من إصدارات

أكاديمية الفيزياء للتعليم الإلكتروني

General Physics (2)

Electrostatic: Principles & Applications

سلسلة محاضرات الفيزياء العامة (٢)

علم الميكانيكا وتطبيقاتها

الدكتور حازم فلاح سكيك

استاذ الفيزياء المشارك

جامعة الأزهر-غزة

أكاديمية الفيزياء

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مقدمة

نقدم لكم أعزائنا طلبة الكليات العلمية هذه السلسلة من محاضرات الفيزياء العامة (٢) بعنوان الكهربية الساكنة وتطبيقاتها في صورة شرائح بوربوينت اعتمادا على كتاب الفيزياء Physics for scientists and engineers لمؤلفه Serway الكتاب المعتمد في معظم الكليات العلمية بمختلف الجامعات لتكون عونا لطلبة المستوى الأول في الكليات العلمية.

تغطي هذه الشرائح مواضيع المنهاج المقرر لفيزياء الكهربية الساكنة حيث تقدم المحاضرات تسلسل تطور علم الكهربية الساكنة من تعريف الشحنة الكهربية وخصائصها وطريقة استخدام قانون كولوم لحساب القوة المتبادلة بين شحنتين وحساب القوة المؤثرة على شحنة الناتج عن شحنات أخرى، ثم نتطرق إلى موضوع المجال الكهربي والجهد الكهربي بالتعريف والتوضيح وشرح كافة المواضيع المتعلقة بها. كما يأتي جزء أخر من المحاضرات حول تطبيقات الكهربية الساكنة من خلال مواضيع تشرح المكثف والسعة الكهربية والمقاومة والتيار والدوائر الكهربية. شرحت هذه المواضيع من خلال تقسيمها إلى محاضرات مدعمة بالصور والشروحات العديد من الأمثلة المحلولة مع الشرح والتوضيح وكذلك في نهاية كل محاضرة مجموعة من الأسئلة التي تتيح للطالب ان يختبر نفسه ومدى استيعابه لما قدم في كل محاضرة.

يصاحب كل محاضرة تسجيل فيديو يشرح المحاضرة ويوضحها بأحدث الوسائل التعليمة على قناة موقع الفيزياء التعليمي على قناة اليوتيوب والتي يمكن ان تجدها بسهولة بالبحث في أداة بحث موقع اليوتيوب عن قناة د. حازم فلاح سكيك وللتسهيل على الطلبة تم تجميع كل المحاضرات المتعلقة بالمنهج في قائمة تشغيل بعنون محاضرات الفيزياء العامة (٢) الكهربية الساكنة وقسمت كل محاضرة إلى مقاطع زمنية لا تتجاوز ٣٠ دقيقة واعطي اسم لكل مقطع يتوافق مع اسم المحاضرة ورقمها. ادرج في تسجيل شرح كل محاضرة العديد من الأفلام العلمية ذات العلاقة مع موضوع المحاضرة.

أتمنى ان تكون هذه المحاضرات مساعدا أساسيا لكل الطلبة الدارسين لمقرر الفيزياء العامة (٢) في فهم هذا المقرر واستيعابه.

كما سوف ندرج على موقع اكاديمية الفيزياء للتعليم الالكتروني www.physicsacademy.org بنك من الأسئلة متعددة الاختيار ليتمكن كل دارس من اختبار مقدار فهمه للمقرر وتقييم مستوى تحصيله في هذا المقرر. يسعدني سماع تعليقاتكم واقتراحاتكم لتطوير هذه السلسلة من المحاضرات داعيا الله عز وجل ان يوفقكم ويسدد خطاكم.

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General Physics II

Electrostatic: Principles & Applications

Lecture (0): Properties of Electrostatic

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Properties of Electrostatic

Electric charge

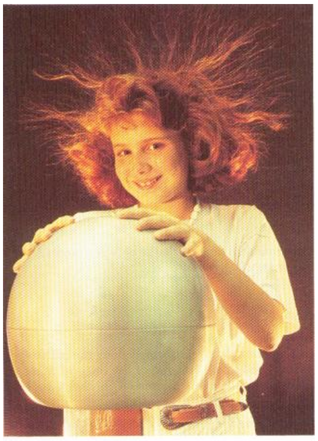
Conductor and insulator

Positive and negative charge

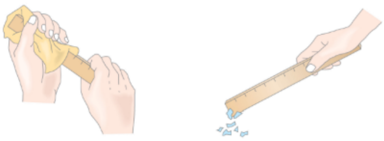
Charge is conserved

Charge and Matter

Charge is Quantized



Electric Charge



The early Greeks knew that if a piece of amber was rubbed, it would attract bits of straw. This is an early example of electrostatics. The English word, electron, is derived from the Greek work for amber.

The ancient Greeks also knew that a certain type of rock, called lodestone, would attract iron and always keep the same orientation if hung from a string and left free to rotate. This is an early example of magnetism.

But only in the 19th century did scientists realize that electrostatics and magnetism were both part of the same phenomena which we call electromagnetism.

James Clerk Maxwell took the ideas of Michael Faraday and some of his original discoveries and put them into mathematical form around the middle of the 19th century. We now know these laws as Maxwell's Equations

Conductor And Insulator

Conductors are materials in which the electrons can move rather freely (i.e. they readily conduct a flow of electrons).

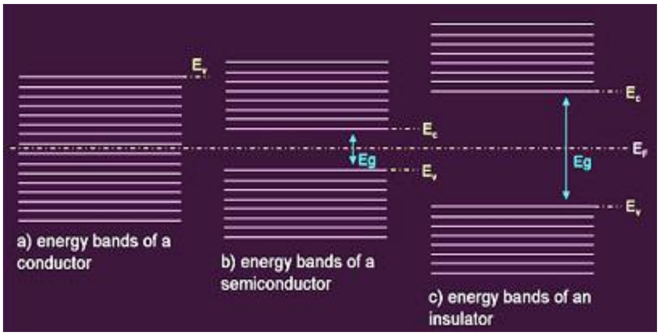
Non-conductors or Insulators are materials in which the electrons are more tightly bound to the atoms and generally are not free to move.

Examples of conductors are metals such as copper, silver, Aluminum plus salt water solutions (the human body falls into this category).

Examples of insulators are wood, plastic, stone; in short, any non-metal.

The earth acts as a large conductor and has a very large capacity to absorb charge concentrations from smaller conductors.

So any charge on a conductor will be lost if there is a path to ground.



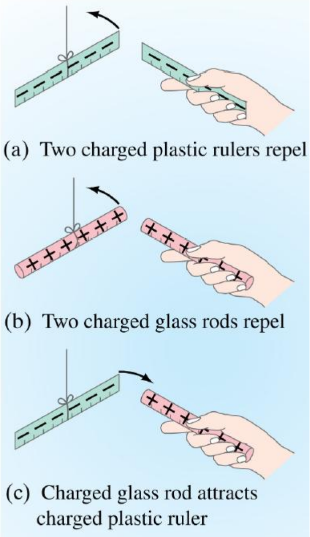
•Substances that fall between the metals and the insulators are called semiconductors.

•Semiconductors such as Silicon and Germanium are widely used in modern electronics since their properties may be radically altered by the addition of small amounts of impurity atoms.

•Superconductors are perfect conductors in the sense that they offer no resistance to the flow of charges.

Positive and Negative Charge

Like charge repel one another and unlike charges attract one another where a suspended rubber rod is negatively charged is attracted to the glass rod. But another negatively charged rubber rod will repel the suspended rubber rod.

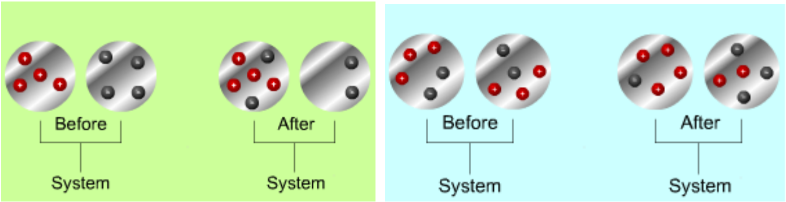

(a) Two charged plastic rulers repel

(b) Two charged glass rods repel

(c) Charged glass rod attracts charged plastic ruler

Charge is Conserved

Electric charge is conserved. The net charge of an isolated system may be positive, negative or neutral. Charge can move between objects in the system, but the net charge of the system remains unchanged.



Charge and Matter

القوى المتبادلة المسئولة عن التركيب الذرى أو الجزئي أو بصفة عامة للمواد هي مبدئيا قوى كهربائية بين الجسيمات المشحونة كهربياً، وهذه الجسيمات هي البروتونات والنيوترونات والإلكترونات.

Particle	Symbol	Charge	Mass
Proton	p	1.6×10^{-19}C	1.67×10^{-27}K
Neutron	n	0	1.67×10^{-27}K
Electron	e	-1.6×10^{-19}C	1.67×10^{-31}K

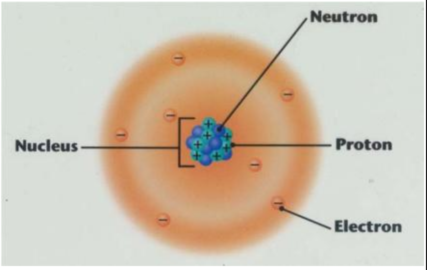
Charge is Quantized

In the early part of the 20th century **Robert Millikan** performed an experiment to determine the smallest possible charge in nature.

Millikan found that that basic charge is 1.6×10^{-19} Coulombs.

This was later found to be the charge on every proton and electron (negative for electrons).

Every experiment since then has observed the basic electron charge or some integral multiple of it.



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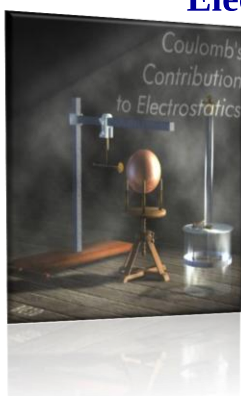
General Physics II

Electrostatic: Principles & Applications



Lecture (1): Coulomb's Law

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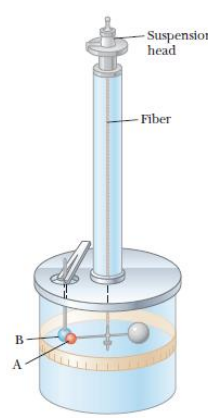
Charles Coulomb
1736 - 1806

Coulomb's Law

In 1785, Coulomb established the fundamental law of *electric force* between two stationary, charged particles. Experiments show that an electric force has the following properties:



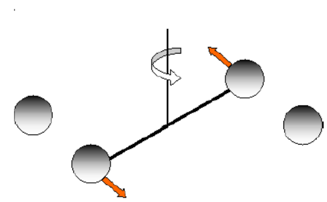
(1) The force is *inversely proportional* to the square of separation, r^2, between the two charged particles.

$$F \propto \frac{1}{r^2}$$

(2) The force is *proportional* to the product of charge q_1 and the charge q_2 on the particles.

$$F \propto q_1 q_2$$

(3) The force is *attractive* if the charges are of opposite sign and *repulsive* if the charges have the same sign.



We can conclude that

$$F \propto \frac{q_1 q_2}{r^2} \quad \Rightarrow \quad \therefore F = K\frac{q_1 q_2}{r^2}$$

where K is the coulomb constant = 9×10^9 N.m^2/C^2.

The above equation is called *Coulomb's law*, which is used to calculate the force between electric charges. In that equation F is measured in Newton (N), q is measured in unit of coulomb (C) and r in meter (m).

The constant K can be written as

$$K = \frac{1}{4\pi\varepsilon_o}$$

where ε_o is known as the *Permittivity constant of free space*.

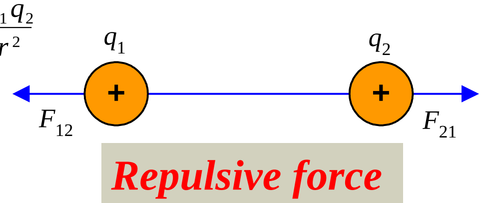
ε_o = 8.85 x 10^{-12} C^2/N.m^2

$$K = \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} = 9 \times 10^9 N.m^2/C^2$$

Coulomb's Law

The electrostatic force of a charged particle exerts on another is proportional to the product of the charges and inversely proportional to the square of the distance between them.

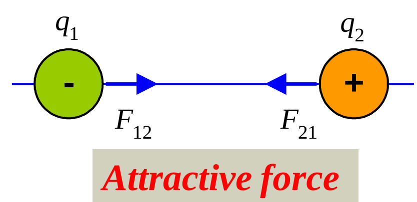
$$\therefore F = K\frac{q_1 q_2}{r^2}$$



$$\vec{F}_{12} = -\vec{F}_{21}$$

أي أن القوتين متساويتان في
المقدار ومتعاكستان في الاتجاه

$$F_{12} = K\frac{q_1 q_2}{r^2} = F_{21}$$



Example 1

Calculate the value of two equal charges if they repel one another with a force of 0.1N when situated 50cm apart in a vacuum.

Solution

$$F = K\frac{q_1 q_2}{r^2}$$

$$0.1 = \frac{9\times 10^9 \times q^2}{(0.5)^2}$$

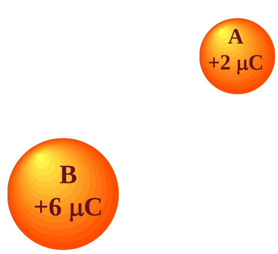
$$q = 1.7\text{x}10^{-6}\text{C} = 1.7\mu\text{C}$$

Quiz 1

One charge of 2.0 C is 1.5m away from a –3.0 C charge. Determine the force they exert on each other.

Quiz 2

Object A has a charge of +2μC and Object B has a charge of +6μC.
Which statement is true?



- **A:** $\mathbf{F}_{AB}=-3\mathbf{F}_{BA}$
- **B:** $\mathbf{F}_{AB}=-\mathbf{F}_{BA}$
- **C:** $3\mathbf{F}_{AB}=-\mathbf{F}_{BA}$
- **D:** $\mathbf{F}_{AB}=12\mathbf{F}_{BA}$

Force From Many Charges

Superposition

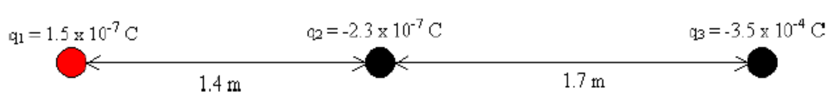
Multiple Charges in One Dimension

كيف نستخدم قانون كولوم لإيجاد القوى الكهربية الناتجة من مجموعة من الشحنات على شحنة محددة.

في المثال التالي لدينا 3 شحنات على استقامة واحدة ونريد حساب القوة الكهربية المؤثرة على واحدة منهم.

Example 2

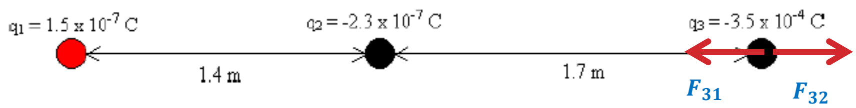
The following three charges are arranged as shown. Determine the net force acting on the charge on the far right (q_3 = charge 3).



Step 1: Calculate the force that charge 1 exerts on charge 3...

Notice that the total distance between charge 1 and 3 is 3.1 m , since we need to add 1.4 m and 1.7 m .

$$F_{31} = \frac{kq_3q_1}{r^2} = \frac{9\times10^9\times3.5\times10^{-4}\times1.5\times10^{-7}}{3.1^2} = 4.9\times10^{-2}\text{N}$$



Step 2: Calculate the force that charge 2 exerts on charge 3...

Same thing as above, only now we are dealing with two negative charges, so the force will be repulsive.

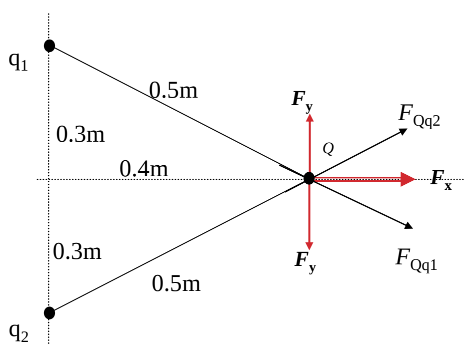
$$F_{32} = \frac{kq_3q_2}{r^2} = \frac{9\times10^9\times3.5\times10^{-4}\times2.3\times10^{-7}}{1.7^2} = 2.5\times10^{-1}\text{N}$$

Step 3: Add you values to find the net force.

$$F_{NET} = F_{32} - F_{31} = 2.0\times10^{-1}\text{N}$$

Example 3

Two equal positive charges *q*=2x10^{-6}C interact with a third charge *Q*=4x10^{-6}C. Find the magnitude and direction of the resultant force on *Q*.



$$F_{Qq1} = K\frac{qQ}{r^2} = 9\times10^9\,\frac{(4\times10^{-6})(2\times10^{-6})}{(0.5)^2} = 0.29N = F_{Qq2}$$

$$F_x = F\cos\theta = 0.29\left(\frac{0.4}{0.5}\right) = 0.23N$$

$$F_y = -F\sin\theta = -0.29\left(\frac{0.3}{0.5}\right) = -0.17N$$

$$\sum F_x = 2\times0.23 = 0.46N$$

$$\sum F_y = 0$$

وبهذا فإن مقدار القوة المحصلة هي 0.46N واتجاهها في اتجاه محور x الموجب.

Example 4

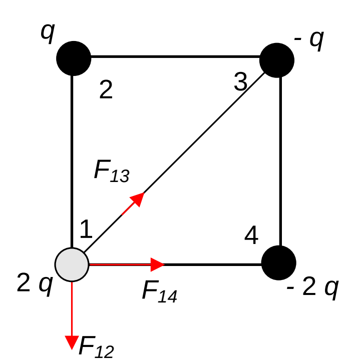
In figure what is the resultant force on the charge in the lower left corner of the square? Assume that q=1×10^{-7} C and a = 5cm

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

$$F_{12} = K\frac{2qq}{a^2} \longrightarrow F_{12} = 0.072\text{ N}$$

$$F_{14} = K\frac{2q2q}{a^2} \longrightarrow F_{13} = 0.036\text{ N},$$

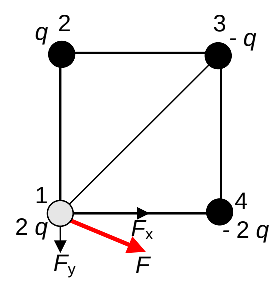
$$F_{13} = K\frac{2qq}{2a^2} \longrightarrow F_{14} = 0.144\text{ N}$$



لاحظ هنا أننا لا نستطيع جمع القوى الثلاث مباشرة لأن خط عمل القوى مختلف، ولذلك لحساب المحصلة نفرض محورين متعامدين x,y ونحلل القوى التي لا تقع على هذين المحورين أي متجه القوة F_{13} ليصبح

$F_{13x} = F_{13} \sin 45 = 0.025$ N & $F_{13y} = F_{13} \cos 45 = 0.025$ N

$F_x = F_{13x} + F_{14} = 0.025 + 0.144 = 0.169$ N
$F_y = F_{13y} - F_{12} = 0.025 - 0.072 = -0.047$ N



The resultant force equals

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} = 0.175 \text{ N}$$

The direction with respect to the x-axis equals

$$\theta = \tan^{-1} \frac{F_y}{F_x} = -15.5°$$

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General Physics II

Electrostatic: Principles & Applications

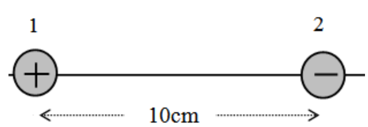
Lecture (2): Examples on Coulomb's Law

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Example (1) Equilibrium

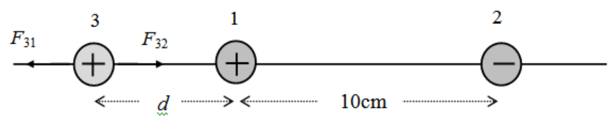
Two fixed charges, 1μC and -3μC are separated by 10cm as shown in figure below (a) where may a third charge be located so that no force acts on it? (b) is the equilibrium stable or unstable for the third charge?



Solution

المطلوب من السؤال هو أين يمكن وضع شحنة ثالثة بحيث تكون محصلة القوى الكهربية المؤثرة عليها تساوى صفراً، أي أن تكون في وضع اتزان equilibrium. (لاحظ أن نوع الشحنة ومقدارها لا يؤثر في تعيين نقطة الاتزان).

حتى يتحقق هذا فإنه يجب أن تكون القوى المؤثرة متساوية في المقدار ومتعاكسة في الاتجاه. وحتى يتحقق هذا الشرط فإن الشحنة الثالثة يجب أن توضع خارج الشحنتين وبالقرب من الشحنة الأصغر. لذلك نفرض شحنة موجبة q_3 كما في الرسم ونحدد اتجاه القوى المؤثرة عليها.



$$F_{31} = F_{32}$$

$$k\frac{q_3 q_1}{r_{31}^2} = k\frac{q_3 q_2}{r_{32}^2}$$

$$\frac{1\times10^{-6}}{d^2} = \frac{3\times10^{-6}}{(d+10)^2}$$

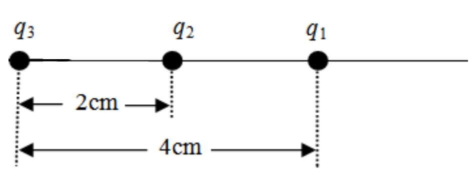
نحل هذه المعادلة ونوجد قيمة d

$$d = 13.7 \text{ cm}$$

(b) This equilibrium is unstable!! Why!!

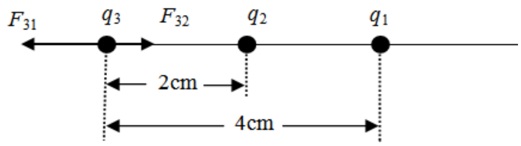
Example (2)

Two charges are located on the positive x-axis of a coordinate system, as shown in figure below. Charge q_1=2nC is 2cm from the origin, and charge q_2= -3nC is 4cm from the origin. What is the total force exerted by these two charges on a charge q_3=5nC located at the origin?



Solution

القوة الكلية المؤثرة على الشحنة q_3 هي المجموع الاتجاهي للقوة الناتجة عن الشحنة q_1 و الشحنة q_2



$$F_{31}=\frac{(9\times10^{9})(2\times10^{-9})(5\times10^{-9})}{(0.02)^2}=2.25\times10^{-4}\,N$$

$$F_{32}=\frac{(9\times10^{9})(3\times10^{-9})(5\times10^{-9})}{(0.04)^2}=0.84\times10^{-4}\,N$$

حيث أن الشحنة q_1 موجبة فإنها تؤثر على الشحنة q_3 بقوة تنافر مقدارها F_{31} واتجاهها كما هو موضح في الشكل، أما الشحنة q_2 سالبة فإنها تؤثر على الشحنة q_3 بقوة تجاذب مقدارها F_{32}.
وبالتالي فإن القوة المحصلة F_3 يمكن حسابها بالجمع الاتجاهي كالتالي:

$$F_3=F_{31}+F_{32}$$

$$\therefore F_3=0.84\times10^{-4}-2.25\times10^{-4}=-1.41\times10^{-4}\,N$$

اذا القوة المحصلة تؤثر على الشحنة q3 في اتجاه اليسار ومقدارها هو $1.41\text{x}10^{-4}\text{N}$

Example (3)

What is the force of repulsion between two electrons held one metre apart in a vacuum? What is the gravitational force of attraction between them? By what factor is the electric repulsion greater than the gravitational attraction?

$k = 1/(4\pi\varepsilon_0) = 9.0\times10^{9}$ N m^2 C^{-2}
mass of an electron = 9.11×10^{-31} kg
mass of a proton = 1.67×10^{-27} kg
G = 6.67×10^{-11} N m^2 kg^{-2}

Solution

يهدف هذا المثال إلى مقارنة القوة الكهربية مع قوى الجاذبية لإلكترونين حيث ان القوة الكهربية تعتمد على شحنة الالكترون وقوى الجاذبية تعتمد على كتلة الالكترون وسوف نجد ان القوة الكهربية اكبر بكثير من قوة التجاذب بين الكتل في حالة الشحنات الذرية

القوة الكهربية بين الالكترونين الذي تفصلهما مسافة مقدراها 1 متر هي

$F = kQ_1Q_2/r^2 = (9.0 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19})/ 1^2 = 2.3 \times 10^{-28}$ N

قوة التجاذب بين كتلتين الكترونين المسافة بينهما 1 متر هي

$F = Gm_1m_2/r^2 = (6.67 \times 10^{-11} \times 9.11 \times 10^{-31} \times 9.11 \times 10^{-31})/ 1^2 = 5.5 \times 10^{-71}$ N

النسبة بين القوة الكهربية على قوة التجاذب بين الكتلتين هي

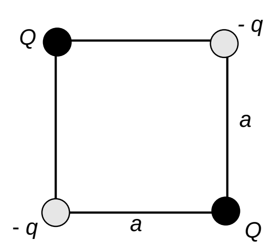
Electrical force/gravitational force = $2.3 \times 10^{-28} / 5.5 \times 10^{-71} = 4 \times 10^{42}$!!!!

ملاحظة: نلاحظ هنا ان القوة الكهربية اكبر بكثير جدا من القوة المتبادلة بين كتلتي الكترونين ومن هنا نستنتج انه عند التعامل مع الشحنات الذرية مثل شحنة الكترون او بروتون نهمل القوى المتبادلة بين الكتل.

Quiz: By what factor is the electric force between two protons greater than the gravitational force between them?

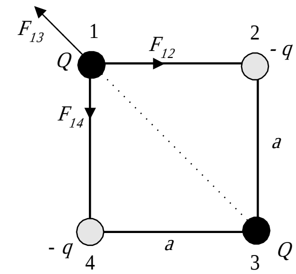
Example (4)

A charge *Q* is fixed at each of two opposite corners of a square as shown in the figure. A charge *q* is placed at each of the other two corners. (a) If the resultant electrical force on *Q* is Zero, how are *Q* and *q* related.

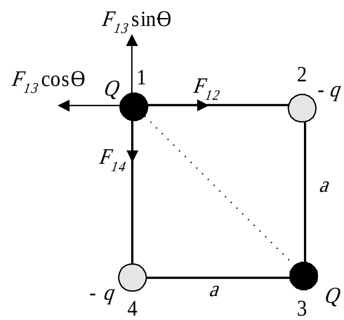


Solution

حتى تكون محصلة القوى الكهربية على الشحنة Q الناتجة عن الشحنات الأخرى مساوية للصفر، فإنه يجب أن تكون تلك القوى متساوية في المقدار ومتعاكسة في الاتجاه عند الشحنة Q رقم (1) مثلا، وحتى يتحقق ذلك نفرض أن كلتي الشحنتين (2) و (4) سالبة و Q (1) و (3) موجبة ثم نعين القوى المؤثرة على الشحنة (1).



نحدد اتجاهات القوى على الشكل (بعد ترقيم الشحنات). بعد تحليل متجه القوة F_{13} نلاحظ أن هناك أربعة متجهات قوى متعامدة، كما هو موضح في الشكل أدناه، وبالتالي يمكن أن تكون محصلتهم تساوى صفراً إذا كانت محصلة المركبات الأفقية تساوى صفراً وكذلك محصلة المركبات الرأسية.



$$F_x = 0 \quad \Rightarrow \quad F_{12} - F_{13x} = 0$$

$$F_{12} - F_{13}\cos 45 = 0$$

$$F_y = 0 \quad \Rightarrow \quad F_{13y} - F_{14} = 0$$

$$F_{13}\sin 45 = F_{14}$$

$$F_{12} = F_{13}\cos 45$$

$$K\frac{Qq}{a^2} = K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} \quad \Rightarrow \quad q = \frac{Q}{2\sqrt{2}}$$

$$F_{13}\sin 45 = F_{14}$$

$$K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} = K\frac{Qq}{a^2} \quad \Rightarrow \quad q = \frac{Q}{2\sqrt{2}}$$

$$\rightarrow Q = 2\sqrt{2}\, q$$

وهذه هي العلاقة بين Q و q التي تجعل محصلة القوى على Q تساوى صفر مع ملاحظة أن إشارة q تعاكس إشارة Q أي أن

$$Q = -2\sqrt{2}\, q$$

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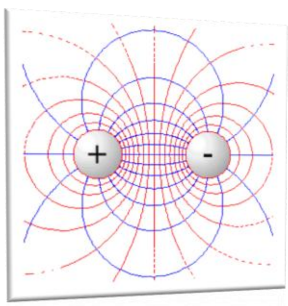
General Physics II

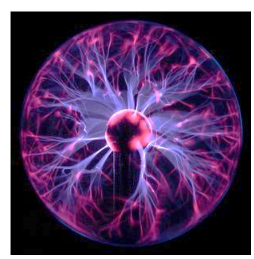
Electrostatic: Principles & Applications



Lecture (3): Electric Field

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

The Electric Field



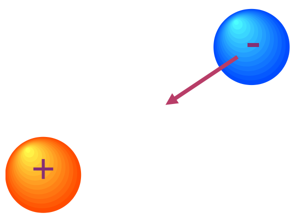
- Definition of the electric field ($\vec{E}$)
- Calculating $\vec{E}$ due to a charged particle
- To find $\vec{E}$ for a group of point charge
- Electric field lines
- Motion of charge particles in a uniform electric field
- The electric dipole in electric field

ELECTRIC FIELD

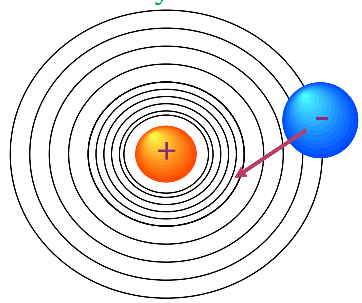
Physicists did not like the concept of "action at a distance" i.e. a force that was "caused" by an object a long distance away.

They preferred to think of an object producing a "field" and other objects interacting with that field.

Thus rather than ...



they liked to think...



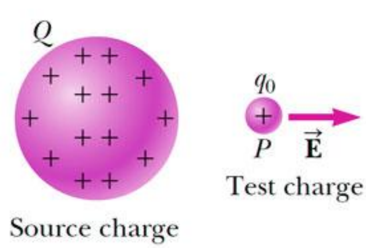
Definition Of The Electric Field

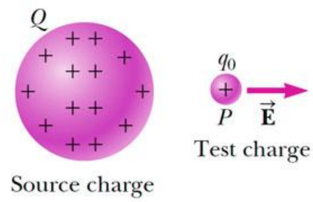
Whenever charges are present and if I bring up another charge, it will feel a net Coulomb force from all the others. It is convenient to say that there is field there equal to the force per unit positive charge.

$$\vec{E} = \frac{\vec{F}}{q_o}$$

The direction of the electric field is along r and points in the direction a positive test charge would move.

This idea was proposed by Michael Faraday in the 1830's.





If q_o were positive

The Coulomb force is $$F = K\frac{Qq_o}{r^2}$$

The force per unit charge is $$E = \frac{F}{q_o}$$

and then the electric field at r is $E = K\frac{Q}{r^2}$ due to the point charge q_o .

The units are N/C.

How do we find the direction.? The direction is the direction a unit positive test charge would move.

أي ان اتجاه المجال الكهربي هو في اتجاه حركة شحنة اختبار موجبة

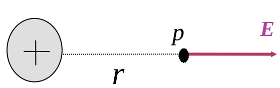
اتجاه المجال الكهربي

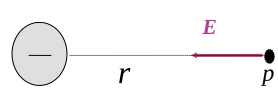
If Q is +ve the electric field at point p in space is radially outward from Q

يكون اتجاه المجال الكهربي الناتج عن شحنة موجبة في اتجاه الخروج من الشحنة

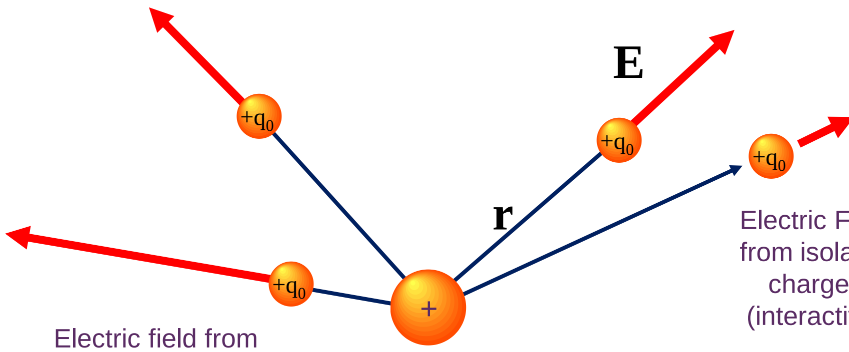
If Q is -ve the electric field at point p in space is radially inward toward Q

يكون اتجاه المجال الكهربي الناتج عن شحنة سالبة في اتجاه الدخول على الشحنة





اتجاه المجال الكهربي الناتج عن شحنة في الفراغ



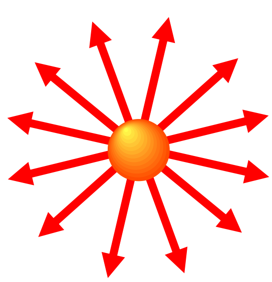
Electric Field from isolated charges (interactive)

Electric field from test particles

Note: the Electric Field is defined everywhere, even if there is no test charge is not there.

Representation Of The Electric Field

- **The lines must begin on positive charges (or infinity)**
- **The lines must end on negative charges (or infinity)**
- **The number of lines leaving a +ve charge (or approaching a -ve charge) is proportional to the magnitude of the charge**
- **electric field lines cannot cross**



We represent the electric field with lines whose direction indicates the direction of the field

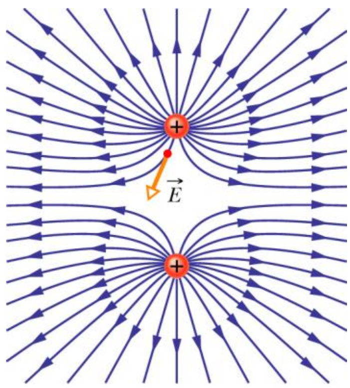
Notice that as we move away from the charge, the density of lines decreases

These are called Electric Field Lines

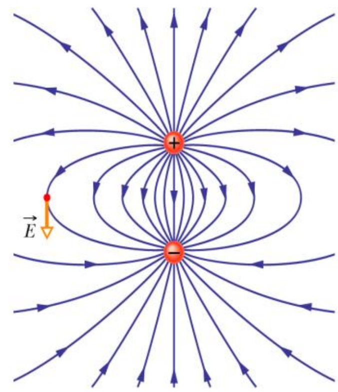
خطوط المجال الكهربي لتوزيعات مختلفة من الشحنات

ELECTRIC FIELD LINES

LIKE CHARGES (++)



OPPOSITE CHARGES (+ -)

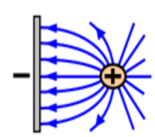


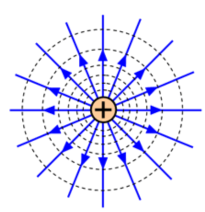
This is called an electric dipole.

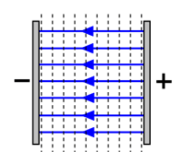
Interpreting Electric Field Lines

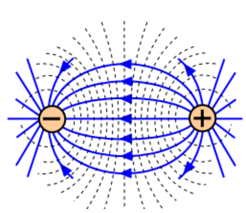
The electric field vector, E, is at a tangent to the electric field lines at each point along the lines

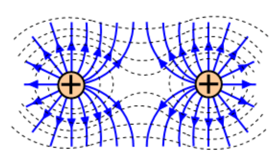
The number of lines per unit area through a surface perpendicular to the field is proportional to the strength of the electric field in that region











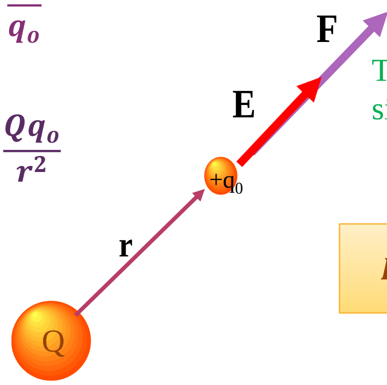
This is called an electric dipole.

Definition of the Electric Field

Electric Field E is defined as the force acting on a test particle divided by the charge of that test particle

$$E = \frac{F}{q_o}$$

$$F = K\frac{Qq_o}{r^2}$$

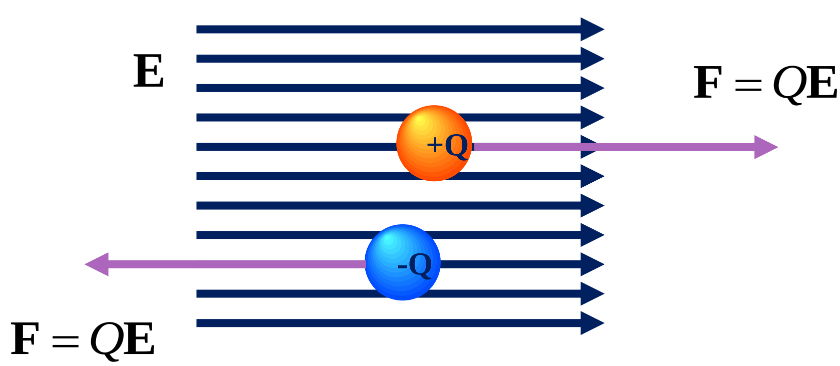


Thus Electric Field from a single charge is

$$E = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}$$

Charged Particles in Electric Field

Using the Field to determine the force



Quiz: The Field Direction

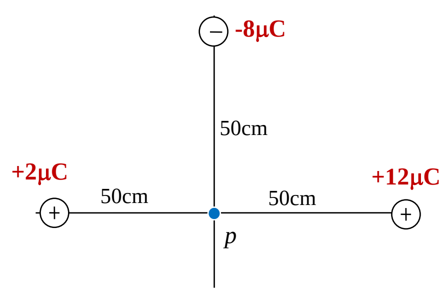
A charge +q is placed at (0,1) and a charge –q is placed at (0,-1)

What is the direction of the field at (1,0)?

- A) **i** + **j**
- B) **i** - **j**
- C) -**j**
- D) -**i**

Example (1)

Find the electric field at point *p* in the figure due to the charges shown.



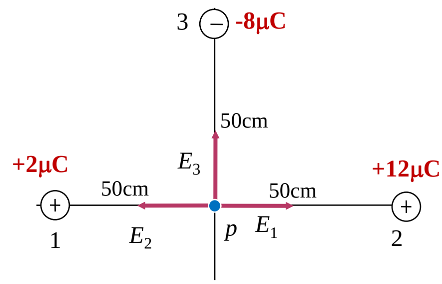
Solution

في البداية نقوم بترقيم الشحنات ثم بعد ذلك نقوم بتحديد اتجاه المجال الناتج عن كل شحنة عند النقطة p مع العلم ان محصلة المجال الكهربي هو الجمع الاتجاهي لها أي ان

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

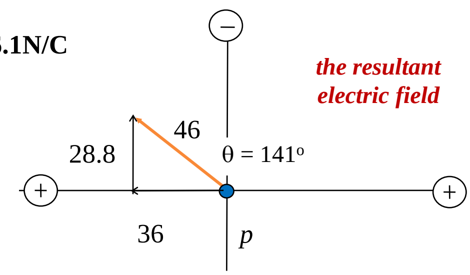
$E_x = E_1 - E_2 = -36\times10^4$ N/C

$E_y = E_3 = 28.8\times10^4$ N/C



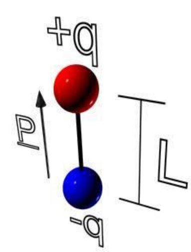
$E_p = \sqrt{(36\times10^4)^2+(28.8\times10^4)^2} = 46.1$N/C

$\theta = 141^o$

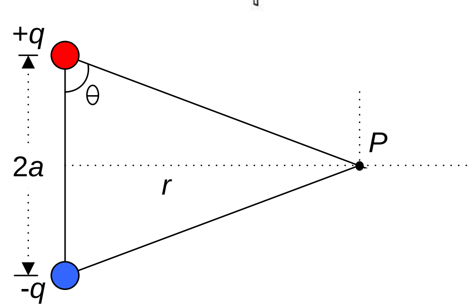


Example (2) Electric Dipole

Find the electric field due to electric dipole along x-axis at point *p*, which is a distance *r* from the origin, then assume *r>>a*



***The electric dipole* is positive charge and negative charge of equal magnitude placed a distance 2*a* apart as shown in figure 3.6**

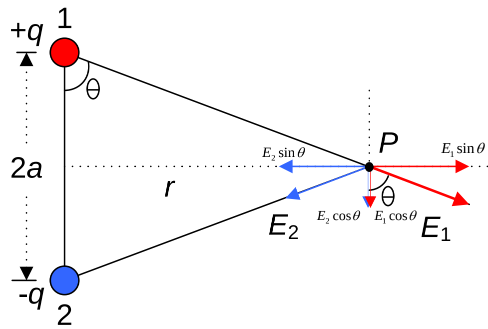


$$\vec{E}_p = \vec{E}_1 + \vec{E}_2$$

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{a^2+r^2} = E_2$$

$$Ex = E_1 \sin\theta - E_2 \sin\theta$$

$$Ey = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$$



$$Ep = 2E_1 \cos\theta$$

$$E_p = \frac{1}{4\pi\varepsilon_o}\frac{q}{a^2+r^2}\cos\theta \longrightarrow \cos\theta = \frac{a}{\sqrt{a^2+r^2}}$$

$$E_p = \frac{1}{4\pi\varepsilon_o}\frac{q}{a^2+r^2}\frac{a}{\sqrt{a^2+r^2}} \longrightarrow \frac{2aq}{4\pi\varepsilon_o(r^2+a^2)^{3/2}}$$

The direction of the electric field in the -ve y-axis.

The quantity 2*aq* is called the *electric dipole momentum* (*P*) and has a direction from the -ve charge to the +ve charge

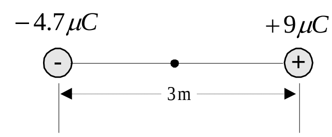
(b) when *r>>a*

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o r^3}$$

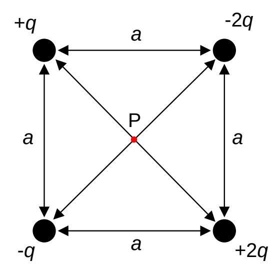
يتضح مما سبق أن المجال الكهربي الناشئ عن electric dipole عند نقطة واقعة على العمود المنصف بين الشحنتين يكون اتجاهه في عكس اتجاه electric dipole momentum وبالنسبة للنقطة البعيدة عن electric dipole فإن المجال يتناسب عكسيا مع مكعب المسافة، وهذا يعنى أن تناقص المجال مع المسافة يكون أكبر منه في حالة شحنة واحدة فقط.

Problems To Solve By Yourself

(1) Find the total electric field along the line of the two charges shown in the figure at the point midway between them.



(2) What is *E* in magnitude and direction at the center of the square shown in the figure? Assume that *q*=1μC and *a*=5cm.



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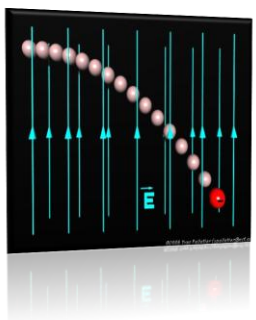
General Physics II

Electrostatic: Principles & Applications

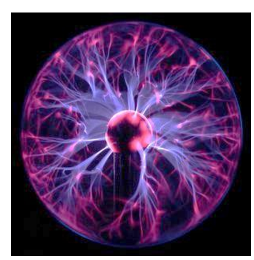


Lecture (4): Motion of charge particles in a uniform electric field

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

The Electric Field

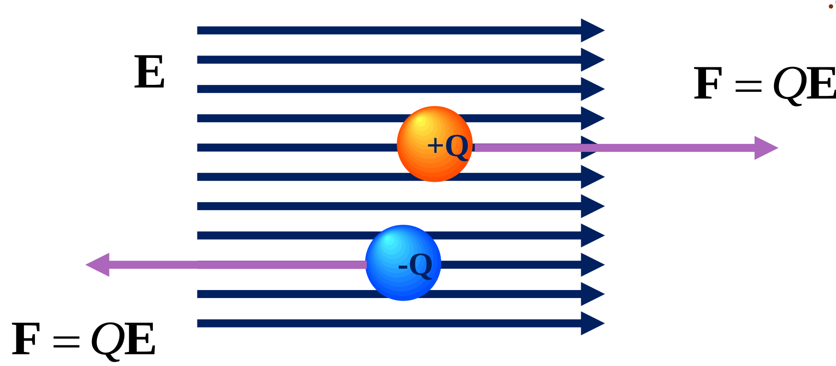


- **Definition of the electric field ($\vec{E}$)**
- **Calculating $\vec{E}$ due to a charged particle**
- **To find $\vec{E}$ for a group of point charge**
- **Electric field lines**
- **Motion of charge particles in a uniform electric field**
- **The electric dipole in electric field**

Motion Of Charge Particles In A Uniform Electric Field

If we are given a field $\vec{E}$, what forces will act on a charge placed in it?

عند وضع شحنة كهربية q كتلتها m في مجال كهربي E فان المجال الكهربي يبذل قوة كهربية مقدارها $q\vec{E}$.



أي الشحنة عندما تكون في مجال كهربي تتعرض لقوة كهربية تعمل هذه القوة الكهربية على اكساب الشحنة عجلة ولهذا فان

$$\vec{F}_e = q\vec{E} = m\vec{a}$$

وبالتالي فان العجلة التي يتحرك بها الجسم المشحون في مجال كهربي هي

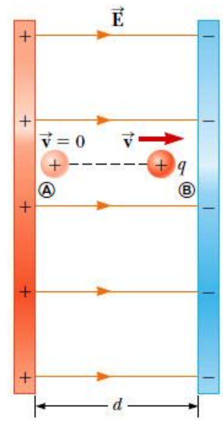
$$\vec{a} = \frac{q\vec{E}}{m}$$

اذا كان المجال الكهربي منتظم أي ان المجال له مقدار ثابت واتجاه ثابت كما في المجال الكهربي الناتج عن لوحين مشحونين احدهما بشحنة موجبة والاخر بشحنة سالبة. فان القوة الكهربية على الجسم المشحون تكون ثابتة وعندها يمكن ان نطبق قوانين الحركة تحت تأثير عجلة ثابتة.

اذا كان الجسم مشحون بشحنة موجبة فانه يتحرك مع اتجاه المجال واذا كان الجسم مشحون بشحنة سالبة فانه يتحرك في عكس اتجاه المجال.

Example (1)

A positive point charge *q* of mass m is released from rest in a uniform electric field directed along the x-axis as shown in the figure, describe its motion.



Solutions:

The acceleration is given by

$$a = \frac{qE}{m}$$

Since the motion of the particle in one dimension, then we can apply the equations of kinematics in one dimension

$$x - x_o = v_0 t + \tfrac{1}{2} a t^2 \qquad v = v_0 + at \qquad v^2 = v_o^2 + 2a(x - x_o)$$

Taking $x_o = 0$ and $v_0 = 0$

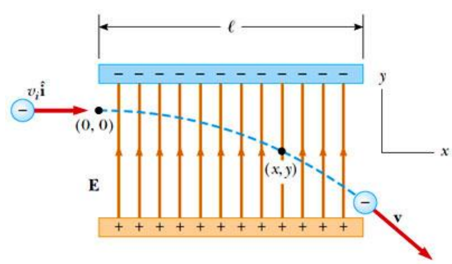
$$x = \tfrac{1}{2} a t^2 = (qE/2m)\, t^2$$

$$v = at = (qE/m)\, t$$

$$v^2 = 2ax = (2qE/m) x$$

Example (2)

In the above example suppose that a negative charged particle is projected horizontally into the uniform field with an initial velocity v_o as shown in the figure. describe its motion.



Solutions:

Since the direction of electric field in the y direction, and the charge is negative, then the acceleration of charge is in the direction of -y.

$$a = -\frac{qE}{m}$$

The motion of the charge is in two dimension with constant acceleration, with
$v_{xo} = v_o$ & $v_{yo} = 0$

The components of velocity after time t are given by

$$v_x = v_o = \text{constant}$$

$$v_y = at = -\,(qE/m)\,t$$

The coordinate of the charge after time t are given by

$$x = v_o t$$

$$y = \tfrac{1}{2}\,at^2 = -\,1/2\,(qE/m)\,t^2$$

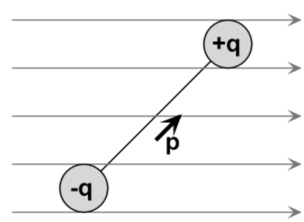
Eliminating t we get

$$y = \frac{qE}{2mv_0^2}x^2$$

we see that y is proportional to x^2. Hence, the trajectory is parabola.

The Electric Dipole In Electric Field

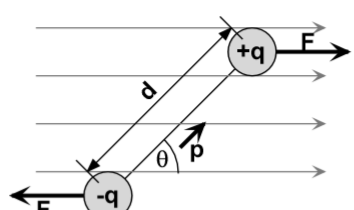
If an electric dipole placed in an external electric field *E* as shown in the figure, then a torque will act to align it with the direction of the field.



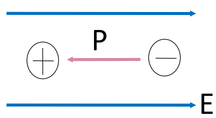
$$\vec{\tau} = \vec{P} \times \vec{E}$$

$$\tau = P\,E\,\sin\theta$$

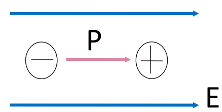
where *P* is the electric dipole momentum, θ the angle between *P* and *E*



يكون ثنائي القطب في حالة اتزان equilibrium عندما يكون الازدواج مساويا للصفر وهذا يتحقق عندما تكون (θ= zero , π)



(ii)



(i)

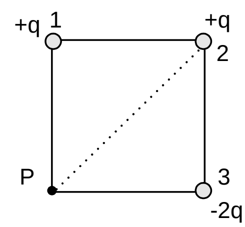
في الوضع الموضح في الشكل (i) عندما θ =0 يقال إن الـ dipole في وضع اتزان مستقر stable equilibrium لأنه إذا أزيح بزاوية صغيرة فانه سيرجع إلى الوضع θ =0،

بينما في الوضع الموضح في الشكل (ii) يقال إن الـ dipole في وضع اتزان غير مستقر unstable equilibrium لأن إزاحة صغيرة له سوف تعمل على أن يدور الـ dipole ويرجع إلى الوضع θ =0 وليس $\theta=\pi$

Some Solved Problems

مسائل متنوعة محلولة على المجال الكهربي

Example (3)



What is the electric field in the lower left corner of the square as shown in figure? Assume that *q* = 1x10^{-7}C and *a* = 5cm.

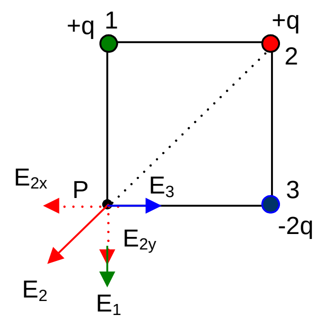
Solutions:

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{a^2}$$

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{2a^2}$$

$$E_3 = \frac{1}{4\pi\varepsilon_o}\frac{2q}{a^2}$$



Evaluate the value of E_1, E_2, & E_3

- $E_1 = 3.6\text{x}10^5$ N/C,
- $E_2 = 1.8 \text{ x } 10^5$ N/C,
- $E_3 = 7.2 \text{ x } 10^5$ N/C

We find the vector E_2 need analysis to two components

- $E_{2x} = E_2 \cos 45$
- $E_{2y} = E_2 \sin 45$

$E_x = E_3 - E_2\cos45 = 7.2\text{x}10^5 - 1.8\text{x}10^5 \cos45 = 6\text{x}10^5$N/C

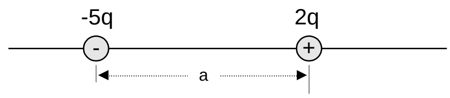
$E_y = -E_1 - E_2\sin45 = -3.6\text{x}10^5 - 1.8 \text{ x}10^5 \sin45 = -4.8\text{x}10^5$ N/C

$$E = \sqrt{E_x^2 + E_y^2} \quad = 7.7 \text{ x } 10^5 \text{ N/C}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} \quad = -38.6^o$$

Example (4)

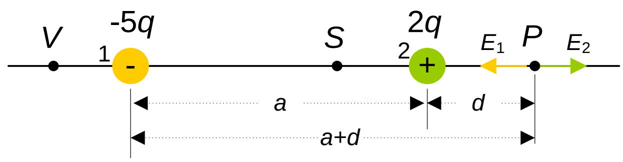
In figure shown, locate the point at which the electric field is zero? Assume a = 50cm



Solutions:

لا يمكن ان تكون النقطة التي ينعدم عندها المجال الكهربي عن النقطة V او النقطة S. لماذا؟

لاحظ هنا أنه في حالة الشحنتين المتشابهتين فإن النقطة التي ينعدم عندها المجال تكون بين الشحنتين، أما إذا كانت الشحنتان مختلفتين في الإشارة فإنها تكون خارج إحدى الشحنتين وعلى الخط الواصل بينهما وبالقرب من الشحنة الأصغر.



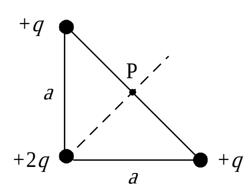
$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_o}\frac{2q}{(0.5+d)^2} = \frac{1}{4\pi\varepsilon_o}\frac{5q}{(d)^2}$$

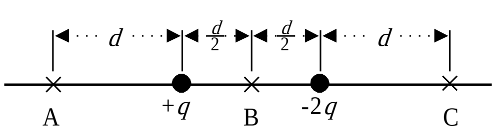
$$d = 30\text{cm}$$

Problems To Solve By Yourself

(1) Calculate E (direction and magnitude) at point P in the figure.



(2) Charges $+q$ and $-2q$ are fixed a distance d apart as shown in the figure. Find the electric field at points A, B, and C.



(3) A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0cm away, in a time 1.5×10^{-8}s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field E?

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General Physics II

Electrostatic: Principles & Applications

Lecture (5): Discussion on Coulomb's Law and electric field

Dr. Hazem Falah Sakeek

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Multiple Choice Questions

1. The magnitude of the electric force between two protons is 2.30×10^{-26} N. How far apart are they? (a) 0.100m (b) 0.022m (c) 3.10m (d) 0.0057m (e) 0.48m

2. Estimate the magnitude of the electric field due to the proton in a hydrogen atom at a distance of 5.29×10^{-11} m, the expected position of the electron in the atom. (a) 10^{-11} N/C (b) 10^{8} N/C (c) 10^{14} N/C (d) 10^{6} N/C (e) 10^{12} N/C

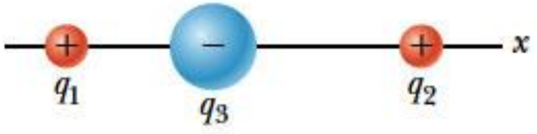
3. A very small ball has a mass of 5.00×10^{-3} kg and a charge of 4.00 μC. What magnitude electric field directed upward will balance the weight of the ball so that the ball is suspended motionless above the ground? (a) 8.21×10^{2} N/C (b) 1.22×10^{4} N/C (c) 2.00×10^{-2} N/C (d) 5.11×10^{6} N/C (e) 3.72×10^{3} N/C.

4. An electron with a speed of 3.00×10^{6} m/s moves into a uniform electric field of magnitude 1.00×10^{3} N/C. The field lines are parallel to the electron's velocity and pointing in the same direction as the velocity. How far does the electron travel before it is brought to rest? (a) 2.56 cm (b) 5.12 cm (c) 11.2 cm (d) 3.34 m (e) 4.24 m

5. A point charge of -4.00 nC is located at (0, 1.00) m. What is the *x* component of the electric field due to the point charge at (4.00, -2.00) m? (a) 1.15 N/C (b) -0.864 N/C (c) 1.44 N/C (d) -1.15 N/C (e) 0.864 N/C.

9. (i) A metallic coin is given a positive electric charge. Does its mass (a) increase measurably, (b) increase by an amount too small to measure directly, (c) remain unchanged, (d) decrease by an amount too small to measure directly, or (e) decrease measurably? **(ii)** Now the coin is given a negative electric charge. What happens to its mass? Choose from the same possibilities as in part (i).

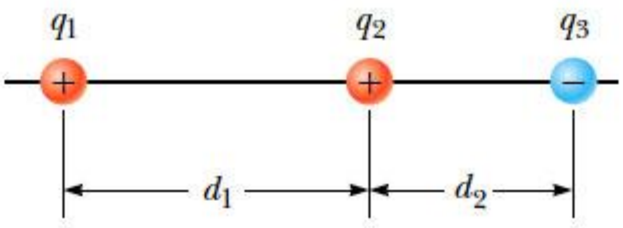
10. Assume the charge objects in the figure are fixed. Notice that there is no sight line from the location of q_2 to the location of q_1. If you were at q_1, you would be unable to see q_2 because it is behind q_3. How would you calculate the electric force exerted on the object with charge q_1? (a) Find only the force exerted by q_2 on charge q_1. (b) Find only the force exerted by q_3 on charge q_1. (c) Add the force that q_2 would exert by itself on charge q_1 to the force that q_3 would exert by itself on charge $q1$. (d) Add the force that q_3 would exert by itself to a certain fraction of the force that q_2 would exert by itself. (e) There is no definite way to find the force on charge q_1.



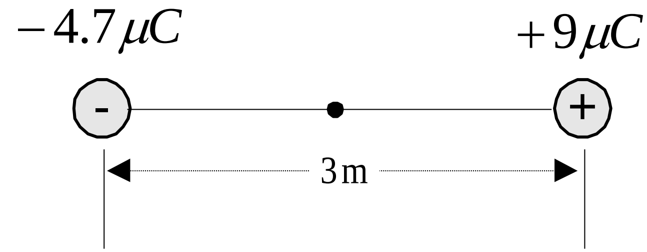
14. An object with negative charge is placed in a region of space where the electric field is directed vertically upward. What is the direction of the electric force exerted on this charge? (a) It is up. (b) It is down. (c) There is no force. (d) The force can be in any direction.

Problems To Solve By Yourself

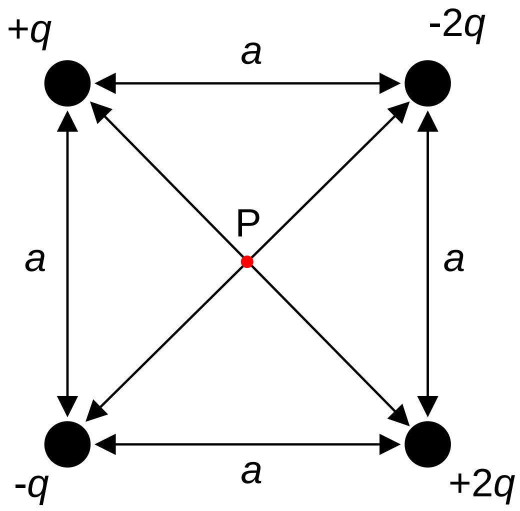
(1) Three point charges lie along a straight line as shown in the figure, where q_1 = 6.00 μC, q_2 = 1.50 μC, and q_3 = -2.00 μC. The separation distances are d_1 = 3.00 cm and d_2 = 2.00 cm. Calculate the magnitude and direction of the net electric force on (a) q_1, (b) q_2, and (c) q_3.



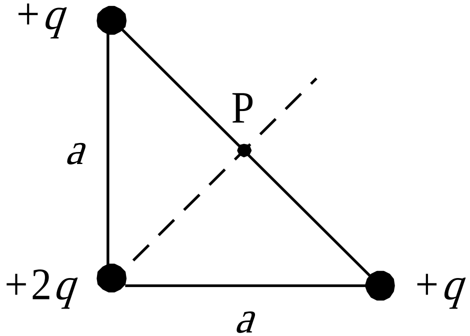
(2) Find the total electric field along the line of the two charges shown in the figure at the point midway between them.



(3) What is E in magnitude and direction at the center of the square shown in the figure? Assume that q=1μC and a=5cm.



(4) Calculate E (direction and magnitude) at point P in the figure.



(5) A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0cm away, in a time 1.5×10^{-8}s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field E?

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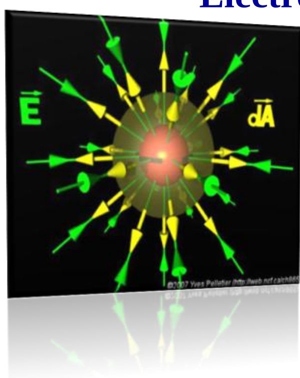
General Physics II

Electrostatic: Principles & Applications

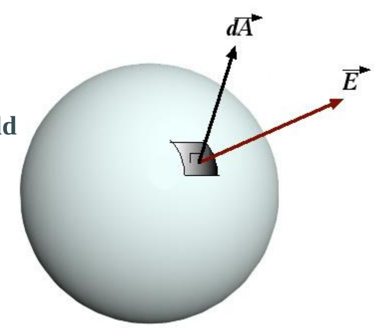


Lecture (6): Gauss's Law

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Electric Flux



يوهان كارل فريدريش غاوس

Carl Friedrich Gauss

30 أبريل 1777 – 23 فبراير 1855) الملقب بأمير الرياضيات ويعتبر واحد من العلماء الثلاثة الأهم في تاريخ الرياضيات، كان رياضياتيًا وفيزيائيا وعالمًا ألمانيًا ساهم بالكثير من الأعمال في نظرية الأعداد، الإحصاء، التحليل الرياضي، الهندسة التفاضلية، علم الاستاتيكا الكهربائية، علم الفلك، والبصريات.

درسنا سابقا كيفية حساب المجال لتوزيع معين من الشحنات باستخدام قانون كولوم. وهنا سنقدم طريقة أخرى لحساب المجال الكهربي باستخدام "قانون جاوس" الذي يسهل حساب المجال الكهربي لتوزيع متصل من الشحنة على شكل توزيع طولي أو سطحي أو حجمي.

يعتمد قانون جاوس أساساً على مفهوم التدفق الكهربي الناتج من المجال الكهربي أو الشحنة الكهربائية، ولهذا سنقوم أولاً بحساب التدفق الكهربي الناتج عن المجال الكهربي، وثانياً سنقوم بحساب التدفق الكهربي الناتج عن شحنة كهربية، ومن ثم سنقوم بإيجاد قانون جاوس واستخدامه في بعض التطبيقات الهامة في مجال الكهربية الساكنة.

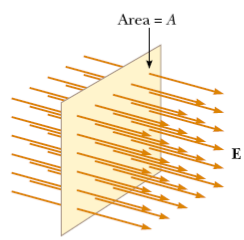
The Electric Flux due to an Electric Field

We have already shown how electric field can be described by lines of force. A line of force is an imaginary line drawn in such a way that its direction at any point is the same as the direction of the field at that point. Field lines never intersect, since only one line can pass through a single point.

***The Electric flux* (Φ) is a measure of the number of electric field lines penetrating some surface of area *A*.**

Case one:

The electric flux for a plan surface perpendicular to a uniform electric field . To calculate the electric flux we recall that the number of lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface of area *A* is proportional to the product *EA*. The product of the electric filed E and the surface area *A* perpendicular to the field is called the electric flux Φ.

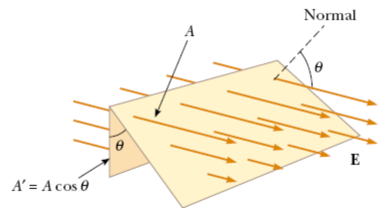


$$\Phi = \vec{E}.\vec{A}$$

The electric flux Φ has a unit of N.m^2/C.

Case Two

The electric flux for a plan surface make an angle θ to a uniform electric field



Note that the number of lines that cross-area is equal to the number that cross the projected area *A`*, which is perpendicular to the field. From the figure we see that the two area are related by *A`=Acosθ*. The flux is given by:

$$\Phi = \vec{E}.\vec{A}' = E\,A\,\cos\theta$$

$$\Phi = \vec{E}.\vec{A}$$

Where θ is the angle between the electric field *E* and the normal to the surface $\vec{A}$

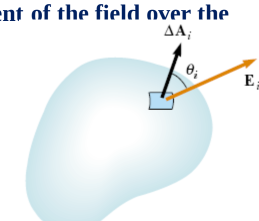
إذاً يكون الفيض ذا قيمة عظمى عندما يكون السطح عمودياً على المجال أي θ = 0 ويكون ذا قيمة صغرى عندما يكون السطح موازياً للمجال أي عندما θ = 90. لاحظ هنا أن المتجه $\vec{A}$ هو متجه المساحة وهو عمودي دائما على المساحة وطوله يعبر عن مقدار المساحة.

Case Three

In general the electric field is nonuniform over the surface

The flux is calculated by integrating the normal component of the field over the surface in question.

$$\Phi = \oint \vec{E}.\vec{A}$$

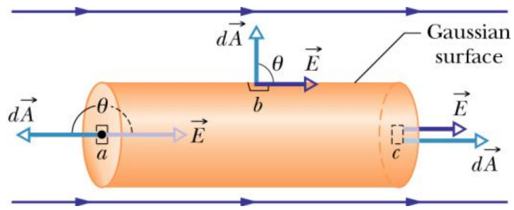


The *net flux* through the surface is proportional to the *net number of lines* penetrating the surface.

والمقصود بـ net number of lines أي عدد الخطوط الخارجة من السطح (إذا كانت الشحنة موجبة) - عدد الخطوط الداخلة إلى السطح (إذا كانت الشحنة سالبة).

Example

What is electric flux Φ for closed cylinder of radius *R* immersed in a uniform electric field as shown in figure



Solution

نطبق قانون جاوس على الأسطح الثلاثة الموضحة في الشكل أعلاه

$$\Phi = \oint \vec{E}.d\vec{A} = \oint_{(1)} \vec{E}.d\vec{A} + \oint_{(2)} \vec{E}.d\vec{A} + \oint_{(3)} \vec{E}.d\vec{A}$$

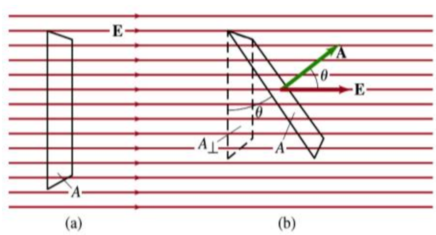
$$= \oint_{(1)} E\cos 180 dA + \oint_{(2)} E\cos 90 dA + \oint_{(3)} E\cos 0 dA$$

Since *E* is constant then

$$\Phi = -EA + 0 + EA = \text{zero}$$

Example

Electric flux. (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux in figure if the angle is 30 degrees?



The electric flux is

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$$

So when (a) **θ=0**, we obtain

$$\Phi_E = EA\cos\theta = EA = (200N/C)\cdot(0.1\times 0.2m^2) = 4.0\,\mathrm{N\cdot m^2}/C$$

And when (b) **θ=30** degrees, we obtain

$$\Phi_E = EA\cos 30^\circ = (200N/C)\cdot(0.1\times 0.2m^2)\cos 30^\circ = 3.5\,\mathrm{N\cdot m^2}/C$$

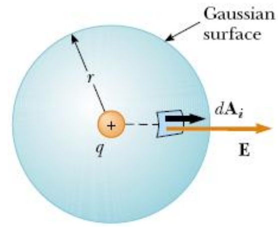
The Electric Flux due to a Point Charge

To calculate the electric flux due to a point charge we consider an imaginary closed spherical surface with the point charge in the center, this surface is called *gaussian surface.* Then the flux is given by

$$\Phi = \oint \vec{E}.d\vec{A} = E\oint dA\cos\theta \quad (\theta = 0)$$

$$\Phi = \frac{q}{4\pi\varepsilon_o r^2}\int dA = \frac{q}{4\pi\varepsilon_o r^2}4\pi r^2$$

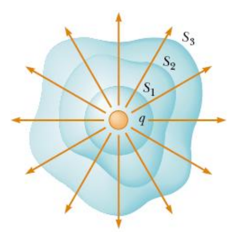
$$\Phi = \frac{q}{\varepsilon_o}$$



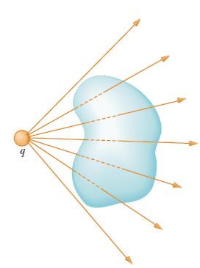
Note that the net flux through a spherical gaussian surface is proportional to the charge *q* inside the surface.

Gaussian Surface

Consider several closed surfaces as shown in figure surrounding a charge *Q* as in the figure below. The flux that passes through surfaces S_1, S_2 and S_3 all has a value q/ε_o. Therefore we conclude that the net flux through any closed surface is independent of the shape of the surface.



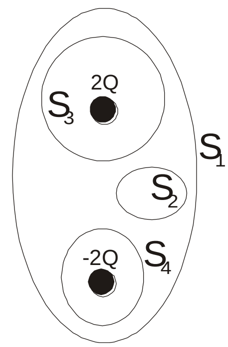
Consider a point charge located outside a closed surface as shown in figure. We can see that the number of electric field lines entering the surface equal the number leaving the surface. Therefore the net electric flux in this case is zero, because the surface surrounds no electric charge.



Example

In figure two equal and opposite charges of 2*Q* and -2*Q* what is the flux Φ for the surfaces S_1, S_2, S_3 and S_4.

Solution



For S_1 the flux Φ = zero

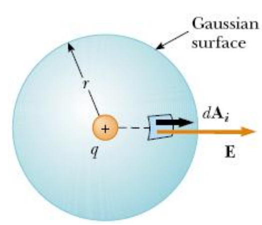
For S_2 the flux Φ = zero

For S_3 the flux Φ = $+2Q/\varepsilon_o$

For S_4 the flux Φ = $-2Q/\varepsilon_o$

Gauss's Law

Gauss law is a very powerful theorem, which relates any charge distribution to the resulting electric field at any point in the vicinity of the charge. As we saw the electric field lines means that each charge q must have q/ε_o flux lines coming from it. This is the basis for an important equation referred to as ***Gauss's law***. Note the following facts:



If there are charges q_1, q_2, q_3, ...q_n inside a closed (gaussian) surface, the total number of flux lines coming from these charges will be

$$(q_1 + q_2 + q_3 + \dots\dots + q_n)/\varepsilon_o$$

The number of flux lines coming out of a closed surface is the integral of over the surface $\oint \vec{E}.d\vec{A}$

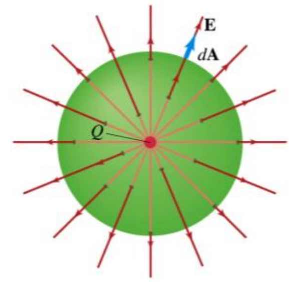
We can equate both equations to get Gauss law which state that the net electric flux through a closed gaussian surface is equal to the net charge inside the surface divided by ε_o

Gauss's law $$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$ where q_{in} is the total charge inside the Gaussian surface.

Gauss's law states that the net electric flux through any closed Gaussian surface is equal to the net electric charge inside the surface divided by the permittivity.

Gauss's Law and Coulomb's Law

We can deduce Coulomb's law from Gauss's law by assuming a point charge *q*, to find the electric field at point or points a distance *r* from the charge we imagine a spherical Gaussian surface of radius *r* and the charge *q* at its center as shown in figure.



$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$\oint E\cos 0 dA = \frac{q_{in}}{\varepsilon_o}$$

Because *E* is constant for all points on the sphere, it can be factored from the inside of the integral sign, then

$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \Rightarrow EA = \frac{q_{in}}{\varepsilon_o} \Rightarrow E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o} \Rightarrow \therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

Now put a second point charge q_o at the point, which *E* is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

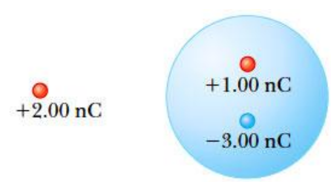
Coulomb's law

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

Problems To Solve By Yourself

(1) A flat surface of area 3.20 m^2 is rotated in a uniform electric field of magnitude E 5 6.20 × 10^5 N/C. Determine the electric flux through this area (a) when the electric field is perpendicular to the surface and (b) when the electric field is parallel to the surface.

(2) Find the net electric flux through the spherical closed surface shown in the figure.



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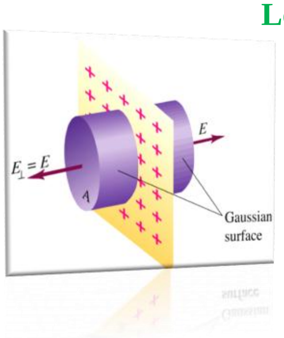
General Physics II

Electrostatic: Principles & Applications

Lecture (7): Gauss's Law Applications



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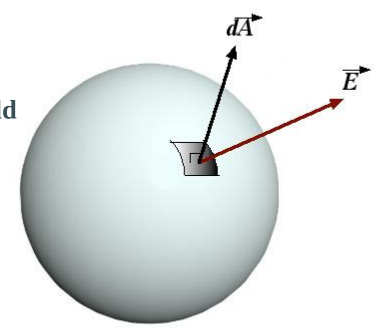
Electric Flux



4.1 The Electric Flux due to an Electric Field
4.2 The Electric Flux due to a point charge
4.3 Gaussian surface
4.4 Gauss's Law
4.5 Gauss's law and Coulomb's law
4.6 Conductors in electrostatic equilibrium
4.7 Applications of Gauss's law
4.8 Solution of some selected problems
4.9 Problems

في هذه المحاضرة سوف نقوم بدراسة العديد من التطبيقات لقانون جاوس مثل تأثير الشحنة الكهربية على الموصل المعزول وسوف نتعلم كيف نحسب المجال الكهربي لتوزيع متصل من الشحنة على شكل خطي أو سطحي أو حجمي.

Conductors in Electrostatic Equilibrium

A good electrical conductor, such as copper, contains charges (electrons) that are free to move within the material. When there is no net motion of charges within the conductor, the conductor is in electrostatic equilibrium.

Conductor in electrostatic equilibrium has the following properties:

Any excess charge on an isolated conductor must reside entirely on its surface. *(Explain why?)* ***The answer is when an excess charge is placed on a conductor,***

It will set-up electric field inside the conductor.

These *fields act on the charge carriers of the conductor (electrons) and cause them to move* ***i.e. current flow inside the conductor.***

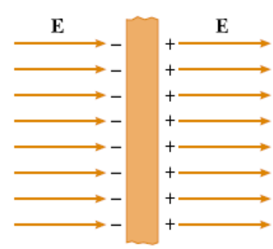
These currents redistribute the excess charge on the surface in such away that the internal electric fields reduced to become zero and the currents stop, and the electrostatic conditions restore.

The electric field is zero everywhere inside the conductor. *(Explain why?) Same reason as above.*

Conducting Slab in an External Electric Field

In the figure it shows a conducting slab in an external electric field *E*.

The charges induced on the surface of the slab produce an electric field, which opposes the external field, giving a resultant field of zero in the conductor.



في الشكل المقابل تم وضع شريحة معدنية موصلة في مجال كهربي خارجي ماذا يحدث؟

مادة الشريحة موصلة وهذا يعني أن الشحنات حرة الحركة فتتحرك الشحنات إلى السطح الخارجي كما في الشكل لينتج عنها مجالاً كهربيا يعاكس المجال الكهربي الخارجي وهذا يعني أن المجال الكهربي داخل مادة الموصل تساوي صفر.

دائما تذكر إن المجال الكهربي داخل مادة الموصل تساوي صفر والشحنة الكهربية الإضافية تستقر على السطح الخارجي للموصل.

Applications of Gauss's Law

Gauss's law can be used to calculate the electric field if the symmetry of the charge distribution is high. Here we concentrate in three different ways of charge distribution

يستخدم قانون جاوس لحساب المجال الكهربي الناتج عن توزيع متصل للشحنة حيث يصعب إيجاد المجال الكهربي باستخدام قانون كولوم. ومن أمثلة التوزيع المتصل للشحنة سلك مشحون أو سطح لانهائي مشحون أو كرة مشحونة، وفي هذه الحالات نفترض إن توزيع الشحنة هو توزيع متجانس ونعبر عنه بكثافة الشحنة.

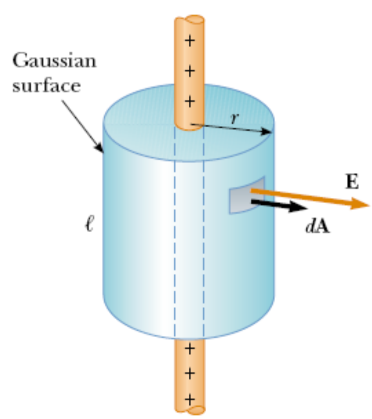
تكون كثافة الشحنة **charge density** على النحو التالي:

3	2	1	
Volume	Surface	Linear	Charge distribution
ρ	σ	λ	Charge density
C/m^3	C/m^2	C/m	Unit

A Linear Charge Distribution

In the figure calculate the electric field at a distance *r* from a uniform positive line charge of infinite length whose charge per unit length is λ=constant.

The electric field E is perpendicular to the line of charge and directed outward. Therefore for symmetry we select a cylindrical gaussian surface of radius r and length L.



The electric field is constant in magnitude and perpendicular to the surface.

The flux through the end of the gaussian cylinder is zero since E is parallel to the surface.

The total charge inside the gaussian surface is λL.

A Linear Charge Distribution, *Continue*

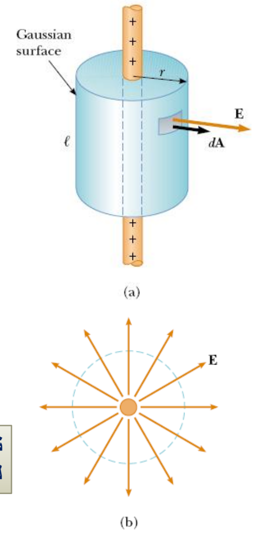
Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$ نطبق قانون جاوس على سطح اسطواني يحيط بالسلك

$$E\oint dA = \frac{\lambda L}{\varepsilon_o}$$ نعوض عن الشحنة بكثافة الشحنة

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o}$$ نجري عملية التكامل

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r}$$



نلاحظ هنا أنه باستخدام قانون جاوس سنحصل على نفس النتيجة التي توصلنا لها بتطبيق قانون كولوم وبطريقة أسهل.

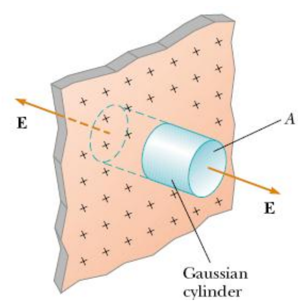
A Surface Charge Distribution

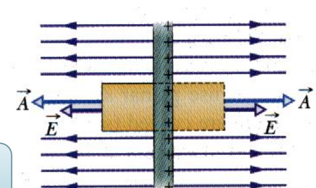
In the figure calculate the electric field due to non-conducting, infinite plane with uniform charge per unit area σ.

The electric field E is constant in magnitude and perpendicular to the plane charge and directed outward for both surfaces of the plane. Therefore for symmetry we select a cylindrical gaussian surface with its axis is perpendicular to the plane, each end of the gaussian surface has area A and are equidistance from the plane.
The flux through the end of the gaussian cylinder is EA since E is perpendicular to the surface.
The total electric flux from both ends of the gaussian surface will be $2EA$.

Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \rightarrow 2EA = \frac{\sigma A}{\varepsilon_o} \rightarrow \therefore E = \frac{\sigma}{2\varepsilon_o}$$

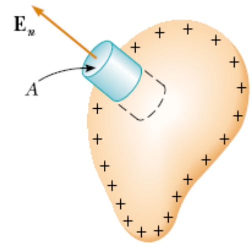




An Insulated Conductor

ذكرنا سابقا أن الشحنة توزع على سطح الموصل فقط، وبالتالي فإن قيمة المجال داخل مادة الموصل تساوى صفراً، وقيمة المجال خارج الموصل تساوى

$$E = \frac{\sigma}{\varepsilon_o}$$



في الشكل الموضح أعلاه نلاحظ أن الوجه الأمامي لسطح جاوس له فيض حيث أن الشحنة تستقر على السطح الخارجي، بينما يكون الفيض مساوياً للصفر للسطح الخلفي الذي يخترق الموصل وذلك لأن الشحنة داخل الموصل تساوي صفراً.

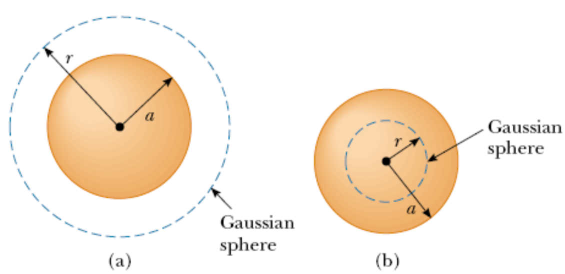
لاحظ هنا أن المجال في حالة الموصل يساوى ضعف قيمة المجال في حالة السطح اللانهائي المشحون، وذلك لأن خطوط المجال تخرج من السطحين في حالة السطح غير الموصل، بينما كل خطوط المجال تخرج من السطح الخارجي في حالة الموصل.

A Volume Charge Distribution

In the figure shows an insulating sphere of radius *a* has a uniform charge density ρ and a total charge *Q*.

1) Find the electric field at point outside the sphere (*r*>*a*)

2) Find the electric field at point inside the sphere (*r*<*a*)



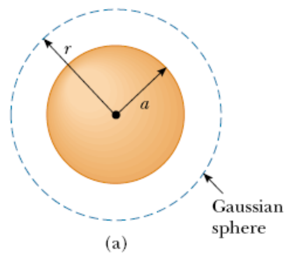
1) Find The Electric Field at Point Outside the Sphere (*R*>*a*)

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where *r*>*a*. The electric field *E* is perpendicular to the gaussian surface as shown in figure. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint A = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_o r^2} \quad \text{(for r>a)}$$



(a)

Note that the result is identical to appoint charge.

2) Find The Electric Field at Point Inside the Sphere (*R*<*a*)

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where *r*<*a*. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.17. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

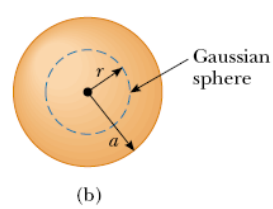
It is important at this point to see that the charge inside the gaussian surface of volume $V`$ is less than the total charge Q.

To calculate the charge q_{in}, we use $q_{in}=\rho V`$, where $V`=4/3\pi r^3$. Therefore,

$$q_{in} = \rho V` = \rho(4/3\pi r^3)$$

$$E\oint A = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2} = \frac{\rho}{3\varepsilon_o} r$$

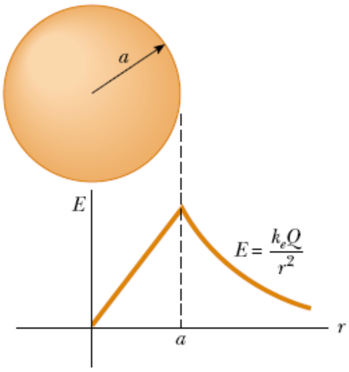


(b)

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2} = \frac{\rho}{3\varepsilon_o} r$$

since $\rho = \frac{Q}{\frac{4}{3}\pi a^3}$

$$\therefore E = \frac{Qr}{4\pi\varepsilon_o a^3}$$ (for $r<a$)



Note that the electric field when $r<a$ is proportional to r, and when $r>a$ the electric field is proportional to $1/r^2$.

Steps Which Should be Followed in Solving Problems

1. **The gaussian surface should be chosen to have the same symmetry as the charge distribution.**
2. **The dimensions of the surface must be such that the surface includes the point where the electric field is to be calculated.**
3. **From the symmetry of the charge distribution, determine the direction of the electric field and the surface area vector *dA*, over the region of the gaussian surface.**
4. **Write *E.dA* as *E dA cos*θ and divide the surface into separate regions if necessary.**
5. **The total charge enclosed by the gaussian surface is *dq* = ∫*dq*, which is represented in terms of the charge density (*dq* = λ*dx* for line of charge, *dq* = σ*dA* for a surface of charge, *dq* = ρ*dv* for a volume of charge).**

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General Physics II

Electrostatic: Principles & Applications

Lecture (8): Discussion on Gauss's Law

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Example 1

If the net flux through a gaussian surface is zero, which of the following statements are true?

a) There are no charges inside the surface.
b) The net charge inside the surface is zero.
c) The electric field is zero everywhere on the surface.
d) The number of electric field lines entering the surface equals the number leaving the surface.

الجملتين (b) و (d) صحيحتين.

الجملة (a) ليست بالضرورة صحيحة لان قانون جاوس ينص على ان الفيض الكلي خلال سطح مغلق يساوي الشحنة الكلية داخل السطح مقسومة على ثابت النفاذية ε_o. على سبيل المثال، يمكن ان يكون ثنائي قطب داخل السطح.

بالرغم من ان الفيض الكلي يمكن ان يكون مساويا للصفر، فانه لا يمكن ان نستنتج ان المجال الكهربي يساوي صفر في هذه المنطقة.

Example 2

A spherical gaussian surface surrounds a point charge *q*. Describe what happens to the: flux through the surface if

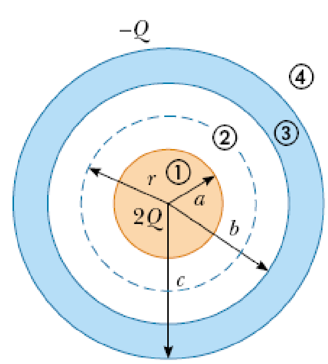
1. **The charge is tripled,**
2. **The volume of the sphere is doubled,**
3. **The shape of the surface is changed to that of a cube,**
4. **The charge is moved to another position inside the surface;**

1. اذا زادت الشحنة 3 مرات فان الفيض الناتج عن السطح يزداد بـ 3 مرات، لان الفيض الكلي يتناسب طرديا مع الشحنة داخل السطح.
2. الفيض لا يتغير عندما يتغير الحجم، لان الحجم لا يزال يحيط بنفس الكمية من الشحنة.
3. الفيض لا يتغير عندما يتغير شكل السطح.
4. الفيض من خلال سطح لا يتغير داخل السطح عندما تتحرك الشحنة داخل السطح.

Example 3 A Sphere Inside a Spherical Shell

A solid conducting sphere of radius *a* has a net charge +2*Q*.

A conducting spherical shell of inner radius *b* and outer radius *c* is concentric with the solid sphere and has a net charge –*Q* as shown in the figure.



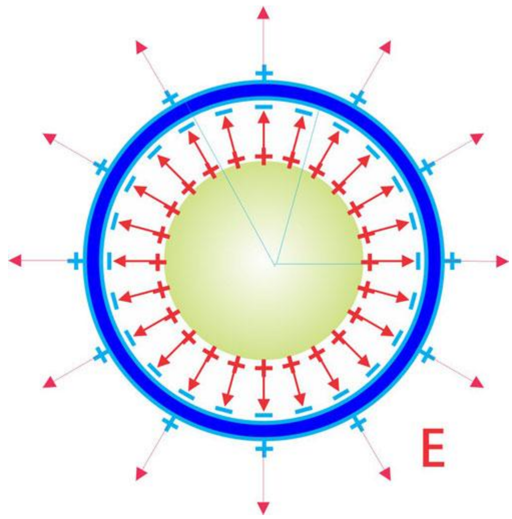
Using Gauss's law find the electric field in the regions labeled 1, 2, 3, 4 and find the charge distribution on the spherical shell.

توزيع الشحنة على الكرة الداخلية والقشرة الكروية

+2Q على السطح الخارجي للكرة المركزية

-2Q على السطح الداخلي للقشرة الكروية

+Qعى السطح الخارجي للقشرة الكروية



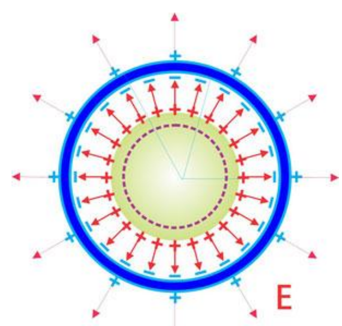
Solution

نلاحظ أن توزيع الشحنة على الكرتين لها تماثل كروي، لذلك لتعيين المجال الكهربي عند مناطق مختلفة فإننا سنفرض أن سطح جاوس كروي الشكل نصف قطره *r*.

Region (1) $r < a$

To find the E inside the solid sphere of radius a we construct a gaussian surface of radius $r < a$

***E* = 0 since no charge inside the gaussian surface.**



Region (2) a < *r* < *b*

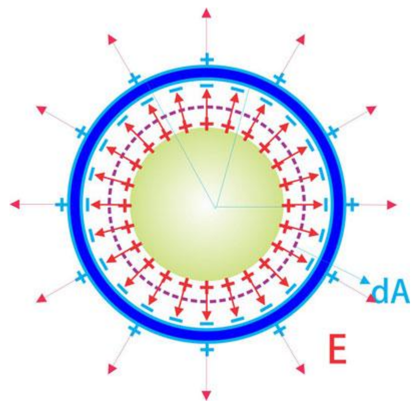
we construct a spherical gaussian surface of radius *r*

لاحظ هنا أن الشحنة المحصورة داخل سطح جاوس هي شحنة الكرة الموصلة الداخلية 2*Q* وأن خطوط المجال في اتجاه أنصاف الأقطار وخارجه من سطح جاوس أي 0 = θ والمجال ثابت المقدار على السطح.

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\, 4\pi\, r^2 = \frac{2Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2} \qquad a < r < b$$



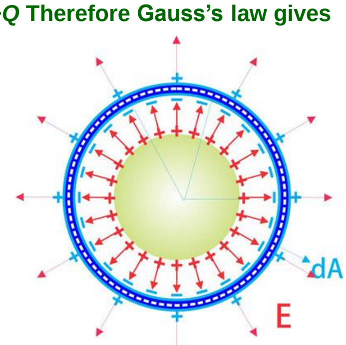
Region (4) *r* > *c*

we construct a spherical gaussian surface of radius *r* > *c*, the total net charge inside the gaussian surface is *q* = 2*Q* + (-*Q*) = +*Q* Therefore Gauss's law gives

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\, 4\pi\, r^2 = \frac{2Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad r > c$$



Region (3) *b* > *r* < *c*

المجال الكهربي في هذه المنطقة يجب أن يكون صفراً لأن القشرة الكروية موصلة أيضا، ولأن الشحنة الكلية داخل سطح جاوس *b*<*r*<*c* يجب أن تساوى صفراً.

إذا نستنتج أن الشحنة -*Q* على القشرة الكروية هي نتيجة توزيع شحنة على السطح الداخلي والسطح الخارجي للقشرة الكروية بحيث تكون المحصلة -*Q* وبالتالي تتكون بالحث شحنة على السطح الداخلي للقشرة مساوية في المقدار للشحنة على الكرة الداخلية ومخالفة لها في الإشارة أي -2*Q* وحيث أنه كما في معطيات السؤال الشحنة الكلية على القشرة الكروية هي -*Q* نستنتج أن على السطح الخارجي للقشرة الكروية يجب أن تكون +*Q*

Example 4

Two large non-conducting sheets of +ve charge face each other as shown in the figure. What is *E* at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?

نعلم ان المجال الكهربي لسطح غير موصل عند أي نقطة فوقه هو

$$E = \frac{\sigma}{2\varepsilon_o}$$

(a) At point to the left of the two parallel sheets

$$E = -E_1 + (-E_2) = -2E$$

$$\therefore E = -\frac{\sigma}{\varepsilon_o}$$

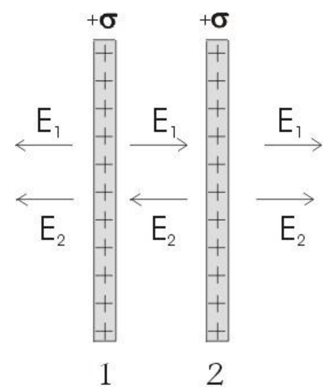
(b) At point between the two sheets

$$E = E_1 + (-E_2) = \text{zero}$$

(c) At point to the right of the two parallel sheets

$$E = E_1 + E_2 = 2E$$

$$\therefore E = \frac{\sigma}{\varepsilon_o}$$

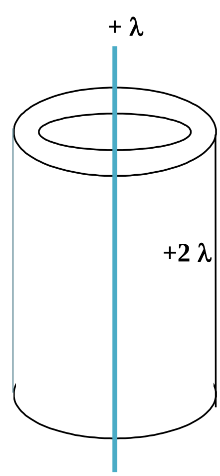


Example 5

A long straight wire is surrounded by a hollow cylinder whose axis coincides with that wire as shown in the figure. The solid wire has a charge per unit length of +λ, and the hollow cylinder has a *net* charge per unit length of +2λ. Use Gauss law to find

(a) the charge per unit length on the inner and outer surfaces of the hollow cylinder and

(b) the electric field outside the hollow cylinder, a distance *r* from the axis.



Solution

(a) Use a cylindrical Gaussian surface S_1 within the conducting cylinder where E=0

$$\text{Thus,} \quad \oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} = 0$$

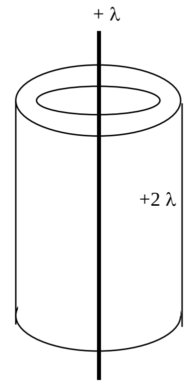
and the charge per unit length on the inner surface must be equal to

$$\lambda_{inner} = -\lambda$$

Also $\lambda_{inner} + \lambda_{outer} = 2\lambda$

Thus $\lambda_{outer} = 3\lambda$

(b) For a gaussian surface S_2 outside the conducting cylinder



$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E(2\pi rL) = \frac{1}{\varepsilon_o}(\lambda - \lambda + 3\lambda)L$$

$$\therefore E = \frac{3\lambda}{2\pi\varepsilon_o r}$$

Example 6

Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where $r<R$.

Solution

If we choose a cylindrical gaussian surface of length L and radius r, Its volume is $\pi r^2 L$, and it encloses a charge $\rho\pi r^2 L$. By applying Gauss's law we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \quad \text{becomes,} \quad E\oint dA = \frac{\rho\pi r^2 L}{\varepsilon_o}$$

$$dA = 2\pi rL \quad \text{therefore,} \quad E(2\pi rL) = \frac{\rho\pi r^2 L}{\varepsilon_o}$$

Thus,

$$E = \frac{\rho r}{2\varepsilon_o} \quad \text{Radially outward from the cylinder axis,}$$

Notice that the electric field will increase as ρ increases, and also the electric field is proportional to r for $r<R$. For the region outside the cylinder ($r>R$), the electric field will decrease as r increases.

Problems to Solve by Yourself

Problem 1

An electric field of intensity 3.5×10^3N/C is applied the x-axis. Calculate the electric flux through a rectangular plane 0.35m wide and 0.70m long if

(a) the plane is parallel to the yz plane,

(b) the plane is parallel to the xy plane, and

(c) the plane contains the y axis and its normal makes an angle of 40^o with the x axis.

Problem 2

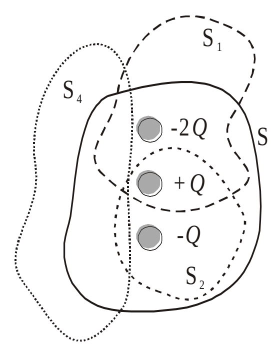
A point charge of +5μC is located at the center of a sphere with a radius of 12cm. What is the electric flux through the surface of this sphere?

Problem 3

(a) Two charges of 8μC and -5μC are inside a cube of sides 0.45m. What is the total electric flux through the cube? (b) Repeat (a) if the same two charges are inside a spherical shell of radius 0. 45 m.

Problem 4

Four closed surfaces, S_1, through S_4, together with the charges *-2Q*, +*Q*, and -Q are sketched in the figure. Find the electric flux through each surface.



Problem 5

The electric field everywhere on the surface of a hollow sphere of radius 0.75m is measured to be equal to 8.90×10^{2}N/C and points radially toward the center of the sphere.

(a) What is the net charge within the surface?

(b) What can you conclude about charge inside the nature and distribution of the charge inside the sphere?

Problem 6

A conducting spherical shell of radius 15cm carries a net charge of -6.4μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

Problem 7

A long, straight metal rod has a radius of 5cm and a charge per unit length of 30nC/m. Find the electric field at the following distances from the axis of the rod: (a) 3cm, (b) 10cm, (c) 100cm.

Problem 8

A square plate of copper of sides 50cm is placed in an extended electric field of 8×10^4N/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

Problem 9

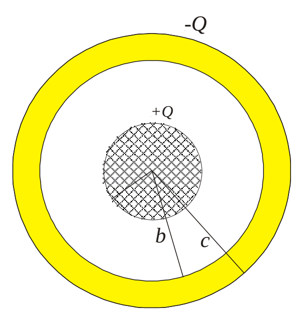
A solid copper sphere 15cm in radius has a total charge of 40nC. Find the electric field at the following distances measured from the center of the sphere: (a) 12cm, (b) 17cm, (c) 75cm. (d) How would your answers change if the sphere were hollow?

Problem 10

A solid conducting sphere of radius 2cm has a positive charge of +8μC. A conducting spherical shell d inner radius 4cm and outer radius 5cm is concentric with the solid sphere and has a net charge of -4μC. (a) Find the electric field at the following distances from the center of this charge configuration: (a) *r*=1cm, (b) *r*=3cm, (c) *r*=4.5cm, and (d) *r*=7cm.

Problem 11

A non-conducting sphere of radius a is placed at the center of a spherical conducting shell of inner radius b and outer radius c, A charge $+Q$ is distributed uniformly through the inner sphere (charge density ρ C/m^3) as shown in the figure. The outer shell carries $-Q$. Find $E(r)$ (i) within the sphere ($r<a$) (ii) between the sphere and the shell ($a<r<b$) (iii) inside the shell ($b<r<c$) and (iv) out side the shell and (v) What is the charge appear on the inner and outer surfaces of the shell?



Problem 12

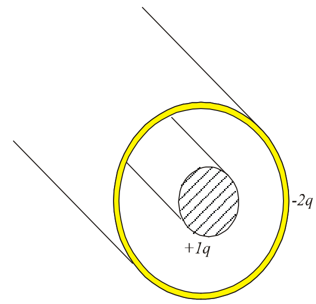
A solid sphere of radius 40cm has a total positive charge of 26μC uniformly distributed throughout its volume. Calculate the electric field intensity at the following distances from the center of the sphere: (a) 0 cm, (b) 10cm, (c) 40cm, (d) 60 cm.

Problem 13

An insulating sphere is 8cm in diameter, and carries a +5.7μC charge uniformly distributed throughout its interior volume. Calculate the charge enclosed by a concentric spherical surface with the following radii: (a) r=2cm and (b) r=6cm.

Problem 14

A long conducting cylinder (length *l*) carry a total charge $+q$ is surrounded by a conducting cylindrical shell of total charge $-2q$ as shown in the figure. Use Gauss's law to find (i) the electric field at points outside the conducting shell and inside the conducting shell, (ii) the distribution of the charge on the conducting shell, and (iii) the electric field in the region between the cylinder and the cylindrical shell?



Problem 15

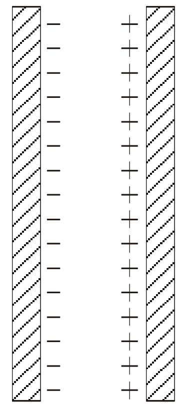
Consider a thin spherical shell of radius 14cm with a total charge of 32μC distributed uniformly on its surface. Find the electric field for the following distances from the center of the charge distribution: (a) r=10cm and (b) r =20cm.

Problem 16

A large plane sheet of charge has a charge per unit area of 9.0μC/m^2. Find the electric field intensity just above the surface of the sheet, measured from the sheet's midpoint.

Problem 18

Two large metal plates face each other and carry charges with surface density +σ and -σ respectively, on their inner surfaces as shown in the figure. What is *E* at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?



يمكنكم الحصول على مساعدة ارشادية في حل هذه المسائل من خلال هذا الرابط

http://www.hazemsakeek.com/Physics_Lectures/Electrostatic/GP2_Problems_4.htm

اتمنى ان تكون الاستفادة من هذه المساعدة الارشادية في الحل بعد التفكير العميق في كل مسألة ومحاولة حلها بنفسك

مع خالص تحياتي ودعواتي لكم بالتوفيق

Physics Academy

www.physicsacademy.org

General Physics II

Electrostatic: Principles & Applications

Lecture (9): Electric Potential

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Al-Azhar University of Gaza

Electric Potential

1. Definition of electric potential difference
2. The Equipotential surfaces
3. Electric Potential and Electric Field
4. Potential difference due to a point charge
5. The potential due to a point charge
6. Electric Potential Energy
7. Calculation of E from V
8. Problems

في هذه الفصل سوف نتعلم كيف يمكننا التعبير عن التأثير الكهربي في الفراغ المحيط بشحنة أو أكثر بواسطة كمية قياسية تسمى الجهد الكهربي The electric potential. وحيث أن الجهد الكهربي كمية قياسية وبالتالي فسيكون التعامل معه أسهل في التعبير عن التأثير الكهربي من المجال الكهربي.

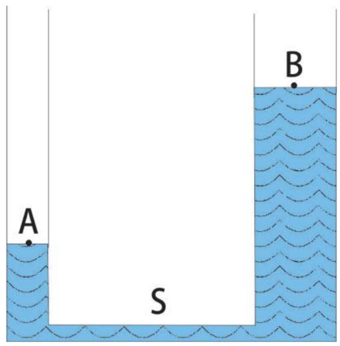
قبل أن نبدأ بتعريف الجهد الكهربي أو بمعنى أصح فرق الجهد الكهربي بين نقطتين في مجال شحنة في الفراغ سوف نضرب بعض الأمثلة التوضيحية.

مثال توضيحي (1)

نفرض إناء على شكل حرف *U* به ماء كما في الشكل.

تكون طاقة الوضع لجزئ الماء عند النقطة B أكبر من طاقة الوضع عند النقطة A.

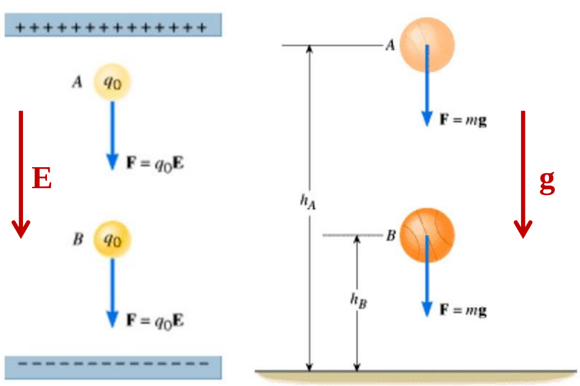
لذلك إذا فتح الصنبور S فإن الماء سوف يتدفق في اتجاه النقطة A إلى أن يصبح الفرق في طاقتي الوضع بين النقطتين A&B مساويا للصفر.



مثال توضيحي (2)

عند رفع جسم كتلته ***m*** إلى ارتفاع ***h*** فوق سطح الأرض فإننا نقول أن شغلا خارجيا (موجبا) تم بذله لتحريك الجسم ضد عجلة الجاذبية الأرضية، وهذا الشغل سوف يتحول إلى طاقة وضع مختزنة في المجموعة المكونة من الجسم ***m*** والأرض. وطاقة الوضع هذه تزداد بازدياد المسافة ***h*** لأنه بالطبع سيزداد الشغل المبذول. إذا زال تأثير الشغل المبذول على الجسم ***m*** فإنه سيتحرك من المناطق ذات طاقة الوضع المرتفعة إلى المناطق ذات طاقة الوضع المنخفضة حتى يصبح فرق طاقة الوضع مساوياً للصفر.

Electrostatic force **Gravitational force**



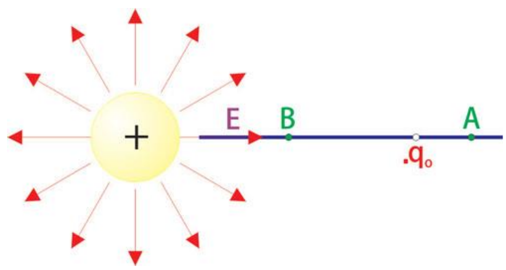
$$W_{AB} = mgh_A - mgh_B$$

$$\Delta U = U_f - U_i = -W$$

Electric energy is one type of energy

مثال توضيحي (3)

هناك حالة مشابهة تماما للحالتين السابقتين في الكهربية، حيث نفترض أن النقطتين A&B موجودتان في مجال كهربي ناتج من شحنة موجبة Q كما في الشكل. إذا كانت هناك شحنة اختبار q_o (مناظرة للجسم m في مجال عجلة الجاذبية الأرضية وكذلك لجزئ الماء عند النقطة B في المثال السابق) موجودة بالقرب من الشحنة Q فإن الشحنة q_o سوف تتحرك من نقطة قريبة من الشحنة إلى نقطة أكثر بعداً أي من B إلى A وفيزيائيا نقول أن الشحنة q_o تحركت من مناطق ذات جهد كهربي مرتفع إلى مناطق ذات جهد كهربي منخفض. ولذلك يكون تعريف فرق الجهد الكهربي بين نقطتين A&B واقعتين في مجال كهربي شدته E بحساب الشغل المبذول بواسطة قوة خارجية (F_{ex}) ضد القوى الكهربية (qE) لتحريك شحنة اختبار q_o من A إلى B بحيث تكون دائما في حالة اتزان (أي التحريك بدون عجلة).

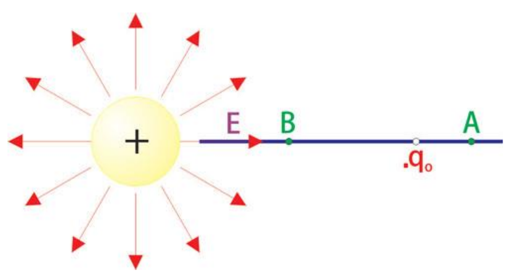


إذا كانت هنالك بطارية فرق الجهد بين قطبيها 1.5volt فهذا يعنى إنها إذا ما وصلت في دائرة كهربية، فإن الشحنات الموجبة ستتحرك من الجهد المرتفع إلى الجهد المنخفض. كما حدث في حالة فتح الصنبور في الأنبوبة U وستستمر حركة الشحنات حتى يصبح فرق الجهد بين قطبي البطارية مساوياً للصفر.

Definition of Electric Potential Difference

We define the potential difference between two points *A* and *B* as the work done by an external agent in moving a test charge q_o from *A* to *B i.e.*

$$V_B - V_A = \frac{W_{AB}}{q_o}$$



The unit of the potential difference is (*Joule/Coulomb*) which is known as *Volt (V)*

Electric Potential Difference is Scalar

The potential difference is independent on the path between *A* and *B*. Since the work (W_{AB}) done to move a test charge q_o from *A* to *B* is independent on the path, otherwise the work is not a scalar quantity. (see example 2)

$$V_B - V_A = \frac{W_{AB}}{q_o}$$

Since the work may be **(a) positive *i.e*** $V_B > V_A$

(b) negative *i.e* $V_B < V_A$

(c) zero *i.e* $V_B = V_A$

You should remember that the work equals

$$W = \vec{F}_{ex}.\vec{l} = F_{ex}\cos\theta\, l$$

If $0 < \theta < 90 \Rightarrow \cos\theta$ **is +ve and therefore the *W* is +ve**

If $90 < \theta < 180 \Rightarrow \cos\theta$ **is -ve and therefore *W* is -ve**

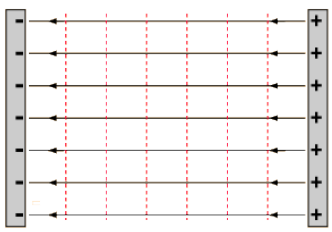
If $\theta = 90$ **between** F_{ex} **and l** $\Rightarrow$ **therefore *W* is zero**

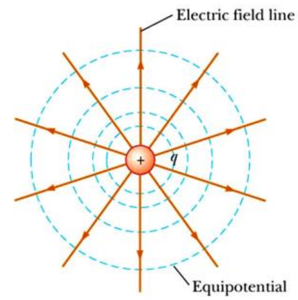
Electric Potential

The Equipotential surfaces

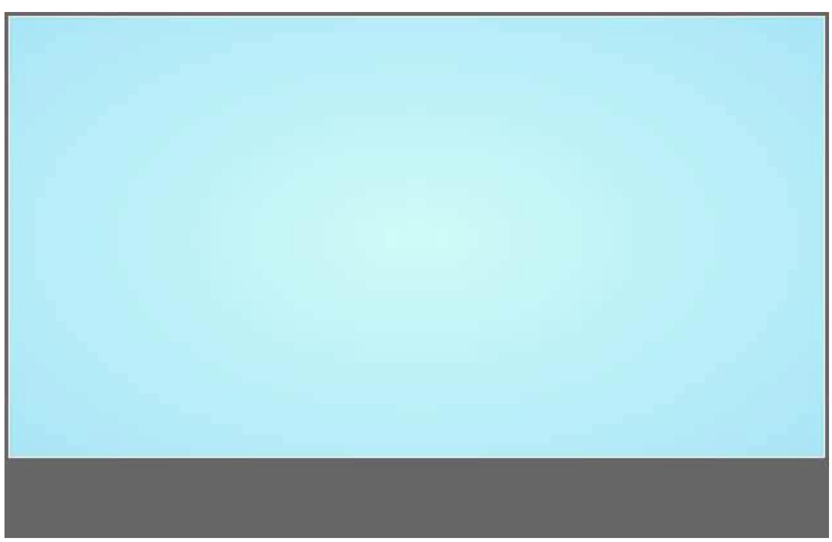
As the electric field can be represented graphically by lines of force, the potential distribution in an electric field may be represented graphically by equipotential surfaces.

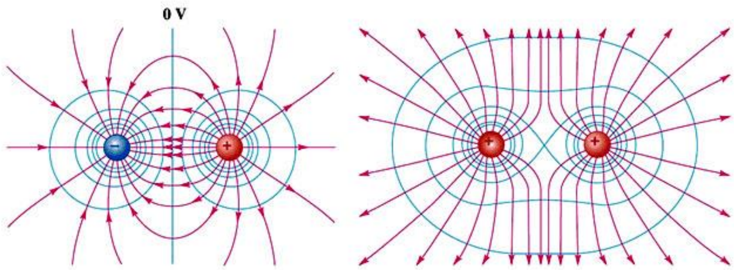
The equipotential surface is a surface such that the potential has the same value at all points on the surface. *i.e.* $V_B - V_A =$ zero for any two points on one surface.





The Equipotential surfaces





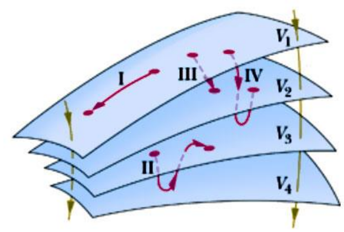
(b) An electric dipole (c) Two equal positive charges

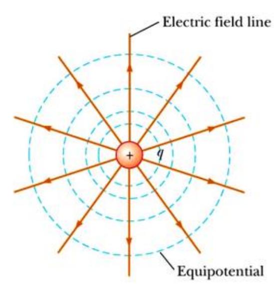
The work is required to move a test charge between any two points on an equipotential surface is zero. (*Explain why*?)

يعود السبب في ذلك إلى ان الشغل يساوي حاصل ضرب الشحنة في فرق الجهد بين النقطتين A وB

$$V_B - V_A = \frac{W_{AB}}{q_o} \rightarrow W_{AB} = q_o(V_B - V_A)$$

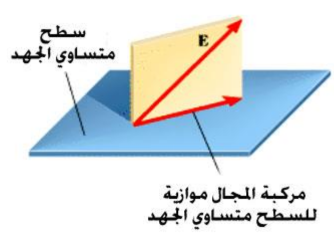
وحيث ان فرق الجهد V_B-V_A يساوي صفر فان الشغل المبذول لتحريك الشحنة q_o يساوي صفر.





In all cases the equipotential surfaces are at right angles to the lines of force and thus to *E*. (*Explain why*?)

- **لنفترض ان المجال الكهربي لم يكن عموديا على السطح متساوي الجهد كما في الشكل الموضح.**
- **يكون في هذه الحالة مركبتين للمجال احدهما مركبة عمودية واخرى موازية للسطح (المركبة المماسية)**
- **نعلم ان الشغل المبذول لتحريك شحنة على سطح متساوي الجهد يساوي صفر ولكن في هذه الحالة المفترض فان هناك شغل يجب ان يبذل لتحريك الشحنة لوجود المركبة المماسية وهذا يتنافي مع كون السطح متساوي الجهد**
- **ولحل هذا التناقض فان المجال الكهربي لا يمتلك أي مركبة مماسية وهذا يتحقق فقط اذا كان المجال عمودي على سطح متساوي الجهد.**

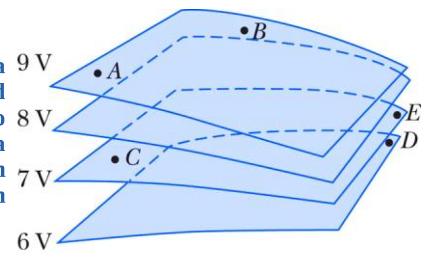


Properties of equipotential surfaces

1. The net electric force does no work as a charge moves on an equipotential surface.
2. The electric field created by any charge or group of charges is everywhere perpendicular to the associated equipotential surfaces.
3. The electric field points in the direction of decreasing potential.

Quiz

The labeled points in the figure are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from A to B, from B to C , from C to D, and from D to E.



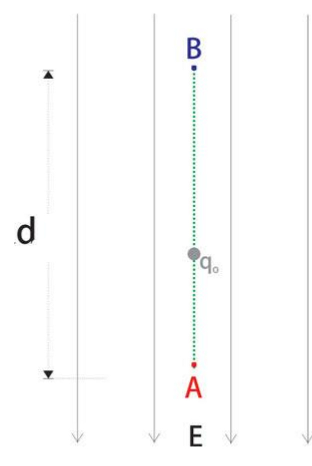
Electric Potential and Electric Field

Simple Case (Uniform electric field):

The potential difference between two points *A* and *B* in a Uniform electric field *E* can be found as follow,

Assume that a positive test charge q_o is moved by an external agent from *A* to *B* in uniform electric field as shown in the figure.

The test charge q_o is affected by electric force of q_oE in the downward direction. To move the charge from *A* to *B* an external force *F* of the same magnitude to the electric force but in the opposite direction.



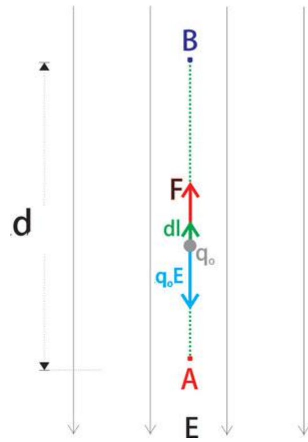
Electric Potential and Electric Field continue

The work W done by the external agent is:

$$W_{AB} = Fd = q_o Ed$$

The potential difference $V_B - V_A$ is

$$V_B - V_A = \frac{W_{AB}}{q_o} = Ed$$

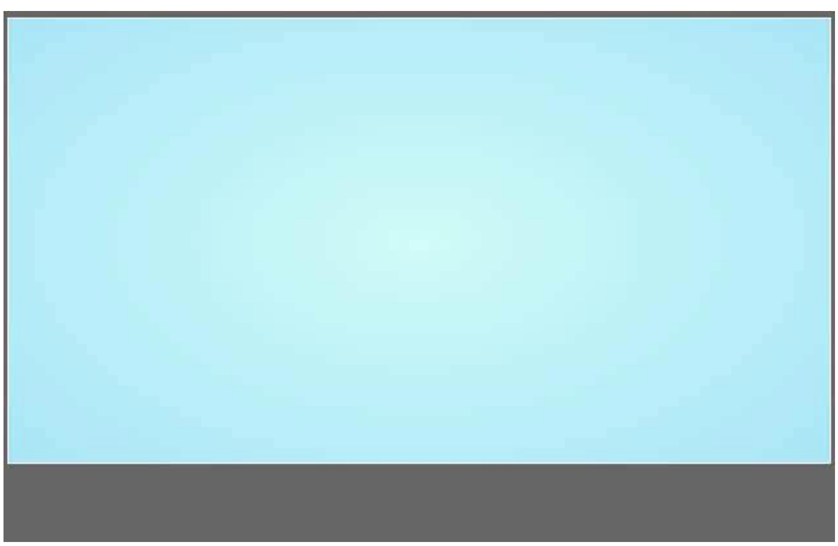


This equation shows the relation between the potential difference and the electric field for a special case (uniform electric field).

Note that E has a new unit (V/m). hence,

$$\frac{Volt}{Meter} = \frac{Newton}{Coulomb}$$

Electric Potential and Electric Field



Electric Potential and Electric Field continue

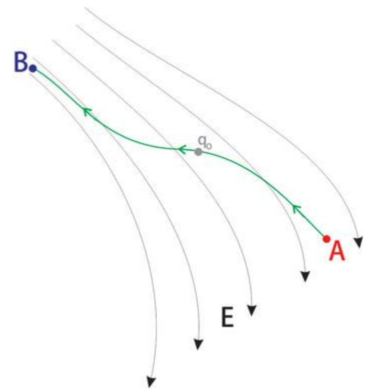
The relation in general case (not uniform electric field):

If the test charge q_o is moved along a curved path from A to B as shown in the figure.

The electric field exerts a force q_oE on the charge. To keep the charge moving without accelerating, an external agent must apply a force F equal to $-q_oE$.

If the test charge moves distance dl along the path from A to B, the work done is $F.dl$.

The total work is given by, $W_{AB} = \int_A^B \vec{F}.d\vec{l}$



Electric Potential and Electric Field continue

$$W_{AB} = \int_A^B \vec{F}.d\vec{l} = -q_o \int_A^B \vec{E}.d\vec{l}$$

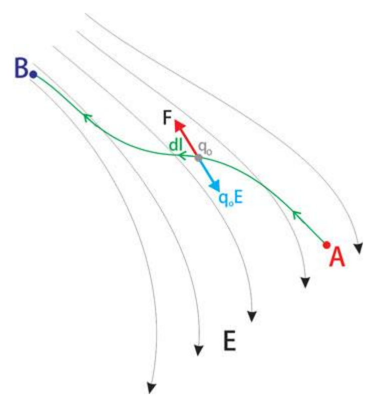
The potential difference V_B-V_A is,

$$V_B - V_A = \frac{W_{AB}}{q_o} = -\int_A^B \vec{E}.d\vec{l}$$

لاحظ هنا أن حدود التكامل من *A* إلى *B* هى التي تحدد المسار ومنه اتجاه متجه الإزاحة *dl* وتكون الزاوية θ هي الزاوية المحصورة بين منتجه الإزاحة ومتجه المجال الكهربي.

If the point A is taken to infinity then V_A=0 the potential V at point B is,

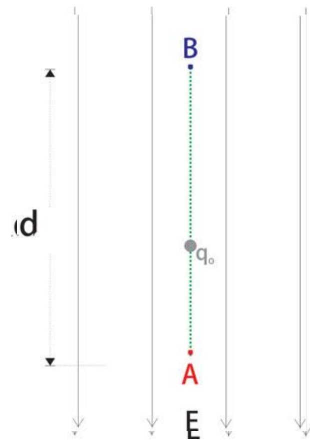
$$V_B = -\int_\infty^B \vec{E}.d\vec{l}$$



This equation gives the general relation between the potential and the electric field.

Example 1

Derive the potential difference between points *A* and *B* in uniform electric field using the general case.



Solution:

E **is uniform (constant) and the integration over the path *A* to *B* is *d*, therefore**

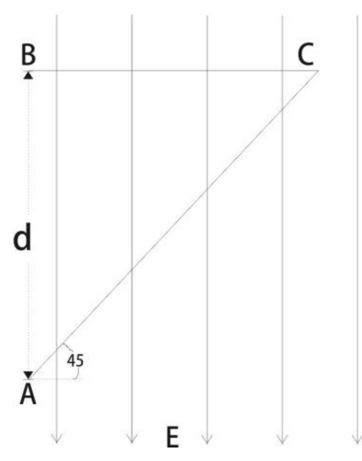
$$V_B - V_A = -\int_A^B \vec{E}.d\vec{l} = -\int_A^B E\cos 180^o\, dl = \int_A^B Edl$$

$$V_B - V_A = E\int_A^B dl = Ed$$

وهي نفس النتيجة التي حصلنا عليها في حالة المجال الكهربي المنتظم.

Example 2

In the figure the test charge moved from *A* to *B* along the path shown. Calculate the potential difference between *A* and *B*.



Solutions

$$V_B - V_A = (V_B - V_C) + (V_C - V_A)$$

For the path *AC* the angle θ is 135º,

$$V_C - V_A = -\int_A^C \vec{E}.d\vec{l} = -\int_A^C E\cos 135^o\, dl = \frac{E}{\sqrt{2}}\int_A^C dl$$

The length of the line *AC* is √2 *d*

$$V_C - V_A = \frac{E}{\sqrt{2}}(\sqrt{2}d) = Ed$$

For the path *CB* the work is zero and *E* is perpendicular to the path therefore, V_C-V_A = 0

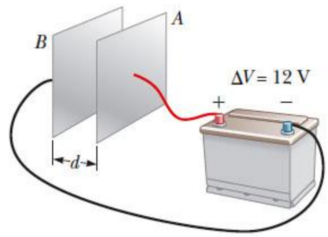
$$V_B - V_A = V_C - V_A = Ed$$

نستنتج ان فرق الجهد الكهربي بين نقطتين لا يعتمد على المسار.

Example 3

A battery has a specified potential difference ΔV between its terminals and establishes that potential difference between conductors attached to the terminals. A 12-V battery is connected between two parallel plates as shown in the figure. The separation between the plates is d = 0.30 cm, and we assume the electric field between the plates to be uniform. Find the magnitude of the electric field between the plates.

Solution



$$V_B - V_A = Ed$$

$$E = \frac{V_B - V_A}{d}$$

$$E = \frac{12}{0.30 \times 10^{-2}} = 4.0 \times 10^{3} V/m$$

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General Physics II

Electrostatic: Principles & Applications

Lecture (10): Electric Potential due to point charge

Dr. Hazem Falah Sakeek
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Electric Potential

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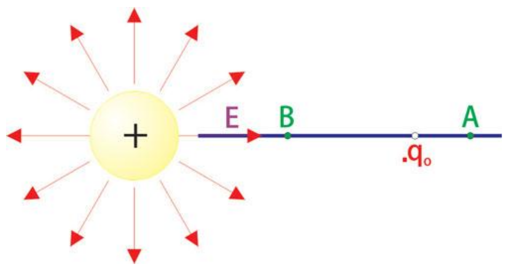
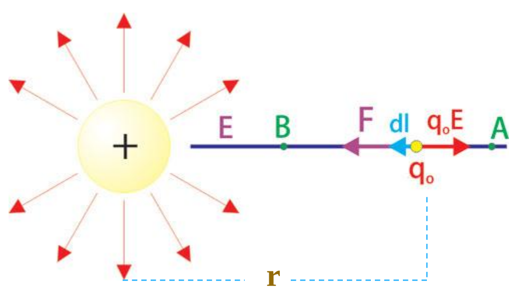
Potential Difference due to a Point Charge

Assume two points *A* and *B* near to a positive charge *q* as shown in the figure.

To calculate the potential difference V_B-V_A we assume a test charge q_o is moved without acceleration from *A* to *B*.



In the figure the electric field *E* is directed to the right and *dl* to the left.



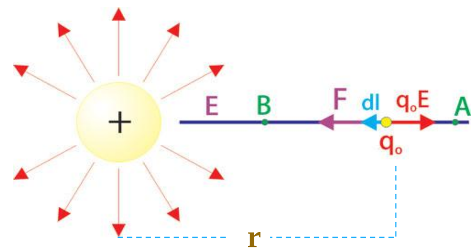
$$\vec{E}.\vec{dl} = E\cos 180^o\, dl = -Edl$$

However when we move a distance *dl* to the left, we are moving in a direction of decreasing *r*. Thus

$$\vec{dl} = -d\vec{r}$$

Therefore

$$-Edl=Edr$$



$$\therefore V_B - V_A = -\int_A^B \vec{E}.d\vec{l} = \int_{r_A}^{r_B} \vec{E}.d\vec{r}$$

$$\because E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

$$\therefore V_B - V_A = -\frac{q}{4\pi\varepsilon_o}\int_{r_A}^{r_B}\frac{dr}{r^2}$$

$$\therefore V_B - V_A = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

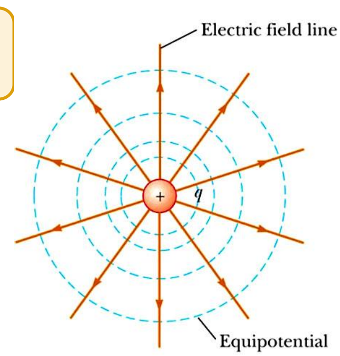
لاحظ هنا أن هذا القانون يستخدم لإيجاد فرق الجهد الكهربي بين نقطتين في الفراغ المحيط بشحنة *q*.

The Potential due to a Point Charge

If we choose *A* at infinity then V_A=0 (*i.e.* $r_A \Rightarrow \infty$) this lead to the potential at distance r from a charge *q* is given by

$$\therefore V_B - V_A = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$



This equation shows that the equipotential surfaces for a charge are spheres concentric with the charge as shown in figure

لاحظ أن المجال الكهربي لشحنة يتناسب عكسيا مع مربع المسافة، بينما الجهد الكهربي يتناسب عكسيا مع المسافة.

Electric Potential due to Several Point Charges

يمكن باستخدام قانون الجهد الكهربي لشحنة في الفراغ من إيجاد الجهد الكهربي لنقطة تبعد عن شحنة أو أكثر من شحنة عن طريق الجمع الجبري للجهد الكهربي الناشئ عن كل شحنة على حده عند النقطة المراد إيجاد الجهد الكلى عندها أي

$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots. + V_n$$

$$\therefore V = \sum_n V_n$$

$$\therefore V = \frac{1}{4\pi\varepsilon_o}\sum_n \frac{q_n}{r_n}$$

عند التعويض عن قيمة الشحنة q تأخذ الإشارة في الحسبان، لاننا نقوم بالجمع هنا جمعا جبرياً وليس جمعاً اتجاهياً كما كنا نفعل في المجال الكهربي.

Example 1

What must the magnitude of an isolated positive charge be for the electric potential at 10 cm from the charge to be +100V?

Solution

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

$$\therefore q = V 4\pi\varepsilon_o r^2 = 100 \times 4\pi \times 8.9 \times 10^{-12} \times 0.1 = 1.1 \times 10^{-9} C$$

Example 2

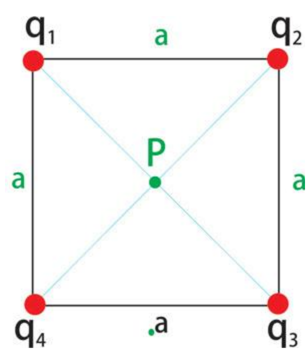
What is the potential at the center of the square shown in the figure? Assume that q_1= +1 ×10^{-8}C, q_2= -2×10^{-8}C, q_3=+3×10^{-8}C, q_4=+2×10^{-8}C, and a=1m.

Solution

$$\therefore V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o}\frac{q_1 + q_2 + q_3 + q_4}{r}$$

The distance r for each charge from P is 0.71m

$$\therefore V = \frac{9\times10^9(1-2+3+2)\times10^{-8}}{0.71} = 500V$$



Example 3

Calculate the electric potential due to an electric dipole as shown in the figure

Solution

$$V = \sum V_n = V_1 + V_2$$

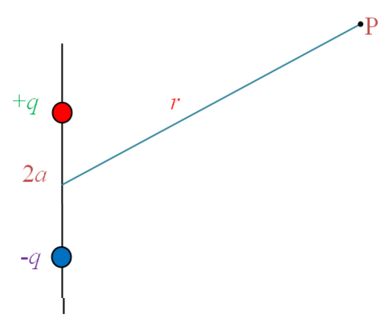
$$V = K\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = Kq\frac{r_2 - r_1}{r_2 r_1}$$

When r>>2a,

$r_2 - r_1 \cong 2a\cos\theta$ and $r_1 r_2 \cong r^2$,

$$V = Kq\frac{2a\cos\theta}{r^2} = K\frac{p\cos\theta}{r^2}$$

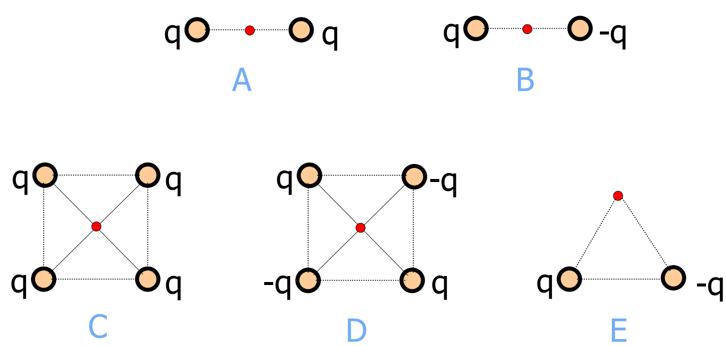
where p is the dipole momentum



Note that V = 0 when θ=90º but V has the maximum positive value when θ=0º and V has the maximum negative value when θ=180º.

Quiz

Which of the following figures have V=0 and E=0 at red point?



Problems to Solve by Yourself

1. **How much energy is gained by a charge of 75 μC moving through a potential difference of 90V?**

2. **An infinite charged sheet has a surface charge density σ of 1.0×10^{-7} C/m^2. How far apart are the equipotential surfaces whose potentials differ by 5.0 V?**

3. **At what distance from a point charge of 8μC would the potential equal 3.6×10^4V?**

4. **At a distance *r* away from a point charge *q*, the electrical potential is *V*=400V and the magnitude of the electric field is *E*=150N/C. Determine the value of *q* and *r*.**

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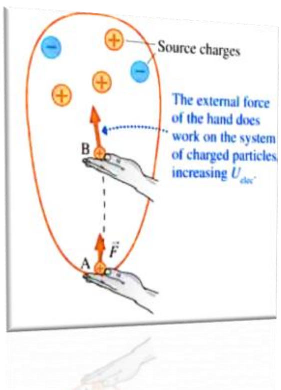
General Physics II

Electrostatic: Principles & Applications



Lecture (11): Electric Potential Energy

Dr. Hazem Falah Sakeek
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Electric Potential

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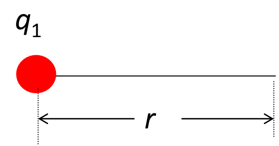
Electric Potential Energy

The definition of the *electric potential energy* of a system of charges is the work required to bring them from infinity to that configuration.

To workout the electric potential energy for a system of charges, assume a charge q_2 at infinity and at rest. As shown in the figure, if q_2 is moved from infinity to a distance r from another charge q_1, then the work required is given by

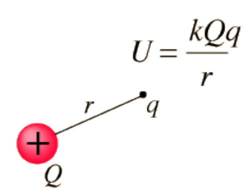
$$W = Vq_2$$



$$\because V = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r}$$

Substitute for V in the equation of work

$$U = W = Vq_2 \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}}$$



$$U = \frac{q_1 q_2}{4\pi\varepsilon_o r} \qquad \textbf{1}$$

To calculate the potential energy for systems containing more than two charges we compute the potential energy for every pair of charges separately and to add the results algebraically.

$$U = \sum \frac{q_i q_j}{4\pi\varepsilon_o r_{ij}} \qquad \textbf{2}$$

القانون الأول يطبق في حالة شحنتين فقط، ولكن إذا كانت المجموعة المراد إيجاد طاقة الوضع الكهربي لها أكثر من شحنتين نستخدم القانون الثاني حيث نوجد الطاقة المختزنة بين كل شحنتين على حده ثم نجمع جمعا جبريا، أي نعوض عن قيمة الشحنة ونأخذ الإشارة بالحسبان في كل مرة.

If the total electric potential energy of a system of charges is positive this correspond to a repulsive electric forces,

If the total electric potential energy is negative this correspond to attractive electric forces.

(explain why?)

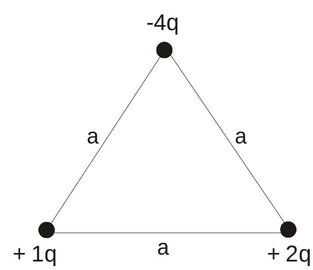
التفسير

عندما تكون طاقة الوضع الكلية لنظام من الشحنات موجبا فان الشغل المبذول لتجميع تلك الشحنات هو شغل موجب وهذا يعني ان القوة المبذولة لتحريك الشحنة في اتجاه الازاحة وهذا يحدث عندما تكون تكون القوى الكهربية المتبادلة بين الشحنات قوى تنافرية.

واذا كانت طاقة الوضع الكلية لنظام من الشحنات سالبا فان الشغل المبذول لتجميع الشحنات سيكون سالبا وهذا يعني ان القوة المبذولة لتحريك الشنحة في عكس اتجاه الازحة وهذا يحدث عندما تكون القوى الكهربية المتبادلة بين الشحنات قوى تجاذبية.

Example 1

Three charges are held fixed as shown in the figure. What is the potential energy? Assume that q=1 ×10^{-7}C and a=10cm.



Solution

$$U=U_{12}+U_{13}+U_{23}$$

$$U=\frac{1}{4\pi\varepsilon_o}\left[\frac{(+q)(-q)}{a}+\frac{(+q)(+2q)}{a}+\frac{(-4q)(+2q)}{a}\right]$$

$$U=-\frac{10}{4\pi\varepsilon_o}\frac{q^2}{a}$$

$$\therefore U=-\frac{9\times10^9(10)(1\times10^{-7})^2}{0.1}=-9\times10^{-3}\,J$$

نلاحظ أن قيمة الطاقة الكلية سالبة، وهذا يعني أن الشغل المبذول للحفاظ على ثبات الشحنات سابقة الذكر سالب أيضاً. نستنتج من ذلك أن القوة المتبادلة بين الشحنات هي قوة تجاذب، أما في حالة أن تكون الطاقة الكلية موجبة فإن هذا يعني أن القوة المتبادلة بين الشحنات هي قوة تنافر.

Calculation of *E* from *V*

As we have learned that both the electric field and the electric potential can be used to evaluate the electric effects. Also we have showed how to calculate the electric potential from the electric field now we determine the electric field from the electric potential by the following relation.

$$V_B - V_A = \Delta V = -\int_A^B \vec{E}.d\vec{l}$$

$$dV = -\vec{E}.\,d\vec{l}$$

If the electric field has only one component E_x

$$dV = -E.\,dx$$

$$E_x = -\frac{dV}{dx}$$

New unit for the electric field is *volt/meter* (v/m)

عمليا يمكن قياس الجهد الكهربي والموقع بطريقة سهلة باستخدام جهاز الفوتميتر ومسطرة. وبالتالي يمكن تحديد شدة المجال الكهربي من خلال قياس الجهد الكهربي عند نقاط مختلفة في المجال ورسم العلاقة بين الجهد الكهربي على محور y والمسافة على محور x ويكون المجال الكهربي هو ميل المنحنى كما في المعادلة اعلاه وتذكر ان اتجاه المجال الكهربي عمودي على اسطح متساوية الجهد.

Calculation of *E* from *V* for spherical charge distribution

If the charge distribution creating an electric field has spherical symmetry such that the volume charge density depends only on the radial distance *r*, the electric field is radial. In this case,

$$\vec{E}.\,d\vec{l} = E_r dr$$

$$dV = -E_r\, dr$$

$$E_r = -\frac{dV}{dr}$$

Example 2

Calculate the electric field for a point charge q, using the equation

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

Solution

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r}\right)$$

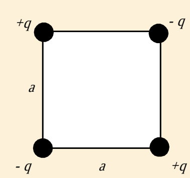
$$E = -\frac{q}{4\pi\varepsilon_o}\frac{d}{dr}\left(\frac{1}{r}\right)$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

Problems to Solve by Yourself

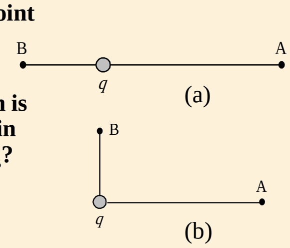
1. **Four equal point charges of charge q=+5μC are located at the corners of a 30cm by 40cm rectangle. Calculate the electric potential energy stored in this charge configuration.**

2. **In the figure prove that the work required to put four charges together on the corner of a square of radius a is given by (w=-0.21q^2/ a).**



3. **Two point charges, Q_1=+5nC and Q_2=-3nC, are separated by 35cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?**

4. **A point charge has q=1.0×10^{-6}C. Consider point A which is 2m distance and point B which is 1m distance as shown in the figure (a). (a) What is the potential difference V_A-V_B? (b) Repeat if points A and B are located differently as shown in figure (b).**



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General Physics II

Electrostatic: Principles & Applications

Lecture (12): Discussion on Electric Potential

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

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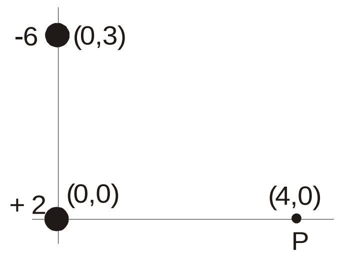
Example 1

Two charges of 2μC and -6μC are located at positions (0,0) m and (0,3) m, respectively as shown in the figure.

(i) Find the total electric potential due to these charges at point (4,0) m.

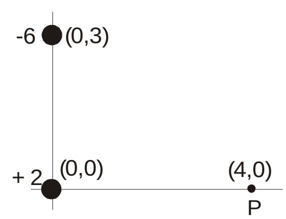
(ii) (ii) How much work is required to bring a 3μC charge from ∞ to the point *P*?

(iii) (iii) What is the potential energy for the three charges?



Solution

(i) the total electric potential due to these charges …….



$$V_p = V_1 + V_2$$

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$$V = 9\times10^{9}\left[\frac{2\times10^{-6}}{4} - \frac{6\times10^{-6}}{5}\right] = -6.3\times10^{3}\ volt$$

(ii) the work required is given by

$$W = q_3\, V_p = 3 \times 10^{-6} \times -6.3 \times 10^{3} = -18.9 \times 10^{-3}\ J$$

The -ve sign means that work is done by the charge for the movement from ∞ to *P*.

(iii) **The potential energy is given by**

$$U = U_{12} + U_{13} + U_{23}$$

$$U = k\left[\frac{(2\times10^{-6})(-6\times10^{-6})}{3} + \frac{(2\times10^{-6})(3\times10^{-6})}{4} + \frac{(-6\times10^{-6})(3\times10^{-6})}{5}\right]$$

$$\therefore U = -5.5\times10^{-2}\ Joule$$

Example 2

A particle having a charge q=3×10^{-9}C moves from point a to point b along a straight line, a total distance d=0.5m. The electric field is uniform along this line, in the direction from a to b, with magnitude E=200N/C. Determine the force on q, the work done on it by the electric field, and the potential difference V_a-V_b.

Solution

The force is in the same direction as the electric field since the charge is positive; the magnitude of the force is given by

$$F = qE = 3\times10^{-9} \times 200 = 600\times10^{-9}\text{N}$$

The work done by this force is

$$W = Fd = 600\times10^{-9} \times 0.5 = 300\times10^{-9}\text{J}$$

The potential difference is the work per unit charge, which is

$$V_a - V_b = W/q = 100\text{V}$$

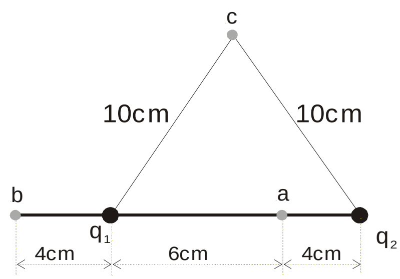
Or

$$V_a - V_b = Ed = 200 \times 0.5 = 100\text{V}$$

Example 3

Point charge of $+12\times10^{-9}$C and -12×10^{-9}C are placed 10cm part as shown in the figure. Compute the potential at point *a*, *b*, and *c*.

Compute the potential energy of a point charge $+4\times10^{-9}$C if it placed at points *a*, *b*, and *c*.



Solution

We need to use the following equation at each point to calculate the potential,

$$V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o}\sum \frac{q_i}{r_i}$$

At point *a*

$$V_a = 9\times10^9\left(\frac{12\times10^{-9}}{0.06} + \frac{-12\times10^{-9}}{0.04}\right) = -900V$$

At point *b*

$$V_b = 9\times10^9\left(\frac{12\times10^{-9}}{0.04} + \frac{-12\times10^{-9}}{0.14}\right) = -1930V$$

At point *c*

$$V_c = 9\times10^9\left(\frac{12\times10^{-9}}{0.1} + \frac{-12\times10^{-9}}{0.14}\right) = 0V$$

We need to use the following equation at each point to calculate the potential energy,

At point *a*

$$U_a = qV_a = 4\times10^{-9}\times(-900) = -36\times10^{-7}J$$

At point *b*

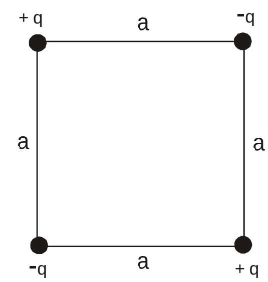
$$U_b = qV_b = 4\times10^{-9}\times1930 = +77\times10^{-7}J$$

At point *c*

$$U_c = qV_c = 4\times10^{-9}\times0 = 0$$

Example 4

Derive an expression for the work required to put the four charges together as indicated in the figure.



Solution

The work required to put these charges together is equal to the total electric potential energy.

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a}\right]$$

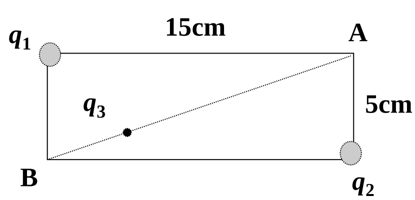
$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-4q^2}{a} + \frac{2q^2}{\sqrt{2}a}\right]$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-\sqrt{2}4q^2 + 2q^2}{\sqrt{2}a}\right] = \frac{-0.2q^2}{\varepsilon_o a}$$

The minus sign indicates that there is attractive force between the charges

Example 5

In the rectangle shown in the figure, q_1 = -5x10^{-6}C and q_2 = 2x10^{-6}C calculate the work required to move a charge q_3 = 3x10^{-6}C from *B* to *A* along the diagonal of the rectangle.



Solution

from the equation $V_B - V_A = W_{AB} / q_o$

$$V_A = V_1 + V_2 \qquad \& \qquad V_B = V_1 + V_2.$$

$$V_A = \frac{q}{4\pi\varepsilon_o}\left[\frac{-5\times10^{-6}}{0.15} + \frac{2\times10^{-6}}{0.05}\right] = 6\times10^4 V$$

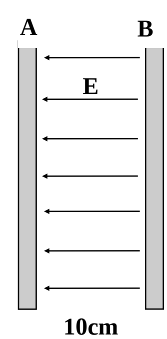
$$V_B = \frac{q}{4\pi\varepsilon_o}\left[\frac{-5\times10^{-6}}{0.05} + \frac{2\times10^{-6}}{0.15}\right] = -7.8\times10^4 V$$

$$W_{BA} = (V_A - V_B)\, q_3$$

$$= (6\times10^4 + 7.8\times10^4)\ 3\times10^{-6} = 0.414 \text{ Joule}$$

Example 6

Two large parallel conducting plates are 10 cm a part and carry equal but opposite charges on their facing surfaces as shown in the figure. An electron placed midway between the two plates experiences a force of 1.6×10^{-15} N.
What is the potential difference between the plates?



Solution

$$V_B - V_A = Ed$$

يمكن حساب المجال الكهربي عن طريق القوى الكهربية المؤثرة على الإلكترون

$$F = eE \quad \Rightarrow \quad E = F/e$$

$$V_B - V_A = 10000 \times 0.1 = 1000 \text{ volt}$$

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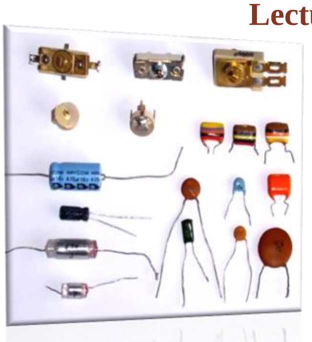
General Physics II

Electrostatic: Principles & Applications

Lecture (13): Capacitors and Capacitance



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Capacitors and Capacitance

Definition of capacitance

Calculation of capacitance

- Parallel plate capacitor
- Cylindrical capacitor
- Spherical capacitor

Combination of capacitors

- Capacitors in parallel
- Capacitors in series

Energy stored in a charged capacitor (in electric field)

Capacitor with dielectric

problems

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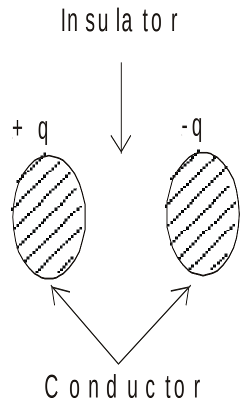
Capacitor

A capacitor consists of two conductors separated by an insulator.

The capacitance of the capacitor depends on the geometry of the conductors and on the material separating the charged conductors, called dielectric that is an insulating material.

The two conductors carry equal and opposite charge +*q* and -*q*.

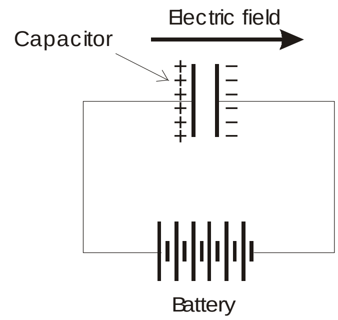


Definition of Capacitance

Definition of Capacitance

The capacitance *C* of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them as shown in the figure.

$$C = \frac{q}{V}$$

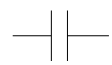


The capacitance *C* has a unit of *C/v*, which is called *farad F*

$$F = C/v$$

$1\mu F = 10^{-6}F$, $1nF = 10^{-9}F$, $1pF = 10^{-12}F$

The capacitor in the circuit is represented by the symbol shown in the figure



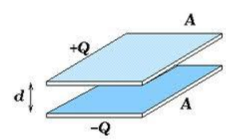
Calculation of Capacitance

The most common type of capacitors are:-

- Parallel-plate capacitor
- Cylindrical capacitor
- Spherical capacitor

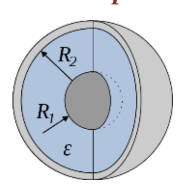
We are going to calculate the capacitance of parallel plate capacitor using the information we learned in the previous chapters and make use of the equation.

$$C = \frac{q}{V}$$



Parallel-plate capacitor

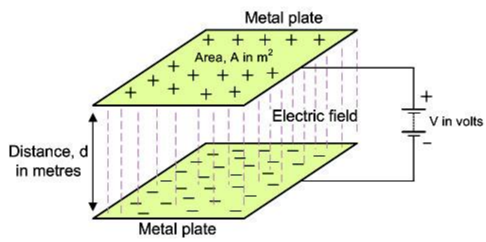
Cylindrical capacitor



Spherical capacitor

Parallel Plate Capacitor

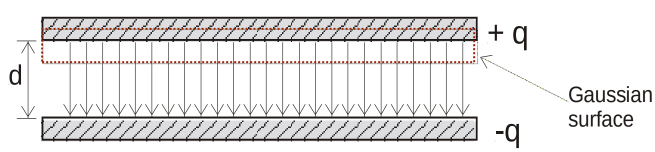
Two parallel plates of equal area *A* are separated by distance *d* as shown in the figure bellow. One plate charged with +*q*, the other -*q*.



The capacitance is given by $C = \dfrac{q}{V}$

First we need to evaluate the electric field *E* to workout the potential *V*.

Using gauss law to find *E*,



The charge per unit area on either plate is $\sigma = q/A$

$$\therefore E = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A}$$

The potential difference between the plates is equal to *Ed*, therefore

$$V = Ed = \frac{qd}{\varepsilon_o A}$$

The capacitance is given by

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_o A} \longrightarrow \therefore C = \frac{\varepsilon_o A}{d}$$

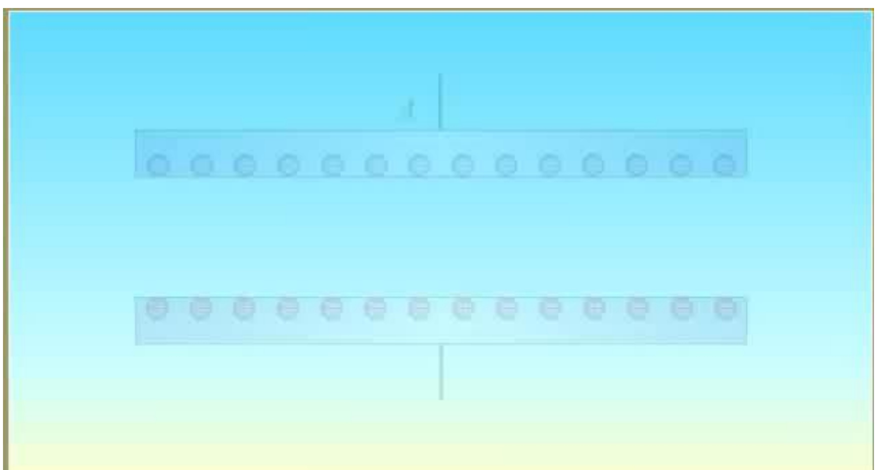
$$\therefore C = \frac{\varepsilon_o A}{d}$$

Notice that the capacitance of the parallel plates capacitor is depends on the geometrical dimensions of the capacitor.

The capacitance is proportional to the area of the plates and inversely proportional to distance between the plates.

تمكننا المعادلة من حساب سعة المكثف من خلال الأبعاد الهندسية له، حيث أن سعة المكثف تتناسب طردياً مع المساحة المشتركة بين اللوحين وعكسياً مع المسافة بين اللوحين.

Parallel Plate Capacitor



A parallel plate capacitor consists of two flat plates of area A separated by a distance d. When each plate is given a charge of magnitude Q,

Example (1)

An air-filled capacitor consists of two plates, each with an area of 7.6cm^2, separated by a distance of 1.8mm. If a 20V potential difference is applied to these plates, calculate,

- the electric field between the plates,
- the surface charge density,
- the capacitance, and
- the charge on each plate.

Solution

(a) $$E = \frac{V}{d} = \frac{20}{1.8\times10^{-3}} = 1.11\times10^{4}\,V/m$$

(b) $$\sigma = \varepsilon_o E = (8.85\times10^{-12})(1.11\times10^{4}) = 9.83\times10^{-8}\,C/m^2$$

(c) $$C = \frac{\varepsilon_o A}{d} = \frac{(8.85\times10^{-12})(7.6\times10^{-4})}{1.8\times10^{-3}} = 3.74\times10^{-12}\,F$$

(d) $$q = CV = (3.74\times10^{-12})(20) = 7.48\times10^{-11}\,C$$

Cylindrical Capacitor

In the same way we can calculate the capacitance of cylindrical capacitor, the result is as follow

$$C = \frac{2\pi\varepsilon_o l}{\ln(b/a)}$$

Where *l* is the length of the cylinder, *a* is the radius of the inside cylinder, and *b* the radius of the outer shell cylinder.

Spherical Capacitor

In the same way we can calculate the capacitance of spherical capacitor, the result is as follow

$$C = \frac{4\pi\varepsilon_o ab}{b-a}$$

Where *a* is the radius of the inside sphere, and *b* is the radius of the outer shell sphere.

Example (2)

An air-filled spherical capacitor is constructed with inner and outer shell radii of 7 and 14cm, respectively. Calculate,

- The capacitance of the device,
- What potential difference between the spheres will result in a charge of 4μC on each conductor?

Solution

(a) $$C = \frac{4\pi\varepsilon_o ab}{b-a} = \frac{(4\pi \times 8.85\times10^{-12})(0.07)(0.14)}{(0.14-0.07)} = 1.56\times10^{-11}F$$

(b) $$V = \frac{q}{C} = \frac{4\times10^{-6}}{1.56\times10^{-11}} = 2.56\times10^{5}V$$

Problem to solve by your self

A parallel-plate capacitor has circular plates of 8.0cm radius and 1.0mm separation. What charge will appear on the plates if a potential difference of 100V is applied?

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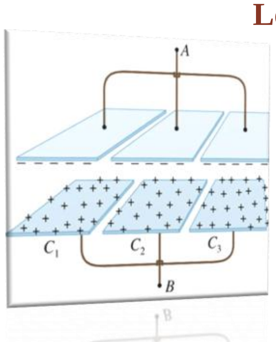
General Physics II

Electrostatic: Principles & Applications

Lecture (14): Combination of capacitors



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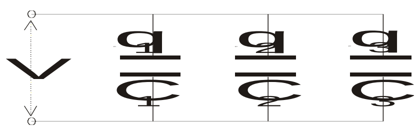
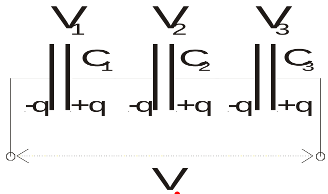
Combination of Capacitors

Some times the electric circuit consist of more than two capacitors, which are, connected either in parallel or in series



parallel



series

في حالة توصيل المكثفات على التوازي يكون فرق الجهد على كل مكثف مساوياً لفرق جهد البطارية، أما الشحنة فتتوزع بنسبة سعة كل مكثف.

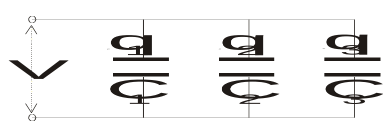
في حالة توصيل المكثفات على التوالي فإن الشحنة تتوزع على كل مكثف بشكل متساو وتساوي الشحنة الكلية. أما مجموع فروق الجهد على كل مكثف يساوي فرق جهد البطارية.

Capacitors in Parallel

In parallel connection the capacitors are connected as shown in the figure where the above plates are connected together with the positive terminal of the battery, and the bottom plates are connected to the negative terminal of the battery.

In this case the potential different across each capacitor is equal to the voltage of the battery *V*

i.e. $V = V_1 = V_2 = V_3$



The charge on each capacitor is

$$q_1 = C_1V_1; \quad q_2 = C_2V_2; \quad q_3 = C_3V_3$$

The total charge is

$$q = q_1 + q_2 + q_3$$

$$q = (C_1 + C_2 + C_3)V$$

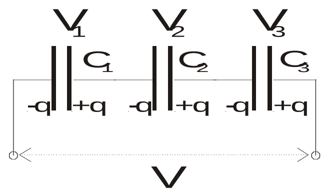
The Equivalent capacitance is $C = C_1 + C_2 + C_3$

Capacitors in Series

In series connection the capacitors are connected as shown in the figure

In this case the magnitude of the charge must be the same on each plate with opposite sign

i.e. $\boldsymbol{q = q_1 = q_2 = q_3}$



The potential across each capacitor is

$$V_1 = q/C_1; \quad V_2 = q/C_2; \quad V_3 = q/C_3$$

The total potential V is equal the sum of the potential across each capacitor

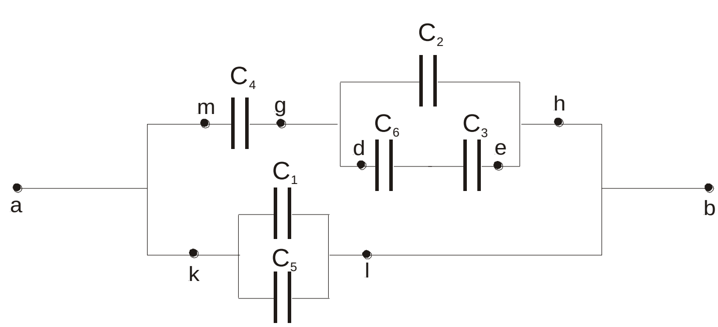
$$V = V_1 + V_2 + V_3 \longrightarrow V = q\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right) \longrightarrow C = \frac{q}{V} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$$

The Equivalent capacitance is $\longrightarrow$ $$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

Example 1

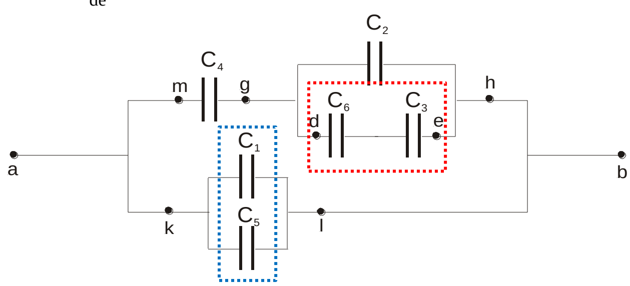
(a) Find the equivalent capacitance between points a and b for the group of capacitors shown in the figure. C_1=1μF, C_2=2μF, C_3=3μF, C_4=4μF, C_5=5μF, and C_6=6μF.

(b) Determine the potential difference across each capacitor and the charge on each capacitor if the total charge on all the six capacitors is 384μC.



Solution (a)

First the capacitor C_3 and C_6 are connected in series so that the equivalent capacitance C_{de} is

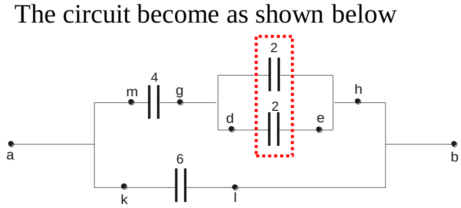


$$\frac{1}{C_{de}} = \frac{1}{6} + \frac{1}{3}; \Rightarrow C_{de} = 2\mu F$$

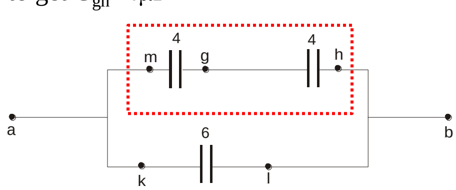
Second C_1 and C_5 are connected in parallel

$$C_{kl} = 1+5 = 6\mu\text{F}$$

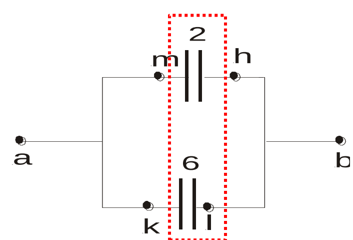
The circuit become as shown below



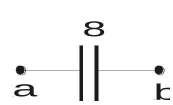
Continue with the same way to reduce the circuit for the capacitor C_2 and C_{de} to get C_{gh}=4μF



Capacitors C_{mg} and C_{gh} are connected in series the result is C_{mh}=2μF, The circuit become as shown below

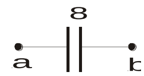


Capacitors C_{mh} and C_{kl} are connected in parallel the result is



C_{eq}=8μF.

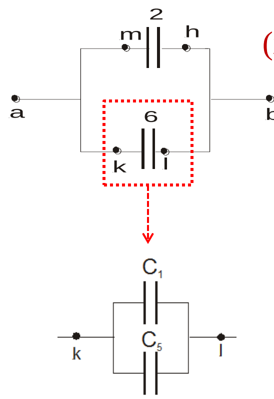
Solution (b)



C_{eq}=8μF.

First consider the equivalent capacitor C_{eq} to find the potential between points a and b (V_{ab})

$$V_{ab} = \frac{Q_{ab}}{C_{ab}} = \frac{384}{8} = 48V$$



(iv)

Second notice that the potential V_{kl}=V_{ab} since the two capacitors between *k* and *l* are in parallel, the potential across the capacitors C_1 and C_5 = 48V.

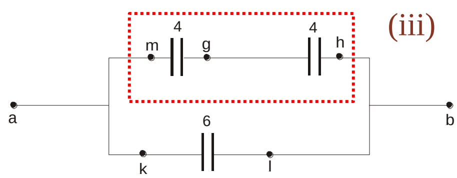
V_1=48V and Q_1=C_1V_1=48μC

And for *C*5

V_5=48V and Q_5=C_5V_5=240μC

For the circuit (iv) notice that V_{mh}=V_{ab}=48V,

and **Q_{mh}= $C_{mh}V_{mh}$= 2×28 = 96μC**



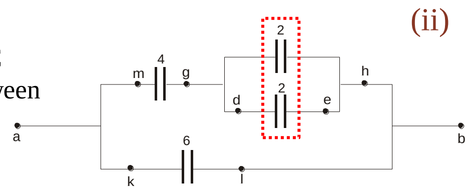
(iii)

Since the two capacitors shown in the circuit (iii) between points *m* and *h* are in series, each will have the same charge as that of the equivalent capacitor, *i.e.*

$$Q_{mh} = Q_{gh} = Q_{mh} = 96\mu C$$

$$V_{mg} = \frac{Q_{mg}}{C_{mg}} = \frac{96}{4} = 24V$$

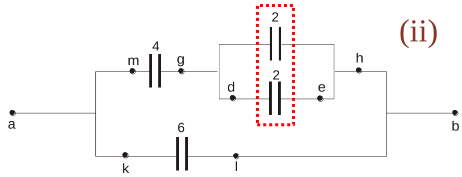
$$V_{gh} = \frac{Q_{gh}}{C_{gh}} = \frac{96}{4} = 24V$$



(ii)

Therefore for C_4, V_4=24 and Q_4=96μC
In the circuit (ii) the two capacitor between points *g* and *h* are in parallel so the potential difference across each is 24V.

Therefore for C_2, V_2=24V and Q_2=C_2V_2=48μC



Also in circuit (ii) the potential difference

$$V_{de} = V_{gh} = 24V$$

And

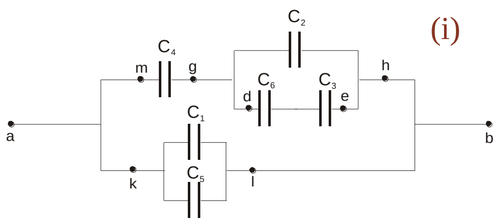
$$Q_{de} = C_{de}V_{de} = 2\times24 = 48\mu C$$

The two capacitors shown in circuit (i) between points *d* and *a* are in series, and therefore the charge on each is equal to Q_{de}.

Therefore for C_6, Q_6=48μC

$$V_6 = \frac{Q_6}{C_6} = 8V$$

For C_3, Q_3=48μC and $V_3=Q_3/C_3$=16V

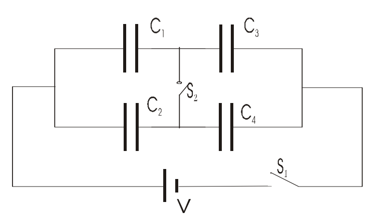


The results can be summarized as follow

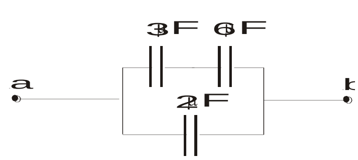
Capacitor	Potential Difference (V)	Charge (μC)
C_1	48	48
C_2	24	48
C_3	16	48
C_4	24	96
C_5	48	240
C_6	8	48
C_{eq}	48	384

Problem to solve by your self

1. Two capacitors, C_1=2μF and C_2=16μF, are connected in parallel. What is the value of the equivalent capacitance of the combination?
2. Calculate the equivalent capacitance of the two capacitors in the previous exercise if they are connected in series.
3. In the figure the battery supplies 12V. (a) Find the charge on each capacitor when switch S_1 is closed, and (b) when later switch S_2 is also closed. Assume C_1=1μF, C_2=2μF, C_3=3μF, and C_4=4μF.



4. (a) Determine the equivalent capacitance for the capacitors shown in figure 6.20. (b) If they are connected to 12V battery, calculate the potential difference across each capacitor and the charge on each capacitor



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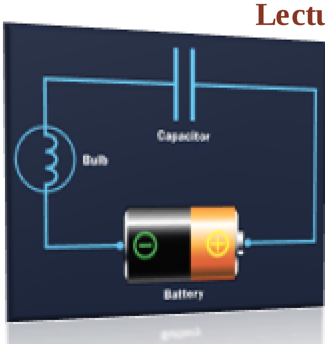
General Physics II

Electrostatic: Principles & Applications



Lecture (15): Energy stored in a charged capacitor (in electric field)

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Capacitor with dielectric

problems

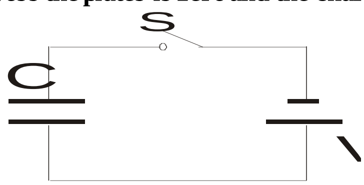
يعتبر هذا الفصل تطبيقاً على المفاهيم الأساسية للكهربية الساكنة، حيث سنركز على التعرف على خصائص المكثفات Capacitors وهي من الأجهزة الكهربية التي لا تخلو منها أية دائرة كهربية. ويعد المكثف بمثابة مخزن للطاقة الكهربية. والمكثف عبارة عن موصلين يفصل بينهما مادة عازلة.

Energy Stored in a Charged Capacitor

If the capacitor is connected to a power supply such as battery, charge will be transferred from the battery to the plates of the capacitor.

This is a charging process of the capacitor which mean that the battery perform a work to store energy between the plates of the capacitor.

Consider uncharged capacitor is connected to a battery as shown in the figure, at start the potential across the plates is zero and the charge is zero as well.

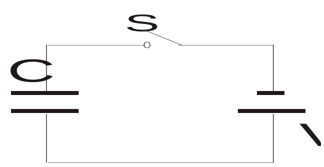


If the switch S is closed then the charging process will start and the potential across the capacitor will rise to reach the value equal the potential of the battery V in time t (called charging time).

بعد إغلاق المفتح S تستمر عملية شحن المكثف حتى يصبح فرق الجهد بين لوحي المكثف مساوياً لفرق جهد البطارية.

Energy Stored in a Charged Capacitor

Suppose that at a time t a charge $q(t)$ has been transferred from the battery to capacitor. The potential difference $V(t)$ across the capacitor will be $q(t)/C$. For the battery to transferred another amount of charge dq it will perform a work dW



$$dW = Vdq = \frac{q}{C}dq$$

The total work required to put a total charge Q on the capacitor is

$$W = \int dW = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C}$$

Using the equation $q=CV$

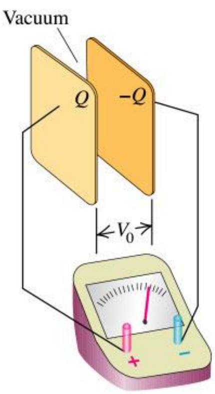
$$W = U = \frac{Q^2}{2C}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

Energy Stored in a Electric Field

The energy per unit volume u (energy density) in parallel plate capacitor is the total energy stored U divided by the volume between the plates Ad

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad}$$



For parallel plate capacitor

$$C = \frac{\varepsilon_o A}{d}$$

$$u = \frac{\varepsilon_o}{2}\left(\frac{V}{d}\right)^2$$

$$u = \frac{1}{2}\varepsilon_o E^2$$

Therefore the electric energy density is proportional with square of the electric field.

$$u = \frac{1}{2}\varepsilon_o E^2$$

لاحظ هنا أن الطاقة الكهربية المخزنة بين لوحي المكثف يمكن التعبير عنها باستخدام الطاقة الكلية U أو من خلال كثافة الطاقة u. الطاقة الكلية تساوي كثافة الطاقة في الحجم المحصور بين لوحي المكثف.

المعادلتان التاليتين توضحان عنوان هذا الموضوع وهو الطاقة المخزنة في المكثف أو في المجال الكهربي.

$$u = \frac{1}{2}\varepsilon_o E^2$$

لحساب كثافة الطاقة المختزنة بين لوحي المكثف
او كثافة الطاقة الكهربية المختزنة في المجال الكهربي

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

لحساب الطاقة الكلية المختزنة بين لوحي المكثف

Example (1)

Three capacitors of 8μF, 10μF and 14μF are connected to a battery of 12V. How much energy does the battery supply if the capacitors are connected (a) in series and (b) in parallel?

Solution

(a) For series combination

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$\frac{1}{C}=\frac{1}{8}+\frac{1}{10}+\frac{1}{14}$$

This gives

C = 3.37 μF

Then the energy U is

$$U=\frac{1}{2}CV^2$$

U = 1/2 (3.37×10^{-6}) $(12)^2$ = 2.43×10^{-4}J

(b) For parallel combination

$$C=C_1+C_2+C_3$$

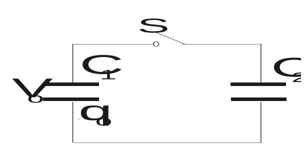
C= 8+10+14=32μF

The energy U is

U = 1/2 (32×10^{-6}) $(12)^2$ = 2.3×10^{-3}J

Example (2)

A capacitor C_1 is charged to a potential difference V_o. This charging battery is then removed and the capacitor is connected as shown in the figure to an uncharged capacitor C_2,



a) What is the final potential difference V_f across the combination?

b) What is the stored energy before and after the switch S is closed?

Solution

(a) The original charge q_o is shared between the two capacitors since they are connected in parallel. Thus

$$q_o = q_1 + q_2$$

$$q = CV$$

$$C_1V_o = C_1V_f + C_2V_f$$

$$C_1V_o = V_f(C_1 + C_2)$$

$$V_f = V_o \frac{C_1}{C_1 + C_2}$$

(b) The initial stored energy is U_o

$$U_o = \tfrac{1}{2}C_1V_o^{\ 2}$$

The final stored energy $U_f = U_1 + U_2$

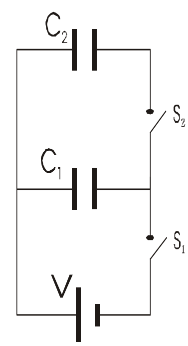
$$U_f = \tfrac{1}{2}C_1V_f^{\ 2} + \tfrac{1}{2}C_2V_f^{\ 2} = \tfrac{1}{2}(C_1 + C_2)\left(\frac{V_oC_1}{C_1 + C_2}\right)^2$$

$$U_f = \left(\frac{C_1}{C_1 + C_2}\right)U_o$$

Notice that U_f is less than U_o (Explain why)

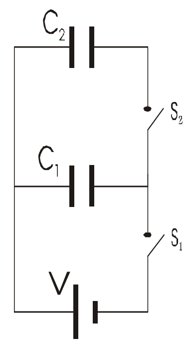
Example (3)

Consider the circuit shown in the figure where C_1=6μF, C_2=3μF, and V=20V. C_1 is first charged by closing switch S_1. S_1 is then opened, and the charged capacitor C_1 is connected to the uncharged capacitor C_2 by closing the switch S_2.



Calculate the initial charge acquired by C_1 and the final charge on each of the two capacitors.

Solution



When S_1 is closed, the charge on C_1 will be

$$Q_1 = C_1 V_1 = 6\mu F \times 20V = 120\mu C$$

When S_1 is opened and S_2 is closed, the total charge will remain constant and be distributed among the two capacitors,

$$Q_1 = 120\mu C - Q_2$$

The potential across the two capacitors will be equal,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{120\mu F - Q_2}{6\mu F} = \frac{Q_2}{3\mu F}$$

Therefore,

$$Q_2 = 40\mu C$$

$$Q_1 = 120\mu C - 40\mu C = 80\mu C$$

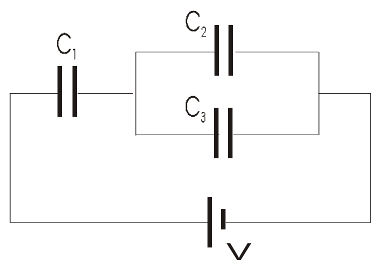
Example (4)

Consider the circuit shown in the figure where C_1=6μF, C_2=4μF, C_3=12μF, and V=12V.

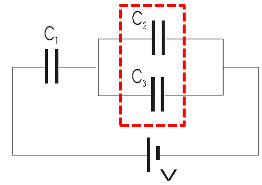
a) Calculate the equivalent capacitance,

b) Calculate the potential difference across each capacitor.

c) Calculate the charge on each of the three capacitors.



Solution



C_2 and C_3 are connected in parallel, therefore

$$C' = C_2 + C_3 = 4 + 12 = 16\mu F$$

Now is connected in series with C_1, therefore the equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_1} = \frac{1}{6} + \frac{1}{16} = \frac{11}{48}$$

$$C = 4.36\mu F$$

The total charge $Q = CV = 4.36 \text{ x } 12 = 52.36\mu C$

The charge will be equally distributed on the capacitor C_1 and

$$Q_1 = Q' = Q = 52.36\mu C$$

Solution

But $Q' = C' V'$, therefore

$$V' = 52.36/16 = 3.27 \text{ volts}$$

The potential difference on C_1 is

$$V_1 = 12 - 3.27 = 8.73 \text{ volts}$$

The potential difference on both C_2 and C_3 is equivalent to V' since they are connected in parallel.

$$V_2 = V_3 = 3.27 \text{volts}$$

$$Q_2 = C_2 V_2 = 13.08\mu C$$

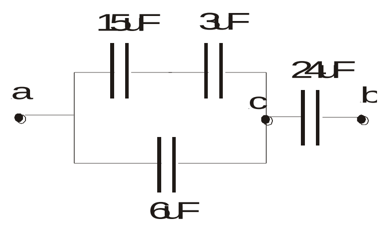
$$Q_3 = C_3 V_3 = 39.24\mu C$$

Example (5)

Four capacitors are connected as shown in the figure.

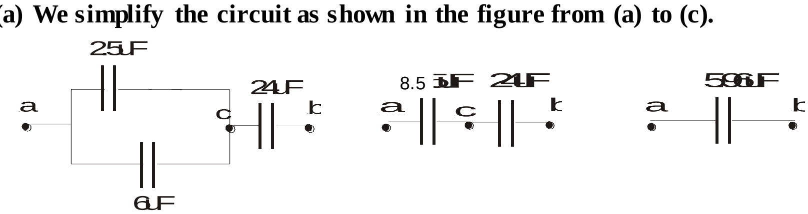
(a) Find the equivalent capacitance between points a and b.

(b) Calculate the charge on each capacitor if V_{ab}=15V.



Solution

(a) We simplify the circuit as shown in the figure from (a) to (c).



(a) **(b)** **(c)**

Firs the 15µF and 3µF in series are equivalent to

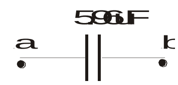
$$\frac{1}{(1/15)+(1/3)} = 2.5\mu F$$

Next 2.5µF combines in parallel with 6µF, creating an equivalent capacitance of 8.5µF. The 8.5µF and 20µF are in series, equivalent to

$$\frac{1}{(1/8.5)+(1/20)} = 5.96\mu F$$

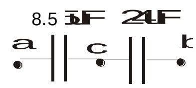
Solution

(b) We find the charge and the voltage across each capacitor by working backwards through solution figures (c) through (a).



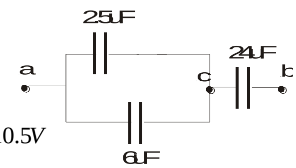
For the 5.96µF capacitor we have $Q = CV = 5.96 \times 15 = 89.5\mu C$

In figure (b) we have, for the 8.5µF capacitor,



$$\Delta V_{ac} = \frac{Q}{C} = \frac{89.5}{8.5} = 10.5V$$

and for the 20µF in figure (b) and (a) $Q_{20} = 89.5\mu C$



$$\Delta V_{cb} = \frac{Q}{C} = \frac{89.5}{20} = 4.47V$$

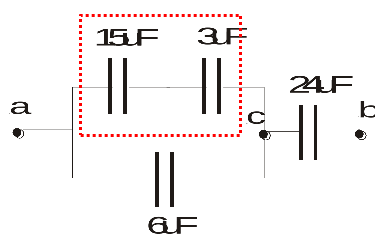
Next (a) is equivalent to (b), so $\Delta V_{cb} = 4.47V$ **and** $\Delta V_{ac} = 10.5V$

Thus for the 2.5µF and 6µF capacitors $\Delta V = 10.5V$

$$Q_{2.5} = CV = 2.5 \times 10.5 = 26.3\mu C$$

$$Q_6 = CV = 6 \times 10.5 = 63.2\mu C$$

Solution



Therefore

$$Q_{15} = 26.3\mu C$$

$$Q_3 = 26.3\mu C$$

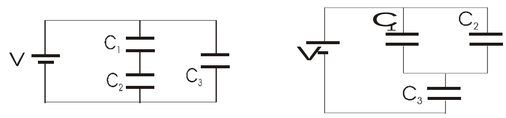
For the potential difference across the capacitors C_{15} and C_3 are

$$\Delta V_{15} = \frac{Q}{C} = \frac{26.3}{15} = 1.75V$$

$$\Delta V_3 = \frac{Q}{C} = \frac{26.3}{3} = 8.77V$$

Problem to solve by your self

(1) In the figure (a)&(b) find the equivalent capacitance of the combination. Assume that C_1=10μF, C_2=5μF, and C_3=4μF.



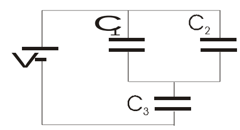
(a) (b)

(2) Two capacitors (2.0μF and 4.0μF) are connected in parallel across a 300V potential difference. Calculate the total stored energy in the system.

(3) A 16pF parallel-plate capacitor is charged by a 10V battery. If each plate of the capacitor has an area of 5cm^2, what is the energy stored in the capacitor? What is the energy density (energy per unit volume) in the electric field of the capacitor if the plates are separated by air?

(4) The energy density in a parallel-plate capacitor is given as 2.1×10^{-9}J/m^3. What is the value of the electric field in the region between the plates?

(5) In the figure find (a) the charge, (b) the potential difference, (c) the stored energy for each capacitor. With V=100V.



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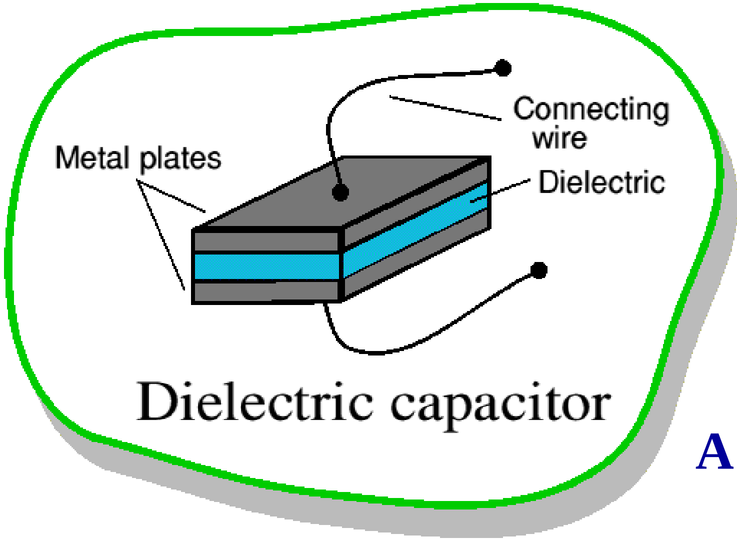
General Physics II

Electrostatic: Principles & Applications

Lecture (16): Capacitor with dielectric



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Capacitors and Capacitance

Definition of capacitance

Calculation of capacitance

- Parallel plate capacitor
- Cylindrical capacitor
- Spherical capacitor

Combination of capacitors

- Capacitors in parallel
- Capacitors in series

Energy stored in a charged capacitor (in electric field)

Capacitor with dielectric

problems

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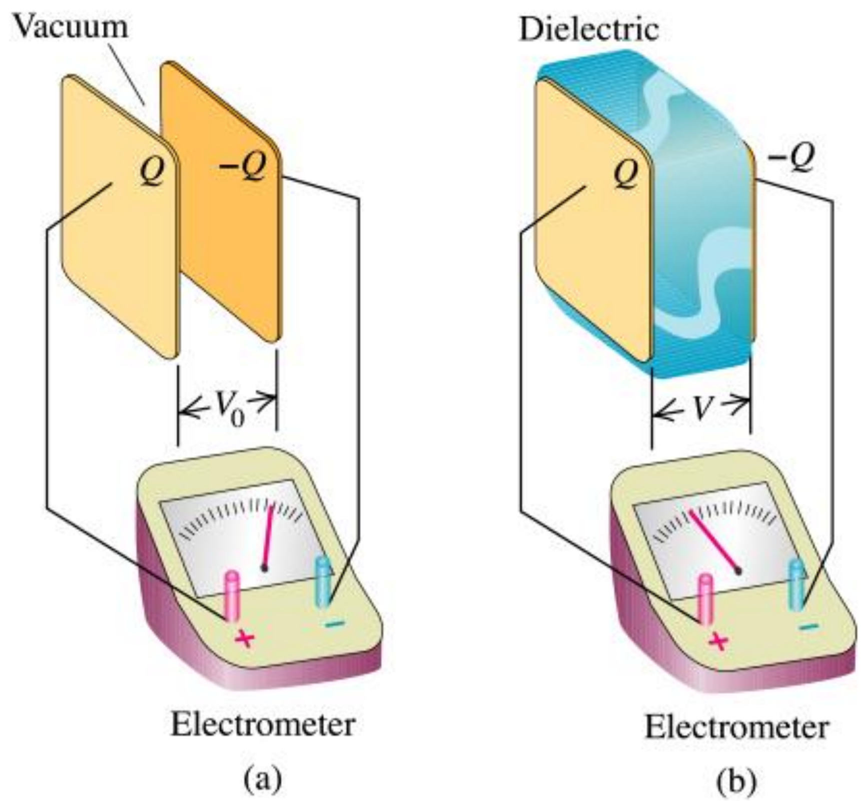
Capacitor with Dielectric

A dielectric is a non-conducting material, such as rubber, glass or paper. Experimentally it was found that the capacitance of a capacitor increased when a dielectric material was inserted in the space between the plates.

The ratio of the capacitance with the dielectric to that without it called the dielectric constant κ of the material.

$$\kappa = \frac{C}{C_o}$$

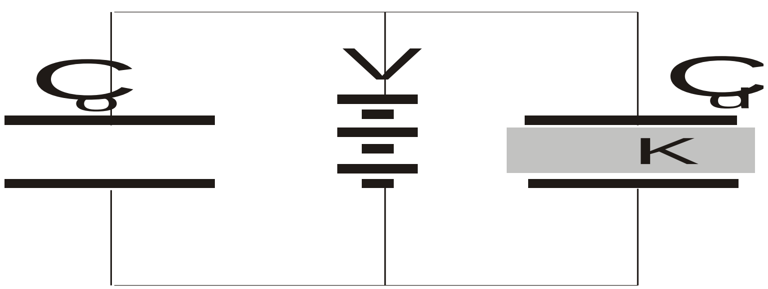


**(a) With a given charge, the potential difference is V_o.
(b) With the same charge but with a dielectric between the plates, the potential difference V is smaller than V_o.**

Experimental result (1)

In the figure below two similar capacitors, one of them is filled with dielectric material, and both are connected in parallel to a battery of potential *V*.

It was found that the charge on the capacitor with dielectric is larger than the on the air filled capacitor, therefore the $C_d > C_o$, since the potential *V* is the same on both capacitors.

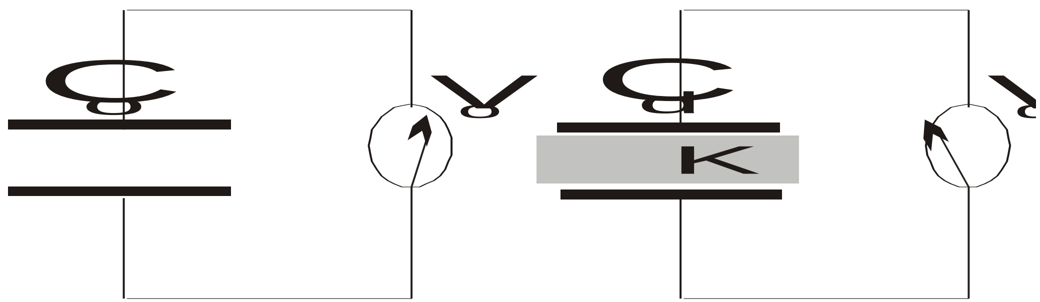


$$C_d = \kappa C_o$$

عند ثبات الفرق الجهد على المكثفين فان الشحنة الكهربية المخزنة في المكثف ذو المادة العازلة اكبر من الشحنة الكهربية على المكثف الهوائي، أي ان السعة الكهربية تزداد بوجود مادة عازلة بين لوحي المكثف

Experimental result (2)

If the experiment repeated in different way by placing the same charge Q_o on both capacitors as shown in the figure. Experimentally it was shown that $V_d < V_o$ by a factor of $1/\kappa$.



$$V_d = \frac{V_o}{\kappa}$$

عند ثبات الشحنة الكهربية على مكثفين متشابهين احدهما يحتوي على مادة عازلة والاخر بدون، وجد ان فرق الجهد الكهربي بين لوحي المكثف الذي يحتوي على مادة عازلة اقل من فرق الجهد على لوحي المكثف بدون المادة العازلة.

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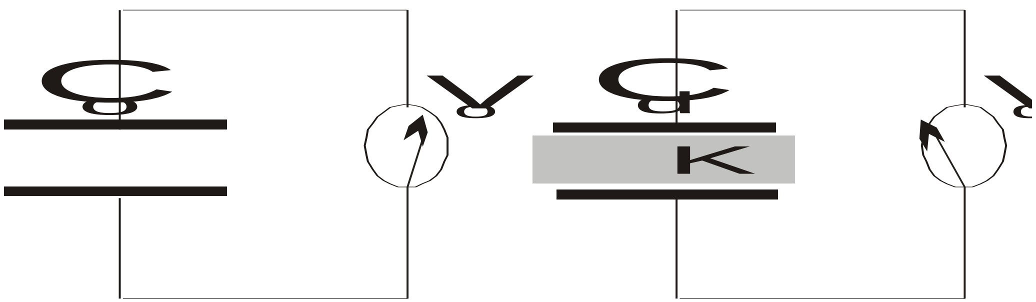

$$V_d = \frac{V_o}{\kappa}$$

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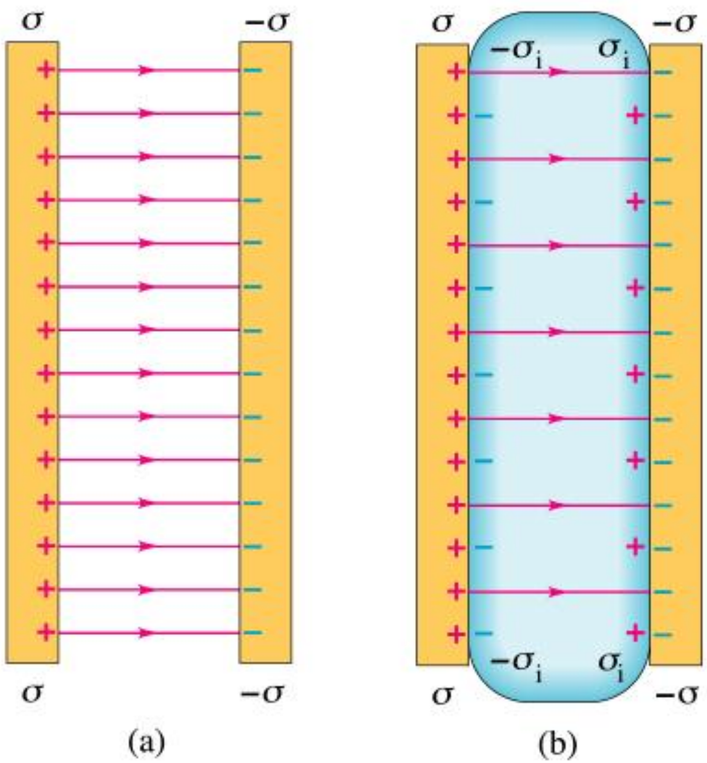
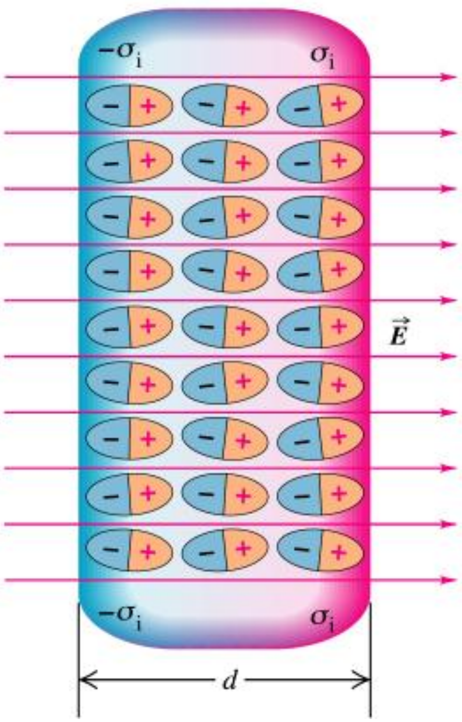
Explanation

Since V = E d, we can conclude that the electric field between the plates must be reduced as a dielectric is inserted Q = (constant).

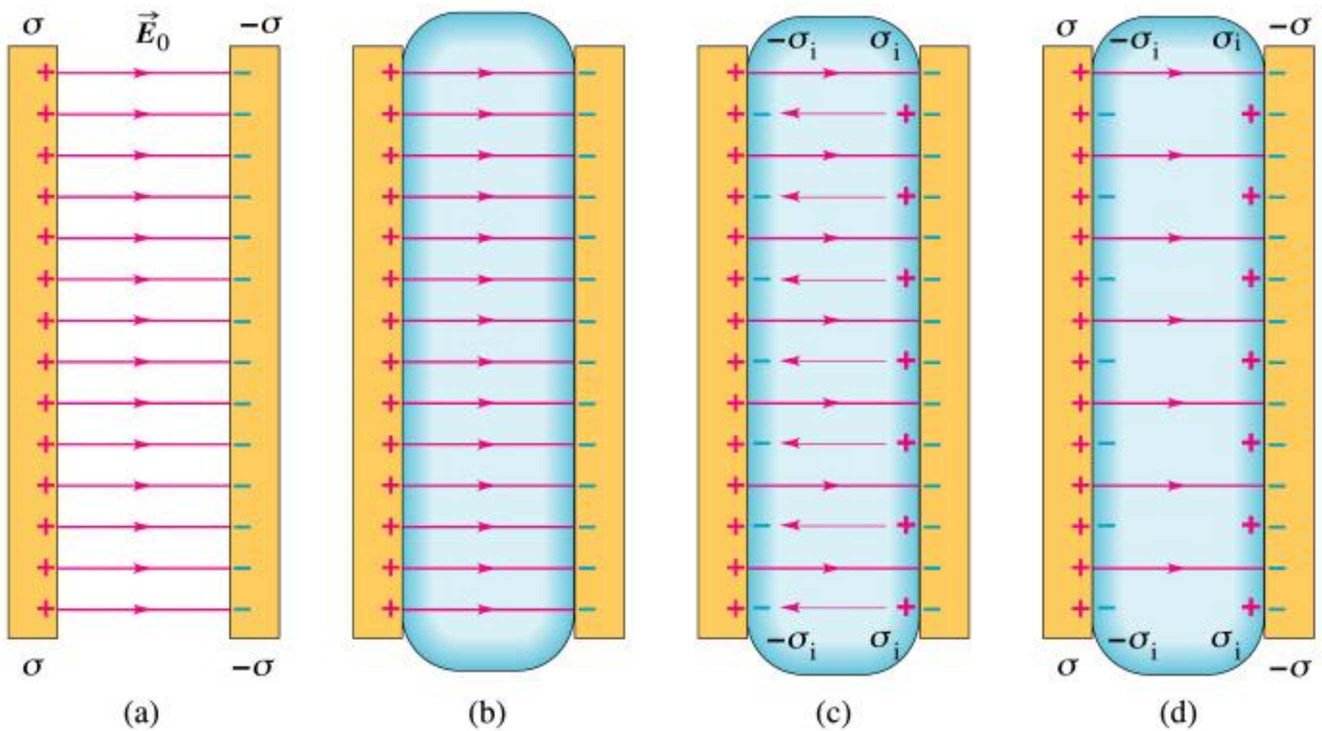




(a) Electric field lines with vacuum between the plates. (b) The induced charges on the faces of the dielectric decrease the electric field.

Molecules in the dielectric material have their positive and negative charges separated slightly, causing the molecules to be oriented slightly in the electric field of the charged capacitor.

The electric field between the capacitor plates is reduced by the presence of the dielectric because the induced surface charges on the dielectric (see figure below) cause an electric field in the opposite direction of the original field in the charged capacitor. These fields tend to cancel each other resulting in a reduction of the original field.



(a) Electric filed of magnitude E_o between two charged plates. (b) Introduction of a dielectric of dielectric constant *K*. (c) the induced surface charges and their field (thinner lines). (d) Resultant filed of magnitude E_o/K when a dielectric is between charged plates.

Example (1)

A parallel plate capacitor of area *A* and separation *d* is connected to a battery to charge the capacitor to potential difference V_o. Calculate the stored energy before and after introducing a dielectric material.

Solution

The energy stored before introducing the dielectric material,

$$U_o = \tfrac{1}{2} C_o V_o^{\,2}$$

The energy stored after introducing the dielectric material,

$$C_d = \kappa C_o \qquad \text{and} \qquad V_d = \frac{V_o}{\kappa}$$

$$U_d = \tfrac{1}{2} C V^2 = \tfrac{1}{2} \kappa C_o \left(\frac{V_o}{\kappa} \right)^2 = \frac{U_o}{\kappa}$$

Therefore, the energy is less by a factor of 1/κ.

Example (2)

A Parallel plate capacitor of area 0.64cm^2. When the plates are in vacuum, the capacitance of the capacitor is 4.9pF.

1. **Calculate the value of the capacitance if the space between the plates is filled with nylon (κ=3.4).**
2. **What is the maximum potential difference that can be applied to the plates without causing discharge (E_{max}=14×10^{6}V/m)?**

Solution

(a) $C_d = \kappa C_o = 3.4\times4.9 = 16.7\text{pF}$

(b) $V_{max} = E_{max} \times d$

To evaluate d we use the equation

$$d = \frac{\varepsilon_o A}{C_o} = \frac{8.85\times10^{-12}\times6.4\times10^{-5}}{4.9\times10^{-12}}$$

$$d = 1.16\times10^{-4}\,m$$

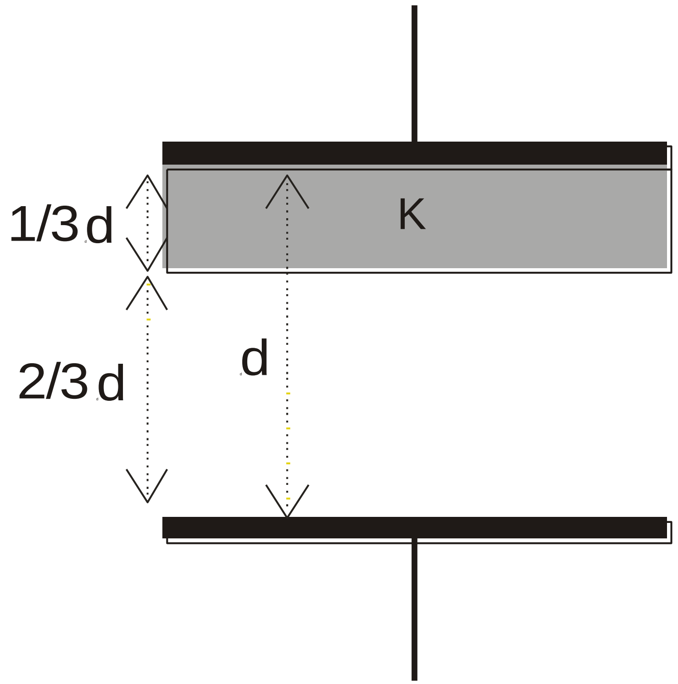
$$V_{max} = 1\times10^6\times1.16\times10^{-4} = 1.62\times10^3\ \text{V}$$

Example (3)

A parallel-plate capacitor has a capacitance C_o in the absence of dielectric.

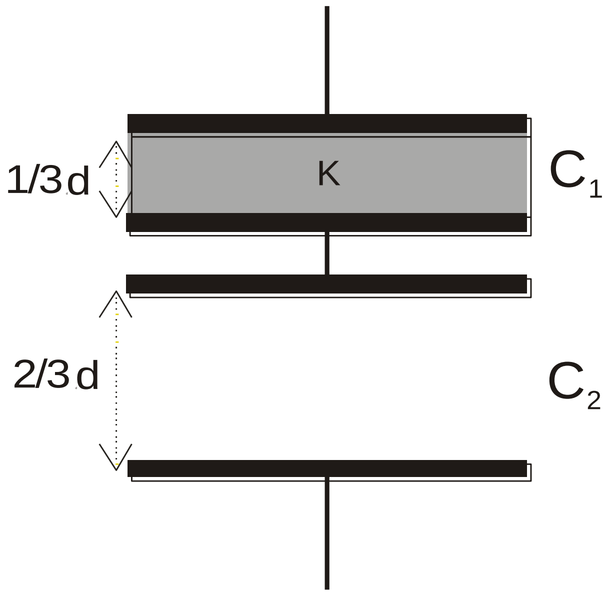
A slab of dielectric material of dielectric constant κ and thickness *d*/3 is inserted between the plates as shown in the Figure.

What is the new capacitance when the dielectric is present?



Solution

We can assume that two parallel plate capacitor are connected in series



$$C_1 = \frac{\kappa\varepsilon_o A}{d/3} \quad \text{and} \quad C_2 = \frac{\varepsilon_o A}{2d/3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{\kappa\varepsilon_o A} + \frac{2d/3}{\varepsilon_o A}$$

$$\frac{1}{C} = \frac{d}{3\varepsilon_o A}\left(\frac{1}{\kappa} + 2\right) = \frac{d}{3\varepsilon_o A}\left(\frac{1+2\kappa}{\kappa}\right)$$

$$C = \left(\frac{3\kappa}{2\kappa+1}\right)\frac{\varepsilon_o A}{d} \quad \Rightarrow \quad C = \left(\frac{3\kappa}{2\kappa+1}\right)C_o$$

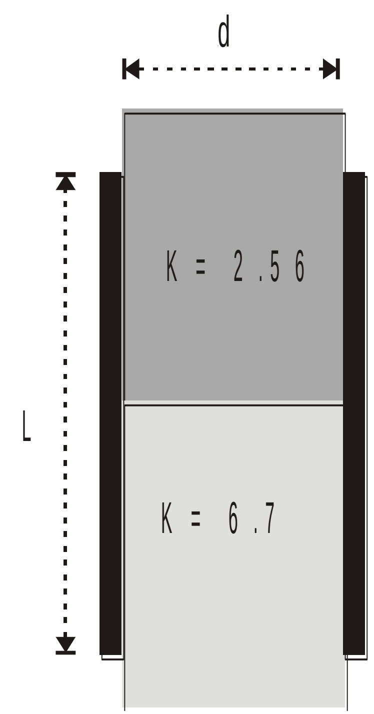
Problem to solve by your self

A capacitor is constructed from two square metal plates of side length *L* and separated by a distance *d*.

One half of the space between the plates (top to bottom) is filled with polystyrene (κ=2.56), and the other half is filled with neoprene rubber (κ=6.7).

Calculate the capacitance of the device, taking *L*=2cm and *d*=0.75mm.

(Hint: The capacitor can be considered as two capacitors connected in parallel.)

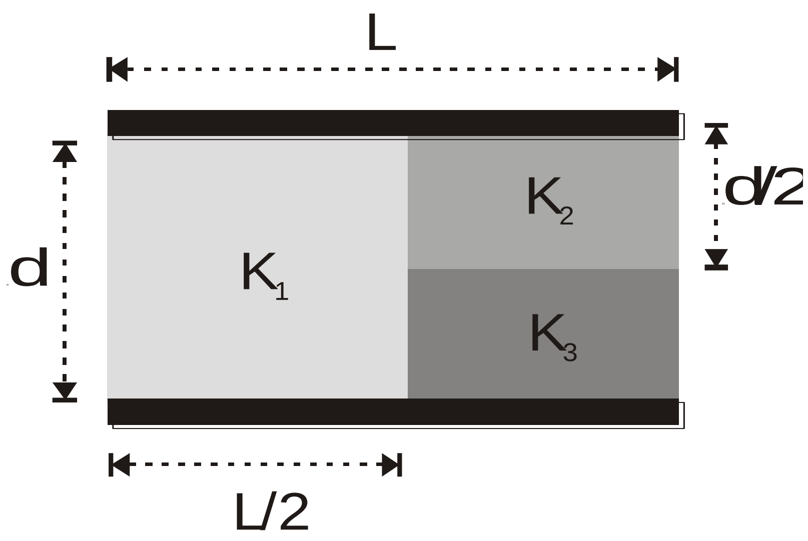


Problem to solve by your self

A parallel plate capacitor is constructed using three different dielectric materials, as shown in the figure.

(a) Find an expression for the capacitance in terms of the plate area *A* and κ_1, κ_2, and κ_3.

(b) Calculate the capacitance using the value *A*=1cm^2, *d*=2mm, κ_1=4.9, κ_2=5.6, and κ_3=2.1.



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General Physics II

Electrostatic: Principles & Applications

Lecture (17): Current and Resistance

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Current and Resistance

- Current and current density
- Definition of current in terms of the drift velocity
- Definition of the current density
- Resistance and resistivity (Ohm's Law)
- Evaluation of the resistance of a conductor
- Electrical Energy and Power
- Combination of Resistors
  - Resistors in Series
  - Resistors in Parallel
- Solution of some selected problems
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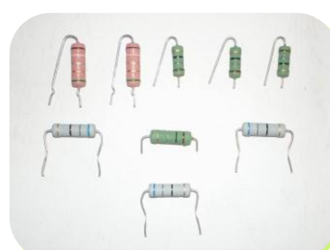


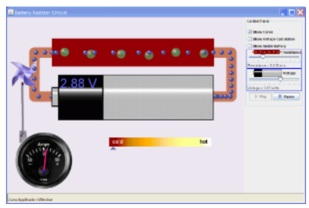
درسنا في الفصول السابقة بعض الظواهر الكهربية المتعلقة بالشحنة الساكنة، وفي هذا الفصل سنركز دراستنا على الشحنات الكهربية في حالة حركة أي "تيار كهربي". حيث نتعامل في حياتنا العملية مع العديد من الأجهزة الكهربية التي تعمل من خلال مرور شحنات كهربية فيها مثل البطارية والضوء وغيرها من الأمثلة الأخرى. ويجب أن نميز بين نوعين من التيار الكهربي وهما التيار الثابت والتيار المتردد، وفي هذا المقرر سنركز على التيار الثابت.

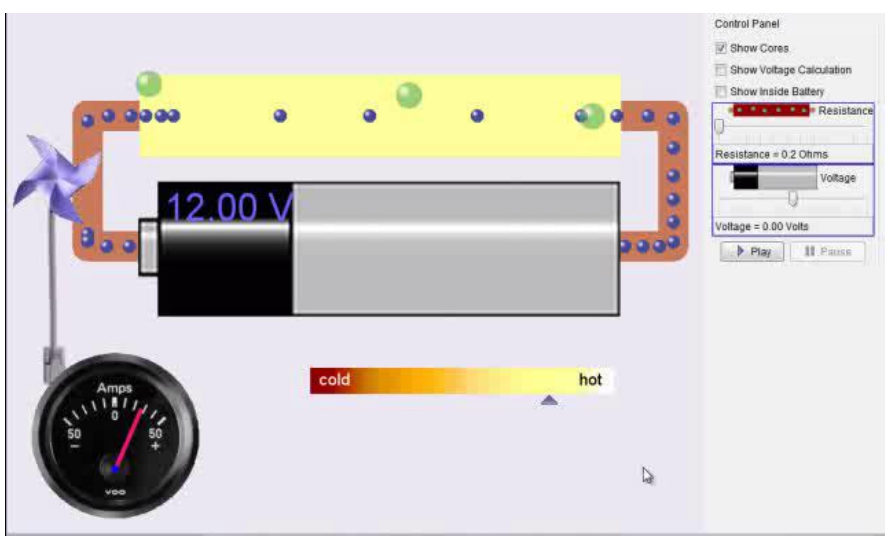
Current and Current Density

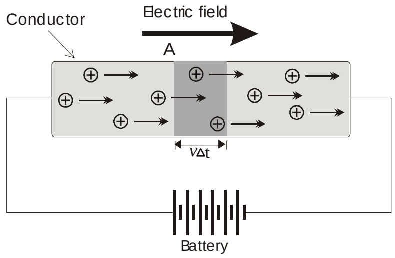
درسنا في الفصل السابق تأثير فرق الجهد الصادر من بطارية على المكثف الكهربي حيث تتراكم الشحنات الموجبة على اللوح المتصل بالقطب الموجب والشحنات السالبة على اللوح المتصل بالقطب السالب للبطارية، وهذا أدى إلى تكون مجال كهربي في الفراغ بين لوحي المكثف. وعرفنا السعة **C** من خلال المعادلة

$$C = \frac{q}{V}$$

سنقوم هنا بتطبيق فرق جهد كهربي صادر من بطارية كهربية على طرفي موصل كهربي مثل سلك من النحاس مساحة مقطعه **A**. وسنتعرف على ظواهر فيزيائية جديدة مثل التيار الكهربي والمقاومة.







يحدد اتجاه التيار الكهربي في الدائرة الكهربية باتجاه التيار الاصطلاحي وهو اتجاه حركة الشحنات الموجبة في الدائرة والذي يكون من القطب الموجب إلى القطب السالب عبر الدائرة.

As shown in the figure above the electric field produces electric force (*F=qE*), this force leads the free charge in the conductor to move in one direction with an average velocity called *drift velocity*.

The current is defined by the net charge flowing across the area *A* per unit time. Thus if a net charge ΔQ flow across a certain area in time interval Δt, the average current I_{av} across this area is

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

In general the current *I* is

$$I = \frac{dQ}{dt}$$

Current is a *scalar* quantity and has a unit of *C/t*, which is called *ampere*

Definition of Current in Terms of the Drift Velocity

Suppose there are *n* positive charge particle per unit volume moves in the direction of the field from the left to the right, all move in drift velocity *v*. In time Δ*t* each particle moves distance *v*Δ*t* the shaded area in the figure, The volume of the shaded area in the figure is equal *A*vΔ*t*, the charge ΔQ flowing across the end of the cylinder in time Δ*t* is

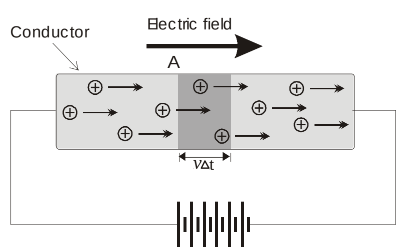
$$\Delta Q = nqvA\Delta t$$

where *q* is the charge of each particle.

Then the current *I* is

$$I = \frac{\Delta Q}{\Delta t} = nqvA$$

تعريف التيار الكهربي بدلالة سرعة الانجراف



Definition of the Current Density

The current per unit cross-section area is called the current density *J*.

$$\vec{J} = \frac{I}{A} = nqv$$

The current density is a vector quantity.

Example

A copper conductor of square cross section 1mm^2 on a side carries a constant current of 20A. The density of free electrons is 8 ×10^{28} electron per cubic meter. Find the current density and the drift velocity.

Solution

The current density is $J = \frac{I}{A} = 20\times10^{6}\,A/m^{2}$

The drift velocity is $v = \frac{J}{nq} = \frac{20\times10^{6}}{(8\times10^{28})(1.6\times10^{-9})} = 1.6\times10^{-3}\,m/s$

This drift velocity is very small compare with the velocity of propagation of current pulse, which is 3 ×10^{8}m/s. The smaller value of the drift velocity is due to the collisions with atoms in the conductor.

Resistance and Resistivity (Ohm's Law)

The resistance *R* of a conductor is defined as the ratio *V/I*, where *V* is the potential difference across the conductor and *I* is the current flowing in it. Thus if the same potential difference *V* is applied to two conductors *A* and *B*, and a smaller current *I* flows in *A*, then the resistance of *A* is grater than *B*, therefore we write,

$$R = \frac{V}{I} \qquad \text{Ohm's law}$$

This equation is known as Ohm's law, which show that a linear relationship between the potential difference and the current flowing in the conductor. Any conductor shows the lineal behavior its resistance is called *ohmic* resistance.

The resistance *R* has a unit of volt/ampere (*v/A*), which is called *Ohm* (Ω).

$$V = IR \qquad \text{and} \qquad I = \frac{V}{R}$$

Fixed resistor *Variable resistor* *Potential divider*

Each material has different resistance; therefore it is better to use the resistivity ρ, it is defined from

$$\rho = \frac{E}{J}$$

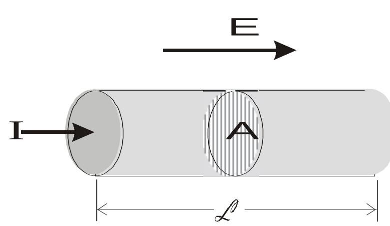
The resistivity has unit of Ω.m

The inverse of resistivity is known as the conductivity σ,

$$\sigma = \frac{1}{\rho}$$

Evaluation of the Resistance of a Conductor

Consider a cylindrical conductor as shown in the figure, of cross-sectional area *A* and length , carrying a current *I*. If a potential difference *V* is connected to the ends the conductor, the electric field and the current density will have the values



Notice that the resistance of a conductor depends on the geometry of the conductor, and the resistivity of the conductor depends only on the electronic structure of the material.

$$E = \frac{V}{\ell} \quad \textbf{and} \quad J = \frac{I}{A}$$

The resistivity ρ is $\rho = \frac{E}{J} = \frac{V/I}{I/A}$

But the *V/I* is the resistance *R* this leads to, $R = \rho\frac{\ell}{A}$

Therefore, the resistance *R* is proportional to the length *l* of the conductor and inversely proportional the cross-sectional area *A* of it

Resistivity of various materials at 20°C

	Material	Resistivity (Ω.m)
1	Silver	1.59×10^{-8}
2	Copper	1.7×10^{-8}
3	Gold	2.44×10^{-8}
4	Aluminum	2.82×10^{-8}
5	Tungesten	5.6×10^{-8}
6	Iron	10×10^{-8}
7	Platinum	11×10^{-8}
8	Lead	20×10^{-8}
9	Nichrome	150×10^{-8}
10	Carbon	3.5×10^{-5}
11	Germanium	0.46
12	Silicon	640
13	Glass	10^{10}-10^{14}

Example (1)

Calculate the resistance of a piece of aluminum that is 20cm long and has a cross-sectional area of 10^{-4}m². What is the resistance of a piece of glass with the same dimensions? $\rho_{Al}=2.82\times10^{-8}\Omega.m$, $\rho_{glass}=10^{10}\Omega.m$.

Solution

The resistance of aluminum

$$R_{Al} = \rho\frac{\ell}{A} = 2.82\times10^{-8}\left(\frac{0.1}{10^{-4}}\right) = 2.82\times10^{-5}\Omega$$

The resistance of glass

$$R_{glass} = \rho\frac{\ell}{A} = 10^{10}\left(\frac{0.1}{10^{-4}}\right) = 10^{13}\Omega$$

Notice that the resistance of aluminum is much smaller than glass.

Example (2)

A 0.90V potential difference is maintained across a 1.5m length of tungsten wire that has a cross-sectional area of 0.60mm^2. What is the current in the wire?

Solution

From Ohm's law

$$I = \frac{V}{R} \qquad \textbf{where} \qquad R = \rho\frac{\ell}{A}$$

therefore,

$$I = \frac{VA}{\rho\ell} = \frac{(0.90)(6.0\times10^{-7})}{(5.6\times10^{-8})(1.5)} = 6.43A$$

Example (3)

(a) Calculate the resistance per unit length of a 22 nichrome wire of radius 0.321mm. (b) If a potential difference of 10V is maintained cross a1m length of nichrome wire, what is the current in the wire. $\rho_{nichromes}$=1.5×10^{-6}Ω.m.

Solution

(a) The cross sectional area of the wire is

$$A = \pi r^2 = \pi\,(0.321\times10^{-3})^2 = 3.24\times10^{-7}m^2$$

The resistance per unit length is *R/l*

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5\times10^{-6}}{3.24\times10^{-7}} = 4.6\Omega/m$$

(b) The current in the wire is

$$I = \frac{V}{R} = \frac{10}{4.6} = 2.2A$$

Nichrome wire is often used for heating elements in electric heater, toaster and irons, since its resistance is 100 times higher than the copper wire.

Problem to solve by your self

1. A current of 5A exists in a 10 Ω resistor for 4min. (a) How many coulombs, and (b) how many electrons pass through any cross section of the resistor in this time?
2. A small but measurable current of 1.0×10^{-10} A exists in a copper wire whose diameter is 0.10in. Calculate the electron drift speed.
3. A square aluminum rod is 1.0m long and 5.0mm on edge. (a) What is the resistance between its ends? (b) What must be the diameter of a circular 1.0m copper rod if its resistance is to be the same?
4. A conductor of uniform radius 1.2cm carries a current of 3A produced by an electric field of 120V/m. What is the resistivity of the material?
5. If the current density in a copper wire is equal to 5.8×10^{6}A/m^2, calculate the drift velocity of the free electrons in this wire.

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General Physics II

Electrostatic: Principles & Applications

Lecture (18): Electrical Energy and Power & Combination of Resistors

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Current and Resistance

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Electrical Energy and Power

If the positive terminal of the battery is connected to *a* and the negative terminal of the battery is connected to *b* of the device.

A charge *dq* moves through the device from *a* to *b*. The battery perform a work $dW = dq\, V_{ab}$.

This work is by the battery is energy *dU* transferred to the device in time *dt* therefore,

$$dU = dW = dq\, V_{ab} = I\, dt\, V_{ab}$$

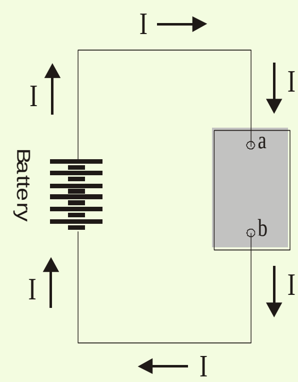
The rate of electric energy (*dU/dt*) is an electric power (*P*).

$$P = \frac{dU}{dt} = IV_{ab}$$

Suppose a resistor replaces the electric device, the electric power is

$$P = I^2 R \quad \text{or} \quad P = \frac{V^2}{R}$$

The unit of power is (*Joule/sec*) ≡ *watt* (*W*).



The current can flow in circuit when a battery is connected to an electrical device through conducting wire.

Example (1)

An electric heater is constructed by applying a potential difference of 110volt to a nichrome wire of total resistance 8Ω. Find the current carried by the wire and the power rating of the heater.

Solution

Since $V = IR$

$$\therefore I = \frac{V}{R} = \frac{110}{8} = 13.8A$$

The power *P* is

$$P = I^2R = (13.8)^2 \times 8 = 1520W$$

Example (2)

A light bulb is rated at 120v/75*W*. The bulb is powered by a 120v. Find the current in the bulb and its resistance.

Solution

$$P = IV$$

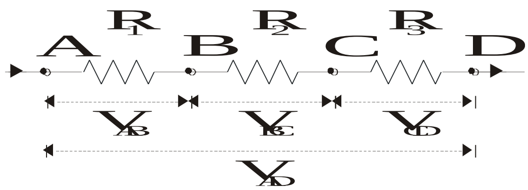
$$\therefore I = \frac{P}{V} = \frac{75}{120} = 0.625A$$

The resistance is

$$R = \frac{V}{I} = \frac{120}{0.625} = 192\Omega$$

Resistors in Series

The figure shows three resistor in series, carrying a current *I*.



For a series connection of resistors, the current is the same in each resistor.

If V_{AD} is the potential deference across the whole resistors, the electric energy supplied to the system per second is IV_{AD}. This is equal to the electric energy dissipated per second in all the resistors.

$$IV_{AD} = IV_{AB} + IV_{BC} + IV_{CD}$$

Resistors in Series

Hence

$$V_{AD} = V_{AB} + V_{BC} + V_{CD}$$

The individual potential differences are

$$V_{AB} = IR_1, \qquad V_{BC} = IR_2, \qquad V_{CD} = IR_3$$

Therefore

$$V_{AD} = IR_1 + IR_2 + IR_3$$

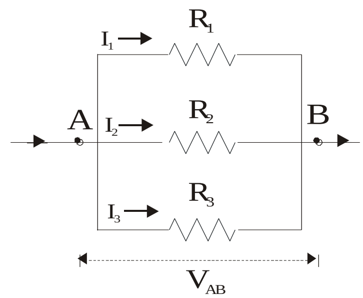
$$V_{AD} = I\,(R_1 + R_2 + R_3)$$

The equivalent resistor is

$$R = R_1 + R_2 + R_3$$

Resistors in Parallel

The figure shows three resistor in parallel, between the points A and B, A current *I* enter from point *A* and leave from point *B*, setting up a potential difference V_{AB}.



For a parallel connection of resistors, the potential difference is equal across each resistor.

The current branches into I_1, I_2, I_3, through the three resistors and,

$$I=I_1+I_2+I_3$$

The current in each branch is given by

$$I_1=\frac{V_{AB}}{R_1} \qquad I_2=\frac{V_{AB}}{R_2} \qquad I_3=\frac{V_{AB}}{R_3}$$

$$\therefore I=V_{AB}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

The equivalent resistance is

$$\therefore \frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

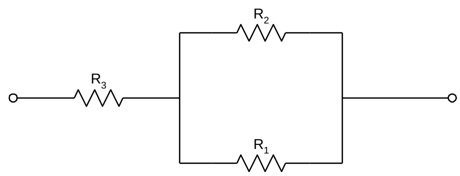
Physical facts for the series and parallel combination of resistors

No.	Series combination	Parallel combination
1	Current is the same through all resistors	Potential difference is the same through all resistors
2	Total potential difference = sum of the individual potential difference	Total Current = sum of the individual current
3	Individual potential difference directly proportional to the individual resistance	Individual current inversely proportional to the individual resistance
4	Total resistance is greater than greatest individual resistance	Total resistance is less than least individual resistance

Notice that parallel resistors combine in the same way that series capacitors combine, and vice versa.

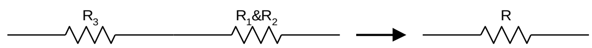
Example (3)

Find the equivalent resistance for the circuit shown in figure 7.6. R_1=3Ω, R_2=6Ω, and R_3=4Ω.



Solution

Resistance R_1 and R_2 are connected in parallel therefore the circuit is simplify as shown



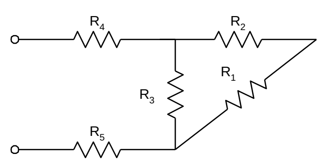
$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow \frac{1}{R'} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6} \longrightarrow R' = 2\Omega$$

Then the resultant resistance of R_1&R_2 (R') are connected in series with resistance R_3

$$R=R'+R_3=2+4=6\Omega$$

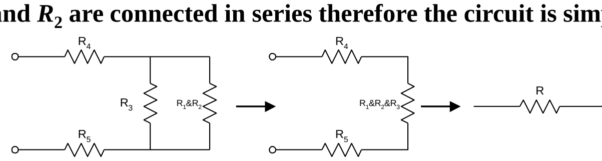
Example (4)

Find the equivalent resistance for the circuit shown in the figure. R_1=4Ω, R_2=3Ω, R_3=3Ω, R_4=5Ω, and R_5=2.9Ω.



Solution

Resistance R_1 and R_2 are connected in series therefore the circuit is simplify as shown below



$$R'=R_1+R_2=4+3=7\Omega$$

Then the resultant resistance of R_1&R_2 (R') are connected in parallel with resistance R_3

$$\frac{1}{R''} = \frac{1}{R'} + \frac{1}{R_3} = \frac{1}{7} + \frac{1}{3} = \frac{10}{21} \longrightarrow R'' = 2.1\Omega$$

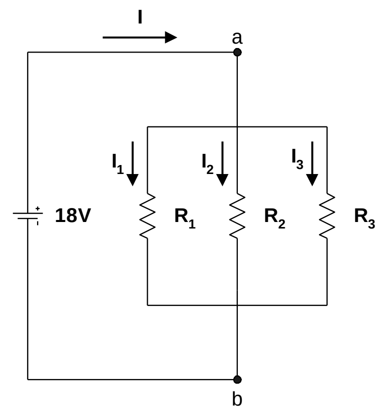
The resultant resistance R for R_5&R_4&R'' are connected in series.

$$R=R''+ R_5+R_4=2.1+5+2.9=10\Omega$$

Example (5)

Three resistors are connected in parallel as in shown in the figure.

A potential difference of 18V is maintained between points *a* and *b*.



(a) find the current in each resistor.

(b) Calculate the power dissipated by each resistor and the total power dissipated by the three resistors.

(c) Calculate the equivalent resistance, and the from this result find the total power dissipated.

Solution

To find the current in each resistors, we make use of the fact that the potential difference across each of them is equal to 18v, since they are connected in parallel with the battery.

Applying *V=IR* to get the current flow in each resistor and then apply *P = I²R* to get the power dissipated in each resistor.

$$I_1 = \frac{V}{R_1} = \frac{18}{3} = 6A \longrightarrow P_1 = I_1^2 R_1 = 108W$$

$$I_2 = \frac{V}{R_2} = \frac{18}{6} = 3A \longrightarrow P_2 = I_2^2 R_2 = 54W$$

$$I_3 = \frac{V}{R_3} = \frac{18}{9} = 2A \longrightarrow P_3 = I_3^2 R_3 = 36W$$

The equivalent resistance R_{eq} is

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18} \longrightarrow R_{eq} = 1.6\ \Omega$$

Example (6)

A 2.4m length of wire that is 0.031cm^2 in cross section has a measured resistance of 0.24Ω. Calculate the conductivity of the material

Solution

$$R = \rho \frac{L}{A} \qquad \text{and} \qquad \rho = \frac{1}{\sigma}$$

$$\sigma = \frac{L}{RA} = \frac{2.4}{(0.24)(3.1\times10^{-6})} = 3.23\times10^{6}/\Omega.m$$

Example (7)

A 0.9V potential difference is maintained across a 1.5m length of tungsten wire that has cross-sectional area of 0.6mm^2. What is the current in the wire?

Solution

$$I = \frac{V}{R} \qquad \text{where} \qquad R = \rho \frac{L}{A}$$

$$I = \frac{VA}{\rho L} = \frac{0.9\times 6\times10^{-7}}{5.6\times10^{-8}\times1.5} = 6.43A$$

Problem to solve by your self

1. **A 2.4m length of wire that is 0.031cm^2 in cross section has a measured resistance of 0.24Ω. Calculate the conductivity of the material.**
2. **Aluminium and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?**
3. **What is the resistance of a device that operates with a current of 7A when the applied voltage is 110V?**
4. **A copper wire and an iron wire of the same length have the same potential difference applied to them. (a) What must be the ratio of their radii if the current is to be the same? (b) Can the current density be made the same by suitable choices of the radii?**
5. **A 0.9V potential difference is maintained across a 1.5m length of tungsten wire that has a cross-sectional area of 0.6mm^2. What is the current in the wire?**
6. **A wire with a resistance of 6.0Ω is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire, assuming that the resistivity and density of the material are not changed during the drawing process.**

7. A wire of Nichrome (a nickel-chromium alloy commonly used in heating elements) is 1.0 m long and 1.0mm^2 in cross-sectional area. It carries a current of 4.0A when a 2.0V potential difference is applied between its ends. What is the conductivity σ, of Nichrome?
8. A copper wire and an iron wire of equal length *l* and diameter *d* are joined and a potential difference V is applied between the ends of the composite wire. Calculate (a) the potential difference across each wire. Assume that *l*=10m, *d*=2.0mm, and *V*=100V. (b) Also calculate the current density in each wire, and (c) the electric field in each wire.
9. Thermal energy is developed in a resistor at a rate of 100W when the current is 3.0A. What is the resistance in ohms?
10. How much current is being supplied by a 200V generator delivering 100kW of power?
11. An electric heater operating at full power draws a current of 8A from 110V circuit. (a) What is the resistance of the heater? (b) Assuming constant *R*, how much current should the heater draw in order to dissipate 750W?

12. If a 55Ω resistor is rated at 125*W*, what is the maximum allowed voltage?
13. A 500W heating unit is designed to operate from a 115V line. (a) By what percentage will its heat output drop if the line voltage drops to 110V? Assume no change in resistance. (b) Taking the variation of resistance with temperature into account, would the actual heat output drop be larger or smaller than that calculated in (a)?
14. A 1250W radiant heater is constructed to operate at 115V. *(a)* What will be the current in the heater? (b) What is the resistance of the heating coil?

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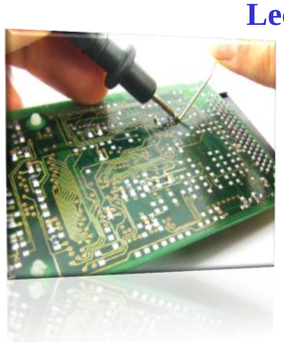
General Physics II

Electrostatic: Principles & Applications

Lecture (19): Direct Current Circuits



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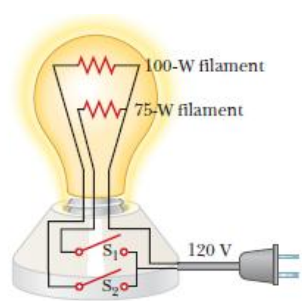
Direct Current Circuits

- Electromotive Force
- Finding the current in a simple circuit
- Kirchhoff's Rules
- Single-Loop Circuit
- Multi-Loop Circuit
- Problems



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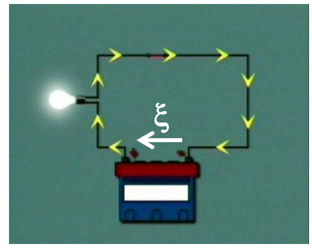
Electromotive Force

In any electrical circuit it must exist a device to provide energy to force the charge to move in the circuit, this device could be battery or generator; in general it is called electromotive force (*emf*) symbol (ξ).



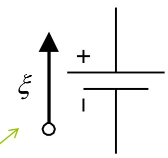
The electromotive force are able to maintain a potential difference between two points to which they are connected.

Then electromotive force (*emf*) (ξ) is defined as the work done per unit charge.

$$\xi = \frac{dW}{dq}$$

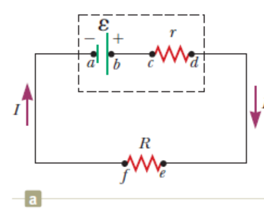
The unit of ξ is *joule*/*coulomb*, which is *volt*.

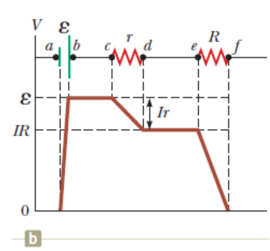
The device acts as an *emf* source is drawn in the circuit as shown in the figure, with an arrow points in the direction which the positive charge move in the external circuit.



Finding the current in a simple circuit

- Consider the circuit shown in figure (a) where a battery is connected to a resistor R with connecting wires assuming the wires has no resistance.
- In the real situation the battery itself has some internal resistance r, hence it is drawn as shown in the rectangle box in the diagram.
- Assume a +ve charge will move from point a along the loop *abcd*. In the graphical representation figure (b) it shows how the potential changes as the charge moves.
- When the charge cross the *emf* from point a to b the potential increases to a the value of *emf* , but when it cross the internal resistance r the potential decreases by value equal Ir. Between the point b to c the potential stay constant since the wire has no resistance. From point c to d the potential decreases by IR to the same value at point a.





The potential difference across the battery between point *a* and *b* is given by

$$V_b - V_a = \xi - Ir$$

Note that the potential difference across points a and b is equal to the potential difference between points *c* and *d* *i.e.*

$$V_b - V_a = V_d - V_c = IR$$

Combining the equations

$$IR = \xi - Ir$$

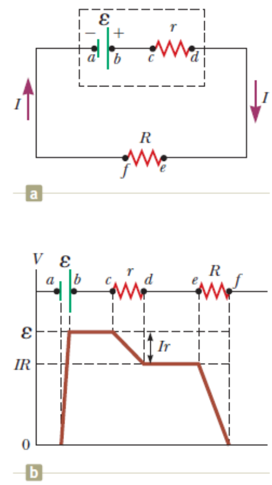
Or

$$\xi = IR + Ir$$

Therefore the current *I* is

$$I = \frac{\xi}{R + r}$$

This equation shows that the current in simple circuit depends on the resistors connected in series with the battery.



Simple Rule

We can reach to the same answer using this rule

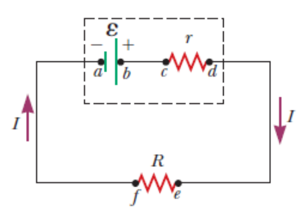
The algebraic sum of the changes in potential difference across each element of the circuit in a complete loop is equal to zero.

By applying the previous rule on the circuit above starting at point *a* and along the loop *abcda*

Here in the circuit we have three elements (one *emf* and two resistors *r*&*R*) applying the rule we get,

$$+\xi - Ir - IR = 0$$

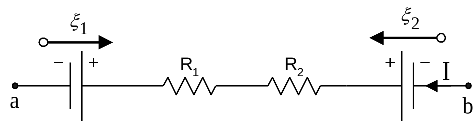
The +ve sine for ξ is because the change in potential from the left to the right across the battery the potential increases, the -ve sign for the change in potential across the resistors is due to the decrease of the potential as we move in the direction *abcda*



$$\therefore I = \frac{\xi}{R + r}$$

Example (1)

Find the current flow in the branch if the potential difference $V_b - V_a = 12v$. Assume ξ_1=10v, ξ_2=25v, R_1=3Ω, and R_2=5Ω.



Solution

We must assume a direction of the current flow in the branch and suppose that is from point *b* to point *a*. To find the current in the branch we need to add all the algebraic changes in the potential difference for the electrical element as we move from point *a* to point *b*.

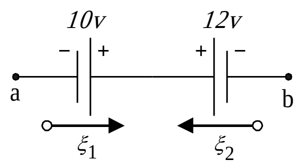
$$V_b - V_a = +\xi_1 + IR_1 + IR_2 - \xi_2$$

لاحظ هنا أننا اخترنا المسار من النقطة a إلى النقطة b. وذلك معتمدين على أن فرق الجهد الكهربي V_b-V_a هو الشغل المبذول لتحريك شحنة من النقطة a إلى النقطة b.

$$I = \frac{(V_b - V_a) + (\xi_2 - \xi_1)}{R_1 + R_2} = \frac{(12) + (25 - 10)}{8} = 3.375A$$

Example (2)

Find the potential difference V_a-V_b for the branches shown in the figure.



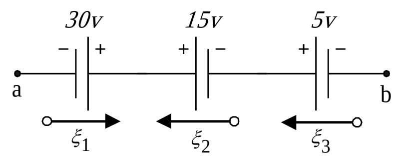
Solution

To find the potential difference V_a-V_b we should add the algebraic change in the potential difference for the two batteries as we move from point *b* to point *a*

$$V_a - V_b = +\xi_2 - \xi_1 = 12 - 10 = 2v$$

Example (3)

Find the potential difference V_a-V_b for the branches shown in the figure.



Solution

$$V_a - V_b = +\xi_3 + \xi_2 - \xi_1 = 5 + 15 - 30 = -10v$$

وهذا يعني أن النقطة *b* لها جهد أعلى من النقطة *a* وحيث أن التيار الكهربي يسري من الجهد المرتفع إلى الجهد المنخفض لذا فإن البطاريتين ξ_3 و ξ_2 تكونان في حالة شحن بواسطة البطارية ξ_1.

Problem to solve by your self

1. **A battery with an *emf* of 12V and internal resistance of 0.9Ω is connected across a load resistor *R*. If the current in the circuit is 1.4A, what is the value of *R*?**
2. **What power is dissipated in the internal resistance of the battery in the circuit described in Problem 8.1?**
3. **(a) What is the current in a 5.6Ω resistor connected to a battery with an 0.2Ω internal resistance if the terminal voltage of the battery is 10V? (b) What is the *emf* of the battery?**
4. **If the emf of a battery is 15V and a current of 60A is measured when the battery is shorted, what is the internal resistance of the battery?**
5. **The current in a loop circuit that has a resistance of R_1 is 2A. The current is reduced to 1.6A when an additional resistor R_2=3Ω is added in series with R_1. What is the value of R_1?**
6. **A battery has an emf of 15V. The terminal voltage of the battery is 11.6V when it is delivering 20W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance of the battery?**

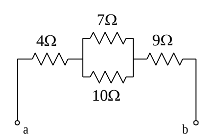
7. A certain battery has an open-circuit voltage of 42V. A load resistance of 12Ω reduces the terminal voltage to 35V. What is the value of the internal resistance of the battery?

8. Two circuit elements with fixed resistances R_1 and R_2 are connected in *series* with a 6V battery and a switch. The battery has an internal resistance of 5Ω, R_1= 32Ω, and R_2=56Ω. (a) What is the current through R_1 when the switch is closed? (b) What is the voltage across R_2 when the switch is closed?

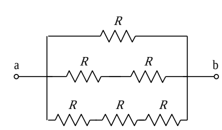
9. The current in a simple series circuit is 5.0A. When an additional resistance of 2.0Ω is inserted, the current drops to 4.0 A. What was the resistance of the original circuit?

10. Three resistors (10Ω, 20Ω, and 30Ω) are connected in parallel. The total current through this network is 5A. (a) What is the voltage drop across the network (b) What is the current in each resistor?

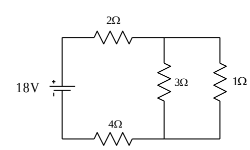
11. Find the equivalent resistance between points a and b in Figure 8.27. (b) A potential difference of 34V is applied between points a and b in the Figure. Calculate the current in each resistor.



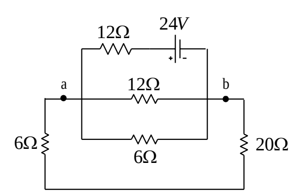
12. Evaluate the effective resistance of the network of identical resistors, each having resistance *R*, shown in the figure



13. Calculate the power dissipated in each resistor in the circuit of the figure.



14. Consider the circuit shown in the figure. Find (a) the current in the 20Ω resistor and (b) the potential difference between points a and b.



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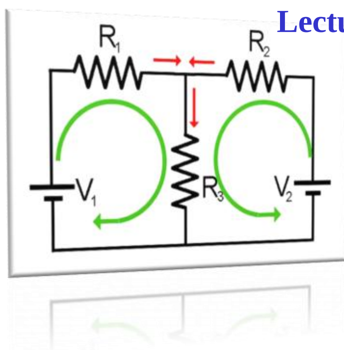
General Physics II

Electrostatic: Principles & Applications



Lecture (20): Direct Current Circuits

Kirchhoff's Rules

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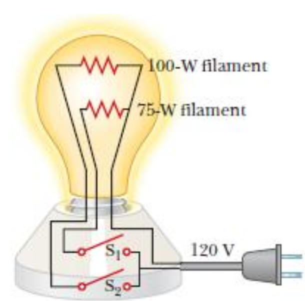
Direct Current Circuits

- Electromotive Force
- Finding the current in a simple circuit
- Kirchhoff's Rules
- Single-Loop Circuit
- Multi-Loop Circuit
- Problems



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Kirchhoff's Rules

A practical electrical circuit is usually complicated system of many electrical elements. Kirchhoff extended Ohm's law to such systems, and gave two rules, which together enabled the current in any part of the circuit to be calculated.

Statements of Kirchhoff's Rules

(1) The algebraic sum of the currents entering any junction must equal the sum of the currents leaving that junction.

$$\sum_i I_i = 0 \qquad \text{at the junction}$$

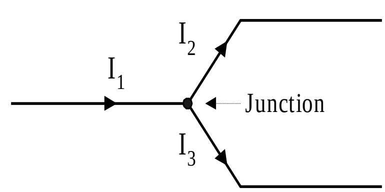
(2) The algebraic sum of the changes in potential difference across all of the elements around any closed circuit loop must be zero.

$$\sum_i \Delta V_i = 0 \qquad \text{for the loop circuit}$$

Note that the first Kirchhoff's rule is for the current and the second for the potential difference.

Kirchhoff's Rules

Applying the first rule on the junction shown below



$$I_1 = I_2 + I_3$$

Kirchhoff's Rules

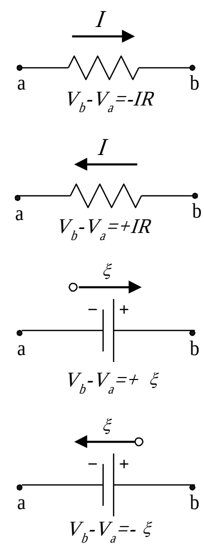
Applying the second rule on the following cases

(1) If a resistor is traversed in the direction of the current, the change in potential difference across the resistor is -*IR*.

(2) If a resistor is traversed in the direction opposite the current, the change in potential difference across the resistor is +*IR*.

(3) If a source of *emf* is traversed in the direction of the *emf* (from - to + on the terminal), the change in potential difference is + .

(4) If a source of *emf* is traversed in the direction opposite the *emf* (from + to - on the terminal), the change in potential difference is - .



Hints for solution of problems using kirchhoff's rules

لاستخدام قاعدة كيرشوف يجب اتباع الخطوات التالية:

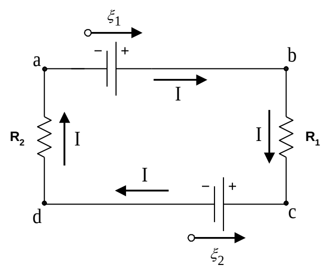
1. حدد اتجاه القوة الدافعة الكهربية *emf* لكل بطارية في الدائرة الكهربية بسهم متجه من القطب السالب إلى القطب الموجب للبطارية.
2. حدد اتجاه التيار الكهربائي لكل عنصر من عناصر الدائرة الكهربية مثل المقاومة بافتراض اتجاه محدد للتيار حتى تتمكن من تطبيق قاعدتي كيرشوف. فإذا كان الحل النهائي يظهر إشارة موجبة للتيار يكون الاتجاه المفترض صحيحاً، أما إذا ظهرت إشارة التيار سالبة فإن قيمة التيار صحيحة، ولكن اتجاه التيار في الاتجاه المعاكس للاتجاه المفترض.
3. نطبق القاعدة الأولى لكيرشوف عند العقدة الموجودة في الدائرة الكهربية بحيث تكون إشارة التيارات الداخلة على العقدة موجبة والخارجة من العقدة سالبة.
4. نطبق القاعدة الثانية لكيرشوف على مسار مغلق محدد لكل فرع من أفرع الدائرة الكهربية ونراعي التغير في فرق الجهد على كل عنصر من عناصر الدائرة الكهربية إذا كان سالباً أو موجباً.
5. نحل المعادلات الرياضية التي نتجت من تطبيق الخطوتين (3) و(4) حلاً جبرياً.

Single-loop Circuit

In a single-loop circuit there is no junctions and the current is the same in all elements of the circuit, therefore we use only the second Kirchhoff rule.

Example (1)

Two battery are connected in opposite in a circuit contains two resistors as shown in figure 8.6 the *emf* and the resistance are ξ_1=6v, ξ_2=12v, R_1= 8Ω, and R_2=10 Ω. (a) Find the current in the circuit. (b) What is the power dissipated in each resistor?



Solution

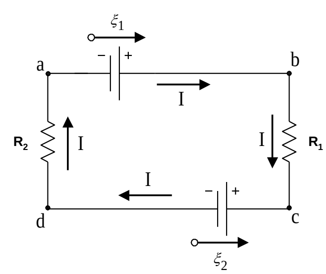
If we assume the current in the circuit is in the clockwise direction (*abcda*).
Applying the second Kirchhoff's rule along arbitrary loop (*abcda*) we get

$$\sum_i \Delta V_i = 0$$

$$+\xi_1 - IR_1 - \xi_2 + IR_2 = 0$$

Solving for the current we get

$$I = \frac{\zeta_1 - \xi_2}{R_1 + R_2} = \frac{6-12}{8+10} = -\frac{1}{3}A$$



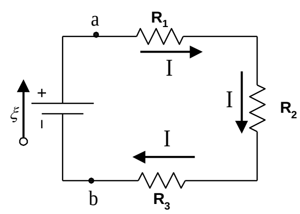
The -ve sign of the current indicates that the correct direction of the current is opposite the assumed direction *i.e.* along the loop (*adba*)
The power dissipated in R_1 and R_2 is

$$P_1 = I^2 R_1 = 8/9\text{W}$$
$$P_2 = I^2 R_2 = 10/9\text{W}$$

In this example the battery ξ_2 is being charged by the battery ξ_1.

Example (2)

Three resistors are connected in series with battery as shown in the figure, apply second Kirchhoff's rule to (a) Find the equivalent resistance and (b) find the potential difference between the points *a* and *b*.



Solution

Applying second Kirchhoff's rule in clockwise direction we get

$$- IR_1 - IR_2 - IR_3 + \xi = 0$$

$$I = \frac{\zeta}{R_1 + R_2 + R_3}$$

$$\because I = \frac{\zeta}{R}$$

therefore,

$$R = R_1 + R_2 + R_3$$

To find the potential difference between points a and b V_{ab} ($=V_a - V_b$) we use the second Kirchhoff's rule along a direction starting from point (b) and finish at point (a) through the resistors. We get

$$V_b + IR = V_a$$

Where *R* is the equivalent resistance for R_1, R_2 and R_3

$$V_{ab} = V_a - V_b = + IR$$

The +ve sign for the answer means that $V_a > V_b$

Substitute for the current *I* using the equation $I = \frac{\zeta}{R}$

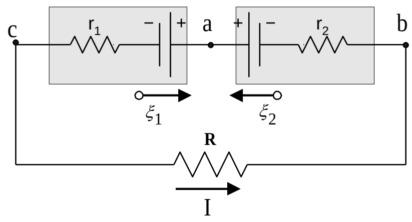
we get

$$V_{ab} = \xi$$

This means that the potential difference between points *a* and *b* is equal to the *emf* in the circuit (when the internal resistance of the battery is neglected).

Example (3)

In the circuit shown in the figure let ξ_1 and ξ_2 be 2v and 4v, respectively; r_1 ,r_2 and R be 1Ω, 2Ω, and 5Ω, respectively. (a) What is the current in the circuit? (b) What is the potential difference V_a-V_b and V_a-V_c?



Solution

$$-\xi_2 + Ir_2 + IR + Ir_1 + \xi_1 = 0$$

Solving the equation for the current we get

$$I = \frac{\zeta_2 - \xi_1}{R + r_1 + r_2} = \frac{4-2}{5+1+2} = +0.25A$$

The potential difference V_a-V_b we apply second Kirchhoff's rule starting at point b to finishing at point a.

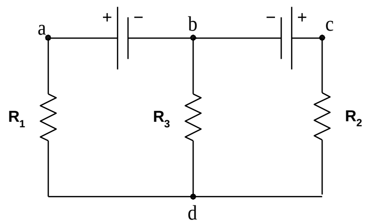
$$V_a - V_b = -Ir_2 + \xi_2 = (-0.25\times2)+4=+3.5v$$

The potential difference V_a-V_c we apply second Kirchhoff's rule starting at point c to finishing at point a.

$$V_a - V_c = +\xi_1 + Ir_1 = +2+(-0.25\times1) = +2.25v$$

Multi-loop Circuit

Some circuits involving more than one current loop, such as the one shown in the figure. Here we have a circuit with three loops: a left inside loop, a right inside loop, and an outside loop.

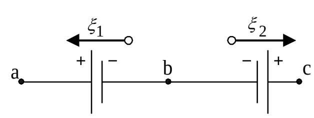


In the circuit shown above there are two junctions *b* and *d* and three branches connecting these junctions. These branches are bad, *bcd*, and *bd*. The problem here is to find the currents in each branch.

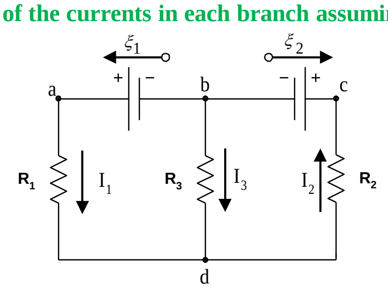
A general method for solving multi-loop circuit problem is to apply Kirchhoff's rules.

You should always follow these steps:

(1) Assign the direction for the *emf* from the -ve to the +ve terminal of the battery.



(2) Assign the direction of the currents in each branch assuming arbitrary direction.



After solving the equations the +ve sign of the current means that the assumed direction is correct, and the -ve sign for the current means that the opposite direction is the correct one.

(3) Chose one junction to apply the first Kirchhoff's rule.

$$\sum_i I_i = 0$$

At junction *d* current I_1 and I_3 is approaching the junction and I_2 leaving the junction therefore we get this equation

$$I_1 + I_3 - I_2 = 0 \qquad (1)$$

(4) For the three branches circuit assume there are two single-loop circuits and apply the second Kirchhoff's rule on each loop.

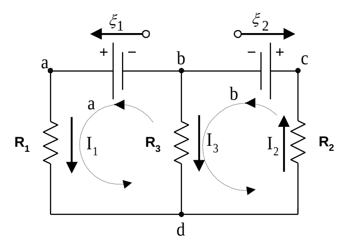
For loop a on the left side starting at point b we get

$$+\xi_1 - I_1R_1 + I_3R_3 = 0 \qquad (2)$$

For loop b on the left side starting at point b we get

$$- I_3R_3 - I_2R_2 - \xi_2 = 0 \qquad (3)$$

Equations (1), (2), and (3) can be solved to find the unknowns currents I_1, I_2, and I_3.



Example (4)

In the circuit shown in the figure, find the unknown current *I*, resistance *R*, and *emf* ξ.

Solution

At junction a we get this equation

$$I + 1 - 6 = 0$$

Therefore the current

$$I = 5A$$

To determine R we apply the second Kirchhoff's rule on the loop (a), we get

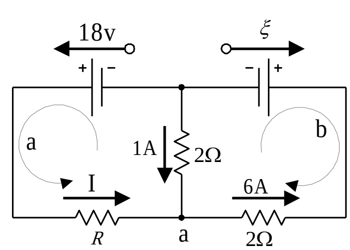
$$18 - 5R + 1 \times 2 = 0$$

$$R = 4\Omega$$

To determine ξ we apply the second Kirchhoff's rule on the loop (a), we get

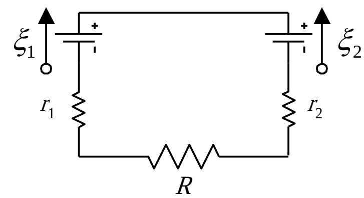
$$\xi + 6 \times 2 + 1 \times 2 = 0$$

$$\xi = -14v$$

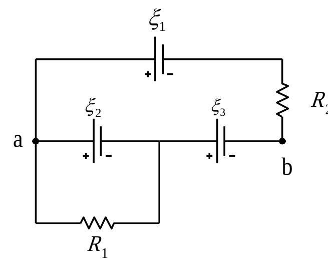


Problem to solve by your self

1. (*a)* In the figure what value must R have if the current in the circuit is to be 0.0010A? Take & ξ_1=2.0V, ξ_2=3.0V, and $r_1=r_2$=3.0Ω. (b) What is the rate of thermal energy transfer in R?

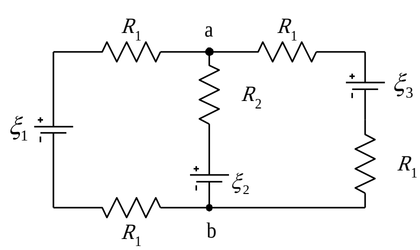


2. In the figure find the current in each resistor and the potential difference between a and b. Put ξ_1=6.0V, ξ_2=5.0V, $_3$=4.0V, R_1=100Ω and R_2 =50Ω.

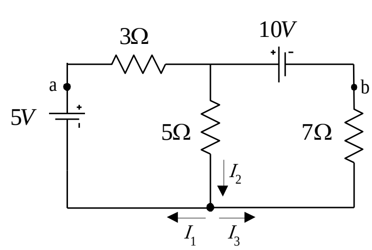


Problem to solve by your self

3. (a) Find the three currents in the figure. (b) Find V_{ab}. Assume that R_1=1.0Ω, R_2=2.0Ω, ξ_1=2.0 V, and $\xi_2=\xi_3$=4.0V.



4. (a) Find the potential difference between points a and b in the circuit in the figure. (b) Find the currents I_1, I_2, and I_3 in the circuit



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