10^{-15}
kilo	k	10^{3}	atto	a	10^{-18}
hecto	h	10^{2}	zepto	z	10^{-21}
deca	da	10^{1}	yocto	y	10^{-24}

SI Derived Units expressed in terms of base units

Quantity	Name	Symbol
area	square meter	m^2
volume	cubic meter	m^3
speed, velocity	meter per second	m/s
acceleration	meter per second square	m/s^2
wave number	1 per meter	m^{-1}
density, mass density	kilogram per cubic meter	kg/m^3
specific volume	cubic meter per kilogram	m^3/kg
current density	ampere per square meter	A/m^2
magnetic field strength	ampere per meter	A/m
concentration	mole per cubic meter	mol/m^3
luminance	candela per square meter	cd/m^2

SI Derived Units with special names

Quantity	Name	Symbol	Expression in terms of other units	Expression in terms of SI base units
frequency	hertz	Hz		s^{-1}
force	newton	N		$m\ kg\ s^{-2}$
pressure, stress	pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
energy, work, quantity of heat	joule	J	N m	$m^2\ kg\ s^{-2}$
power, radiation flux	watt	W	J/s	$m^2\ kg\ s^{-3}$
electric charge	coulomb	C		s A
electric potential difference	volt	V	W/A	$m^2\ kg\ s^{-3}A^{-1}$
electromotive force	volt	V	W/A	$m^2\ kg\ s^{-3}A^{-1}$
capacitance	farad	F	C/V	$m^{-2}\ kg^{-1}\ s^4A^2$
electric resistance	ohm	Ω	V/A	$m^2\ kg\ s^{-3}A^{-2}$
electric conductance	siemens	S	A/V	$m^{-2}\ kg^{-1}\ s^3A^2$
magnetic flux	weber	Wb	V s	$m^2\ kg\ s^{-2}A^{-1}$
magnetic flux density	tesla	T	Wb/m^2	$kg\ s^{-2}A^{-1}$
inductance	herny	H	Wb/A	$m^2\ kg\ s^{-2}A^{-2}$
temperature	degree celsius	^oC		K
luminance flux	lumen	lm		cd sr
luminance	lux	lx	lm/m^2	$m^{-2}\ cd\ sr$
activity of radionuclide	becquerel	Bq		s^{-1}
absorbed dose	gray	Gy	J/kg	m^2s^{-2}
dose equivalent	sievert	Sv	J/kg	m^2s^{-2}

SI Derived Units Expressed by Means of Special Names

Quantity	Name	Symbol	Expression in terms of SI base units
Dynamic Viscosity	pascal second	Pa s	$m^{-1} \, kg \, s^{-1}$
Moment of Force	newton meter	N m	$M^2 \, kg \, s^{-2}$
Surface Tension	newton per metre	N/m	$kg \, s^{-2}$
Heat Flux Density / Irradiance	watt per square metre	W/m^2	$kg \, s^{-3}$
Heat Capacity / Entropy	joule per kelvin	J/K	$M^2 \, kg \, s^{-2} \, K^{-1}$
Specific Heat Capacity / Specific Entropy	joule per kilogram kelvin	J/(kg K)	$m^2 \, s^{-2} \, K^{-1}$
Specific Energy	joule per kilogram	J/kg	$M^2 \, s^{-2}$
Thermal Conductivity	watt per metre kelvin	W/(m K)	$m \, kg \, s^{-3} \, K^{-1}$
Energy Density	joule per cubic metre	J/m^3	$m^{-1} \, kg \, s^{-2}$
Electric Field Strength	volt per metre	V/m	$m \, kg \, s^{-3} \, A^{-1}$
Electric Charge Density	coulomb per cubic metre	C/m^3	$m^{-3} \, s \, A$
Electric Flux Density	coulomb per square metre	C/m^2	$m^{-2} \, s \, A$
Permittivity	farad per metre	F/m	$m^{-3} \, kg^{-1} \, s^4 \, A^2$
Permeability	henry per metre	H/m	$m \, kg \, s^{-2} \, A^{-2}$
Molar Energy	joule per mole	J/mol	$m^2 \, kg \, s^{-2} \, mol^{-1}$
Molar Entropy / Molar Heat Capacity	joule per mole kelvin	J/(mol K)	$m^2 \, kg \, s^{-2} \, K^{-1} \, mol^{-1}$
Exposure (x and γ rays)	coulomb per kilogram	C/kg	$kg^{-1} \, s \, A$
Absorbed Dose Rate	gray per second	Gy/s	$M^2 \, s^{-3}$

SI Supplementary Units

Quantity	Name	Symbol	Expression in terms of SI base units
Plane Angle	radian	rad	$m\ m^{-1} = 1$
Solid Angle	steradian	sr	$m^2\ m^{-2} = 1$

SI Derived Units Formed Using Supplementary Units

Quantity	Name	Symbol
Angular Velocity	radian per second	rad/s
Angular Acceleration	radian per second squared	rad/s^2
Radiant Intensity	watt per steradian	W/sr
Radiance	watt per square metre steradian	$W/(m^2\ sr)$

Metric and Imperial Measures

Length

1 centimeter	cm	= 10 mm	= 0.3937 in
1 meter	m	= 100 cm	= 1.0936 yd
1 kilometer	km	= 1000 m	= 0.6214 mile
1 inch	in		= 2.54 cm
1 yard	yd	= 36 in	= 0.9144 m
1 mile		= 1760 yd	= 1.6093 km

Surface or Area

1 sq cm	cm^2	= 100 mm^2	= 0.1550 in^2
1 sq m	m^2	= 10000 cm^2	= 1.1960 yd^2
1 sq km	km^2	= 100 ha	= 0.3861 $mile^2$
1 sq in	in^2		= 6.4516 cm^2
1 sq yd	yd^2	= 9 ft^2	= 0.8361 m^2
1 sq mile	$mile^2$	= 640 acres	= 2.59 km^2

Volume and Capacity

1 cu cm	cm^3		= 0.0610 in^3
1 cu m	m^3	= 1000 dm^3	= 1.3080 yd^3
1 liter	l	= 1 dm^3	= 0.220 gal
1 hectoliter	hl	= 100 l	= 21.997 gal
1 cu in	in^3		= 16.387 cm^3
1 cu yd	yd^3	= 27 ft^3	= 0.7646 m^3
1 pint	pt	= 20 fl oz	= 0.56831 l
1 gallon	gal	= 8 pt	= 4.546 l

Mass

1 gram	g	= 1000 mg	= 0.0353 oz
1 kilogram	kg	= 1000 g	= 2.2046 lb
1 tonne	t	= 1000 kg	= 0.9842 ton
1 ounce	oz	= 437.5grains	= 28.35 g
1 pound	lb	= 16 oz	= 0.4536 kg
1 ton		= 20cwt	= 1.016 t

US Measures

1 US dry pint	= 33.60 in^3	= 0.5506 l
1 US liquid pint	= 0.8327 imp pt	= 0.4732 l
1 US gallon	= 0.8327 imp gal	= 3.785 l
1 short cwt	100 lb	= 45.359 kg
1 short ton	2000 lb	= 907.19 kg

Symbols
Greek Alphabet
'**http://tcaep.co.uk/science/symbols/greek.htm**

A	α	alpha	Ξ	ξ	xi
B	β	beta	O	o	omicron
Γ	γ	gamma	Π	π	pi
Δ	δ	delta	P	ρ	rho
E	ε	epsilon	Σ	σ	sigma
Z	ζ	zeta	T	τ	tau
H	η	eta	Y	υ	upsilon
Θ	θ	theta	Φ	ϕ	phi
I	ι	iota	X	χ	chi
K	κ	kappa	Ψ	ψ	psi
Λ	λ	lambda	Ω	ω	omega
M	μ	mu	Ξ	ξ	xi
N	ν	nu			

Periodic Table

	1	2		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	I	II		III	IV	V	VI	VII		VIII			I	II	III	IV	V	VI	VII	VIII
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca		21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr		39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	La†	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	Ac‡	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Uun	111 Uuu	112 Uub	113	114 Uuq	115	116 Uuh	117	118 Uuo

† Lanthanides	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	
‡ Actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	

Obtained from http://tcaep.co.uk/science/periodic/index.htm

APPENDIX (D)

Physics Resources on the Web
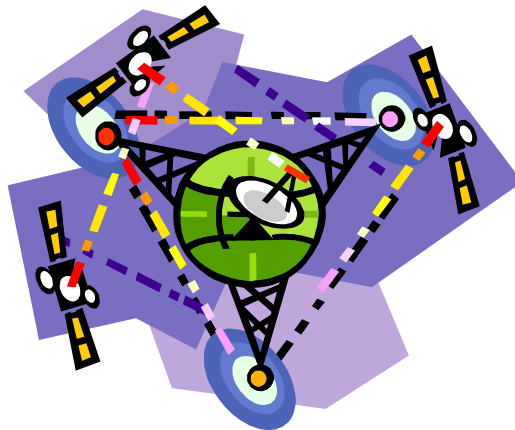
APPENDIX (D)

Physics Resources on the Web

Astronomy and Astrophysics

Center for Particle Astrophysics	http://physics7.berceley.edu/home.hml
NASA Home Page	http://www.gsfc.nasa.gov/NASAhomepage.html
Institutes of Astronomy	http://www.yahoo.com/Science/Astronomy/Institutes
WebStars: Astrophysics on the Web	http:// .gsfc.nasa.gov/WebStars/About.html
Astrotext:(Online hyper-textbook)	http://uu.gna.mit.edu:8001/uu-gna/text/astro
Earth Views	http://images.jsc.nasa.gov/html/earth.htm
National Optical Observatories	http://www.noao.edu/noao.html
Observatories Around the World	http://www.yahoo.com/Science/Astronomy/Observatories
Space Telescope Science Institute	http://marvel.stsci.edu/top.html
Strasburg Astronomical Data Center	http://cdsweb.u-strasbg.fr/CDS.html
WebStars: Astrophysics in Cyberspace	http://guinan.gsfc.nasa.gov
NASA Research Labs	http://www.yahoo.com/Government/Research_Labs/NASA
Astronomy Resources	http://www.stsci.edu/net-resources.html

Computers in Physics and Laboratory Automation

The Computer Society	http://www.computer.org/
Physical Computing A hands-on Tutorial.	http://www.itp.tsoa.nyu.edu/~alumni/dano/physical/physical.html
ADAC Manufacturers of Labtech Notebook.	http://www.adac.com/adac/index.html
Keithely.	http://www.keithley.com/
Strawberry Tree.	http://www.strawberrytree.com/
National Instruments.	http://www.natinst.com/
Links to TEX/LATEX Resources	http://www.it.kth.se/misc/external/teX-links.html
Computers in Physics(Journal)	http://www.aip.org/cip/ciphome.html
IEEE Spectrum(Journal)	http://www.ieee.org/spectrum/spechome.html

Condensed Matter Resources

Condensed Physics Labs	http://www-cmt.physics.brown.edu/cmt/resources/groups.html
Society of Vacuum Coaters	http://www.svc.org
Materials Research Society	http://www.mrs.org

Energy Research Resources

Department of Energy (DOE)	http://www.ofr.er.doe.gov/
DOE Office of Energy Research	http://www.er.doe.gov/
National Renuable Energy Laboratory (NREL)	http://www.nrel.gov/
Solar Energy	www.netins.net/showcase/solarcatalog/

High Energy Physics (HEP)

Fermilab	http://fnnews.fnal.gov/
HEP Labs Around the World	http://www.cern.ch/Physics/HEP.html
HEP Information Center	http://www.hep.net/
Experiments On-Line: home pages of individual HEP experiments	http://www-slac.slac.stanford.edu/find/explist.html
Institute of High Energy Physics (China)	http://www.ihep.ac/ihep.html/
PDG - The Particle Data Group	http://pdg.lbl.gov/
FreeHEP - Library of High Energy Physics Software	http://heplibw3.slac.stanford.edu/find/fhmain.html
HEPIC - High-Energy Physics Information Center	http://www.hep.net/

Nuclear Physics

The American Nuclear Society	http://www.ans.org/
DOE/NSF Nuclear Science Advisory Committee Home Page	http://www.nsf.gov/mps/phy/nuclear.htm
Radioactive Waste	http://www.nea.fr/html/trw/index.html/
Fusion	http://www.yahoo.con/Science/Physics/Fusion/
Austin Fusion Studies	http://w3fusion.ph.utexas.edu/
KVI Research Network	gopher://kviexp.kvi.nl/

Optics

OpticsNet (Optical Society of America)	http://www.osa.org/homepage.html

Particle Physics

Particle Physics and Cosmology	http://www.eagle.co.uk/ppi/home.html
Stanford Linear Accelerator Center	http://www.heplibw3.slac.stanford.edu/FIND/slac.html

Relativity

Einstein in 3D	http://www.ncsa.uiuc.edu/Apps/GenRel/SC93/HOME_sc93.html
Relativity	http://www.yahoo.com/Science/Physics/Relativity

X-ray Resources

Advanced Photon Source Mosaic Home Page	http://www.aps.anl.gov/welcome.html
Cornell High Energy Synchrotron Source	http://www.tn.cornell.edu/
Harvard X-Ray Group - studies various interfaces: liquid helium films, liquid metals, langmuir monolayers etc.	http://hutrough.harvard.edu/
IIT Synchrotron Radiation Program	http://www.csrri.iit.edu/
International XAFS Society(2)	http://www.birzeit.edu/Science/Physics/X_Ray/International_XAFS_Society/"
Real Time Studies of Crystal Growth with X-rays	http://www.tn.cornell.edu/SIG/RTgrowth/RTgrowth.contents.html
SUNY Stony Brook X-ray Physics	http://xray1.physics.sunysb.edu/
Synchrotron Radiation Materials Research Lab	http://syli34.physik.uni-bonn.de
UWXAFS - XAFS analysis programs from the University of Washington	http://krazy.phys.washington.edu/
X-Ray - development of industrial application of radiation after the discovery of x-rays.	http://www.ricoh.co.jp/net-messena/NDTWW/JSNDI/XRAY.html
X-ray absorption data	http://www.csrri.iit.edu/periodic-table.html
X-Ray WWW Server	http://xray.uu.se/
XAFS Database	http://xafsdb.iit.edu/database/

Other Physics Topics on the Web

Electromagnetic Fields and Microwave Electronics	http://www.ifh.ee.ethz.ch/"
Plasma Physics	http://plasma-gate.weizmann.ac.il/PlasmaI.html
Sounds From Chaos	http://www.ccsr.uiuc.edu/People/gmk/Papers/Chua/SndRef.html

Periodic Tables of the Elements

The Periodic Table of Elements	http://www.cs.ubc.ca/elements/periodic-table
Periodic Table in Hypertext Format	http://www.cchem.berkeley.edu/Table/
Periodic Table of the Elements (Online)	http://kuhttp.cc.ukans.edu/cwis/reference/Periodic/periodic.html
Webelements Dept. of Chemistry, Univ. of Scheffield	http://www2.shef.ac.uk/chemistry/

Physics Laboratories Worldwide

Fermilab	http://fnnews.fnal.gov/
High energy physics labs around the world	http://www.cern.ch/Physics/HEP.html
Condensed matter labs around the world	http://www-cmt.physics.brown.edu/cmt/resources/groups
Brookhaven National Laboratory	http://www.bnl.gov/
Stanford Linear Accelerator Center	http://www.heplibw3.slac.stanford.edu/FIND/slac.html
Cornell High Energy Synchrotron Source	http://www.tn.cornell.edu/
NIST Physics Laboratory	http://physics.nist.gov

Physics Related Journals

American Journal of Physics	http://www.amherst.edu/~aip/menu.html
Computers in Physics	http://www.aip.org/cip/ciphome.html
IEEE Spectrum	http://www.ieee.org/spectrum/spechome.html

Physics and Related Societies Worldwide

American Physical Society (APS)	http://aps.org/
American Institute of Physics	http://www.aip.org
American Mathematical Society	http://e-math.ams.org
National Science Foundation	http://stis.nsf.gov
The Electrochemical Society	http://www.electrochem.org
Society of Vacuum Coaters	http://www.electrochem.org
American Nuclear Society	http://www.ans.org
Optical Society of America	http://www.osa.org/homepage.html
The Computer Society	http://www.computer.org
The European Physical Society	http://www.nikhef.nl/www/pub/eps/eps.html
Materials Research Society	http://www.mrs.org

Publishers and Published Resources

Addison Wesley - Physics	http://www.AW.com/he/Physics/PhysIndex.html
John Wiley :Wiley College(Physics)	http://www.wiley.co.uk/college/math/phys/index.html
AIP Press Catalog	http://www.aip.org/aippress
Institute of Physics Publishing	http://iop.org
Index of Abstracts	http://xxx.lanl.gov

Other Physics Resource Sites

TIPTOP: The Internet Pilot TO Physics	http://www.tp.umu.se/TIPTOP
World Wide Web Virtual Library - Physics	http://www.w3.org/vl/Physics/Overview.html
Einet Galaxy - Physics (Science)	http://www.einet.net/galaxy/Science/Physics.html
AIP's LinkLIST	http://www.aip.org/aip/physres.html
Yahoo: A Guide to the WWW - Physics	http://www.yahoo.com/Science/Physics
World Wide Web Index - Physics	http://www.het.brown.edu/Physics/index.html
Physics Servers and Services Around the World	http://www.physics.mcgill.ca/deptdocs/physics services.html
The Virtual Review	http://www.het.brown.edu/physics/review/index.html
Astronomical Internet Resources	http://marvel.stsci.edu/net-resources.html

APPENDIX (E)

Bibliography

APPENDIX (E)
Bibliography

The Subject of this book "***Electrostatic: Principles and Applications***" may, of course, found in many textbooks of general physics. Some of the books, in particular, have good and interesting discussions of electrostatic (for example, R. A. Serway, Physics for scientists and engineering with modern physics. In the following list comprises books that relate either to individual topics or to the whole scope of the present text. In general these references are comparable in level to the present books.

Borowitz and Beiser "*Essentials of physics*". Addison-Wesley Publishing Co., 1971.

Halliday, D. and Resnick, R. "*Physics* (*part two*)". John Wiley & Sons, Inc., 1978.

Kubala, T.S., "*Electricity 2: Devices, Circuits and Materials*", 2001

Nelkon, M. and Parker, P. "*Advanced level physics*". Heinemann Educational Books Ltd., 1982.

Ryan, C.W., "*Basic Electricity : A Self-Teaching Guide*", 1986

Sears, F.W., Zemansky, M.W. and Young, H.D. "*University physics*" Addison-Wesley Publishing Co., 1982.

Serway, R. A. "*Physics for scientists and engineering with modern physics*". Saunders College Publishing, 1990.

Weidner, R.T. and Sells, R.L. "*Elementary physics: classical and modern*". Allyn and Bacon, Inc., 1973.

Valkenburgh, N.V., "*Basic Electricity: Complete Course*", 1993

APPENDIX (F)

Index

APPENDIX (F)

Index