من إصدارات
أكاديمية الفيزياء للتعليم الإلكتروني

General Physics (1)
Mechanics: Principles & Applications

سلسلة محاضرات الفيزياء العامة (١)
علم الميكانيكا وتطبيقاتها

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مقدمة

نقدم لكم أعزائنا طلبة الكليات العلمية هذه السلسلة من محاضرات الفيزياء العامة (١) بعنوان الميكانيكا وتطبيقاتها في صورة شرائح بوربوينت اعتمادا على كتاب Physics for scientists and engineers لمؤلفه Serway الكتاب المعتمد في معظم الكليات العلمية بمختلف الجامعات لتكون عونا لطلبة المستوى الأول في الكليات العلمية. تغطي هذه الشرائح موضوع مقدمة عن علم الفيزياء والقياس وعلم المتجهات وموضوع علم وصف الحركة بأنواعها المختلفة لتشمل الحركة في بعد واحد وفي بعدين عند ثبوت التسارع مع مجموعة من التطبيقات من واقع الحياة العملية وكذلك موضوع قوانين نيوتن للحركة مع تطبيقاتها المتعددة كذلك موضوع القوة والشغل والطاقة وقانون الجذب العام لنيوتن.
شرحت هذه المواضيع من خلال تقسيمها إلى محاضرات مدعمة بالصور والشروحات العديد من الأمثلة المحلولة مع الشرح والتوضيح وكذلك في نهاية كل محاضرة مجموعة من الأسئلة التي تتيح للطالب ان يختبر نفسه ومدى استيعابه لما قدم في كل محاضرة.
يصاحب كل محاضرة تسجيل فيديو يشرح المحاضرة ويوضحها بأحدث الوسائل التعليمية على قناة موقع الفيزياء التعليمي على قناة اليوتيوب والتي يمكن ان تجدها بسهولة بالبحث في أداة بحث موقع اليوتيوب عن قناة د. حازم فلاح سكيك وللتسهيل على الطلبة تم تجميع كل المحاضرات المتعلقة بالمنهج في قائمة تشغيل بعنون محاضرات الفيزياء العامة(١) الميكانيكا وقسمت كل محاضرة إلى مقاطع زمنية لا تتجاوز ٢٠ دقيقة واعطي اسم لكل مقطع يتوافق مع اسم المحاضرة ورقمها. ادرج في تسجيل شرح كل محاضرة العديد من الأفلام العلمية ذات العلاقة مع موضوع المحاضرة.
أتمنى ان تكون هذه المحاضرات مساعدا أساسيا لكل الطلبة الدارسين لمقرر الفيزياء العامة (١) في فهم هذا المقرر واستيعابه.
كما سوف ندرج على موقع اكاديمية الفيزياء للتعليم الالكتروني www.physicsacademy.org بنك من الأسئلة متعددة الاختيار ليتمكن كل دارس من اختبار مقدار فهمه للمقرر وتقييم مستوى تحصيله فيه.

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General Physics I

Mechanics: Principles & Applications

Lecture (1): Units, Dimensions and Vectors

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Units, Dimensions, and Vectors

- Introduction
- Unit systems
- Dimensional Analysis
- Vector and Scalar
- Coordinate system
- Properties of Vectors
- The unit vector
- Components of a vector
- Product of a vector

About Physics

1. Explains Nature
2. Fundamental science
3. Most technology of today (cell phones, DVD player, etc) are result of discoveries that happened in physics last century.
4. Develop problem solving and logical reasoning skills – very important in any field of work!!!

What is Physics?

Physics tries to explain processes we observe in terms of a set of 'laws' or rules

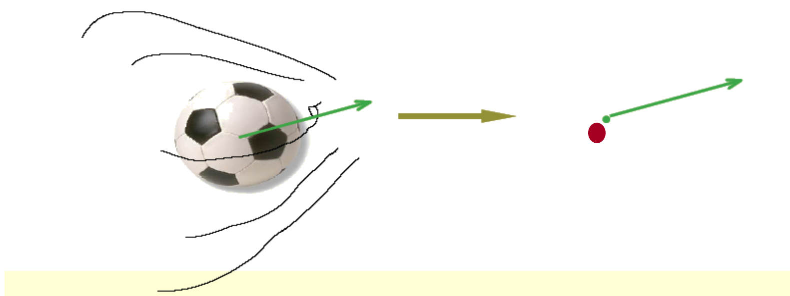
These laws are normally expressed in terms of mathematical equations with which we can predict things

These predictions can be compared to what we see

"Theory & Experiment"

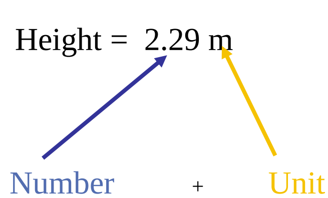
Idealized Models

- A physical system is often too complicated to analyze all at once
- A **model** is a simplified version of a physical system
- Example: We neglect the size and shape of a *ball* and represent it as a *particle* (completely localized at a single point in space)

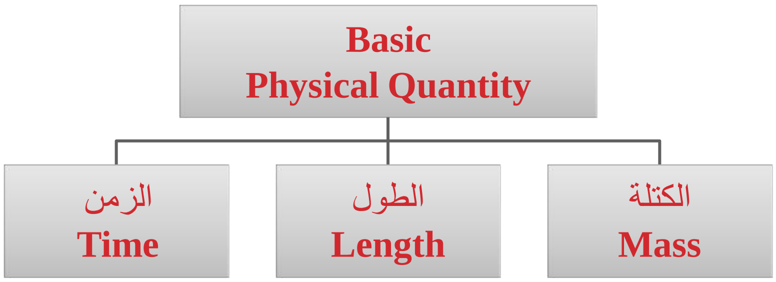


Physics and Measurements

- **Physics is based on experimental observations and quantitative measurements. These observations have described by numbers and units.**
- **Numbers give us how large our measurement was, and the units tell us the nature of this measurement.**



Physical Quantity



Unit systems

Two systems of units are widely used in the world, the metric and the British systems.

The metric system measures the length in meters whereas the British system makes use of the foot, inch, …..

The metric system is the most widely used. Therefore the metric system will be used in this course.

By international agreement the metric system was formalized in 1971 into the *International System of Units* (SI). There are seven basic units in the SI as shown in table Below.

"For this Course only three units are used, the meter, kilogram, and second".

Basic units in the SI

Quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A
Number of particles	mole	mol
Luminous intensity	candela	cd

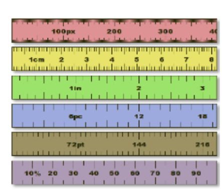
Definition of Basic Quantities

Mass

The SI unit of mass is the *Kilogram*, which is defined as the mass of a specific platinum-iridium alloy cylinder.

Time

The SI unit of time is the *Second*, which is the time required for a cesium-133 atom to undergo 9,192,631,770 vibrations.



Length

The SI unit of length is *Meter*, which is the distance traveled by light in vacuum during a time of 1/299,792,458 second.

Power of ten prefixes

- 3,000 m = 3 × 1,000 m
  = 3 × 10^3 m = 3 km
- 1,000,000,000 = 10^9 = 1G
- 1,000,000 = 10^6 = 1M
- 1,000 = 10^3 = 1k

- 141 kg = ? g
- 1 GB = ? Byte = ? MB

10^x	Prefix	Symbol
x=18	exa	E
15	peta	P
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deca	da

Power of ten prefixes

- 0.003 s = 3 × 0.001 s
  = 3 × 10^{-3} s = 3 ms
- 0.01 = 10^{-2} = centi
- 0.001 = 10^{-3} = milli
- 0.000 001 = 10^{-6} = micro
- 0.000 000 001 = 10^{-9} = nano
- 0.000 000 000 001 = 10^{-12}
  = pico = p

- 3 cm = ? m = ? mm

10^x	Prefix	Symbol
x=-1	deci	d
-2	centi	c
-3	milli	m
-6	micro	μ
-9	nano	n
-12	pico	p
-15	femto	f
-18	atto	a

Role of "UNITS" in problem solving

Need to know conversion.

Do problems with all units in the same system.

Only quantities with same units can be added or subtracted.

Example

$$40\,\text{m} + 11\,\text{cm} = ?$$

The above expression yields:

a) 40.11 m
b) 4011 cm
c) A or B
d) Impossible to evaluate (dimensionally invalid)

$$1.5\,\text{m} \cdot 3.0\,\text{kg} = ?$$

The above expression yields:

a) 4.5 m kg
b) 4.5 g km
c) A or B
d) Impossible to evaluate (dimensionally invalid)

Express a speed of 50 kilometers per hour as meters per second.

Derived Quantities

All physical quantities measured by physicists can be expressed in terms of the three basic unit of length, mass, and time. For example, *speed* is simply length divided by time, and the *force* is actually mass multiplied by length divided by time squared.

$$[\text{Speed}] = L/T = LT^{-1}$$
$$[\text{Force}] = ML/T^2 = MLT^{-2}$$

where [Speed] is meant to indicate the unit of speed, and M, L, and T represents mass, length, and time units.

- **Multiply and divide units just like numbers**
- **Derived quantities: area, speed, volume, density ……**
  - Area = Length × Length — unit for area = m^2
  - Volume = Length × Length × Length — unit for volume = m^3
  - Speed = Length / time — unit for speed = m/s
  - Density = Mass / Volume — unit for density = kg/m^3
- In 2008 Olympic Game, Usain Bolt sets world record at 9.69 s in Men's 100 m Final. What is his average speed?

$$\text{speed} = \frac{100\text{ m}}{9.69\text{ s}} = \frac{100}{9.69} \cdot \frac{\text{m}}{\text{s}} = 10.32\text{ m/s}$$

Dimensional Analysis

The word dimension in physics indicates the physical nature of the quantity. For example the distance has a dimension of *length*, and the *speed* has a dimension of *length*/*time*.

The dimensional analysis is used to check the formula, since the dimension of the left hand side and the right hand side of the formula must be the same.

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Example

Using the dimensional analysis check that this equation $x = \frac{1}{2} at^2$ is correct, where x is the distance, a is the acceleration and t is the time.

Solution

$$x = \frac{1}{2} at^2$$

The right hand side $[x] = L$

The left hand side $[at^2] = \frac{L}{T^2} \times T^2 = L$

This equation is correct because the dimension of the left and right side of the equation have the same dimensions.

Example

Show that the expression $v = v_o + at$ is dimensionally correct, where v and v_o are the velocities and a is the acceleration, and t is the time.

Solutions:

The right hand side $[v] = \frac{L}{T}$

The left hand side $[at] = \frac{L}{T^2} \times T = \frac{L}{T}$

Therefore, the expression is dimensionally correct

Example

Suppose that the acceleration of a particle moving in circle of radius *r* with uniform velocity *v* is proportional to the r^n and v^m. Use the dimensional analysis to determine the power n and m.

Solution

Let us assume *a* is represented in this expression

$$a = k\, r^n\, v^m$$

Where *k* is the proportionality constant of dimensionless unit.

The Left hand side

$$[a] = \frac{L}{T^2}$$

The Right hand side

$$[k\, r^n\, v^m] = L^n\left(\frac{L}{T}\right)^m = \frac{L^{n+m}}{T^m}$$

Therefore

$$\frac{L}{T^2} = \frac{L^{n+m}}{T^m}$$

Hence, *n* + *m* = 1 and *m* = 2

Therefore. *n* = -1 and the acceleration *a* is

$$a = k\, r^{-1}\, v^2$$

***k* = 1**

$$a = \frac{v^2}{r}$$

Problem

Which of the following equations are dimensionally correct?

(a) $v_f = v_i + ax$

(b) y = (2 m) cos(kx), where k = 2 m^{-1}

The position of a particle moving under uniform acceleration is some function of time and the acceleration. Suppose we write this position s = k $a^m t^n$,

1. **where k is a dimensionless constant.**
2. **Show by dimensional analysis that this expression is satisfied if m = 1 and n =2. Can this analysis give the value of k?**

If the frequency (*f*) of a simple harmonic motion of a pendulum can be written as,

$$f = m^{\alpha}\, g^{\beta}\, l^{\gamma}$$

where is *m* the mass of the oscillating pendulum, *g* is the acceleration due gravity, and *l* is the length of the pendulum. Find α, β, and γ.

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General Physics I

Mechanics: Principles & Applications

Lecture (2): Units, Dimensions and Vectors

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VECTOR AND SCALAR

Scalars: One that can be described by a single number (along with the unit)

- Water freezes at a temperature of 0^o C or 32^o F
- The mass of a book is 198.2 g
- The length of room is 5 m
- The car kinetic energy was 0.345 J

Vectors: A quantity that deals with magnitude and direction is called a vector quantity.

- The wind had a velocity of 25 km/h from the North
- The momentum was 1.234 kg m/s to the left

Textbooks use either A or $\vec{A}$

Vector and scalar quantities

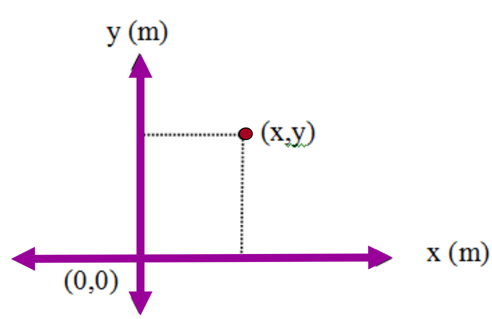
Vector Quantity	Scalar Quantity
Displacement	Length
Velocity	Mass
Force	Speed
Acceleration	Power
Field	Energy
Momentum	Work

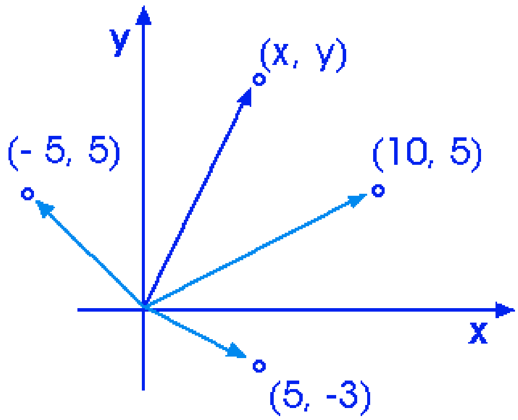
COORDINATE SYSTEM (1)

(1) The rectangular coordinates

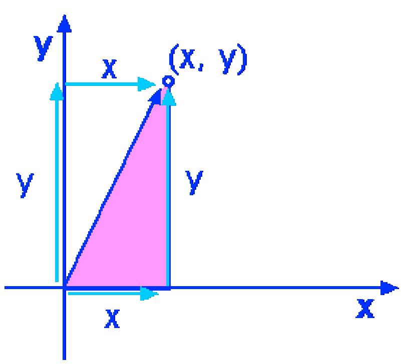
This coordinate system is consist of a fixed reference point (0,0) which called the origin.

A set of axis with appropriate scale and label.





The x- and y-coordinates may be either positive or negative



We can call this x-component, a vector along the x-direction of length x, and indicate that it is a vector by x.

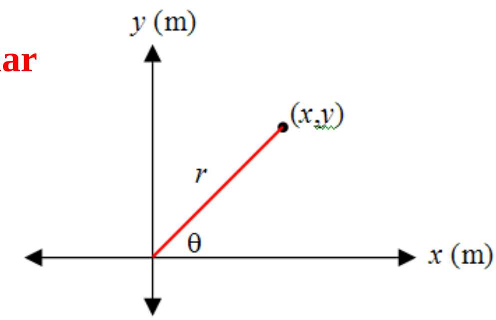
Likewise, we can call this y-component, a vector along the y-direction of length y, and indicate that it is a vector by y .

r = x + y

Coordinate system (2)

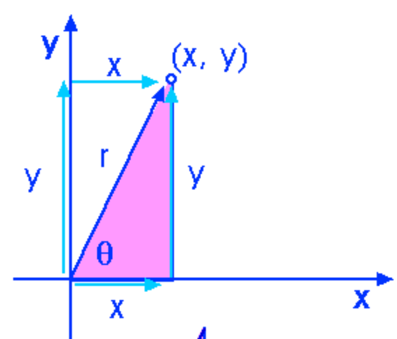
(2) The polar coordinates

Sometimes it is more convenient to use the polar coordinate system (r,θ), where r is the distance from the origin to the point of rectangular coordinate (x,y), and θ is the angle between r and the x axis.



THE RELATION BETWEEN COORDINATES

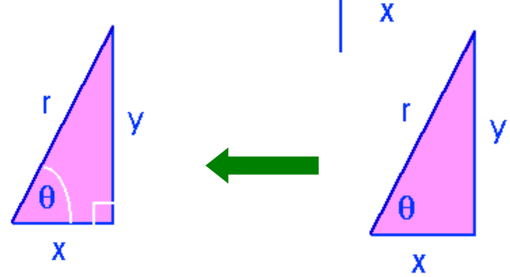
The relation between the rectangular coordinates (x,y) and the polar coordinates (r,θ) is shown in Figure,

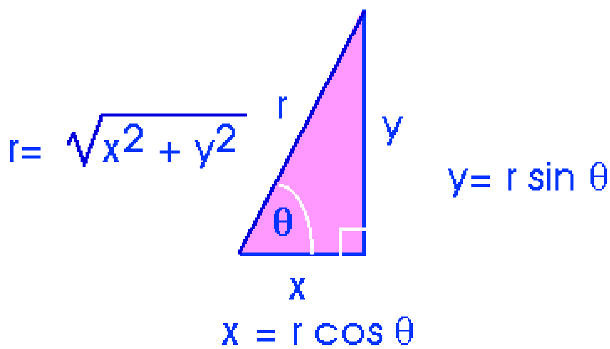


$\sin\theta = y/r$

$\cos\theta = x/r$

$\tan\theta = y/x$





It is common practice to measure the angle from the positive x-axis and to measure it positive for a counterclockwise direction.

Example 1

The example shown below might be for an angle of $\theta = 53^o$ .Then, if r = 10, the components will be

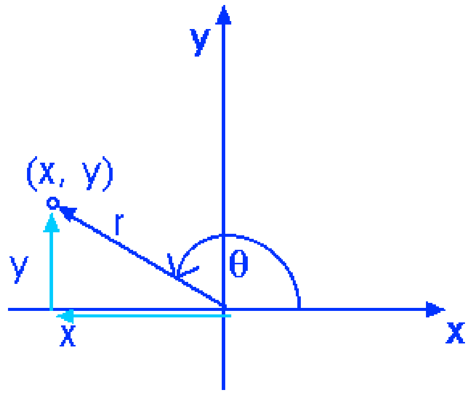
$x = r \cos \theta = (0.6)\ (10) = 6$

$y = r \sin \theta = (0.8)\ (10) = 8$

Of course, angle θ does not need to be limited to the first quadrant. for $\theta = 150^o$, r = 10 for this numerical example. For that case,

$x = r \cos \theta = (10)(\cos 150^o) = (10)(-0.866) = -8.66$

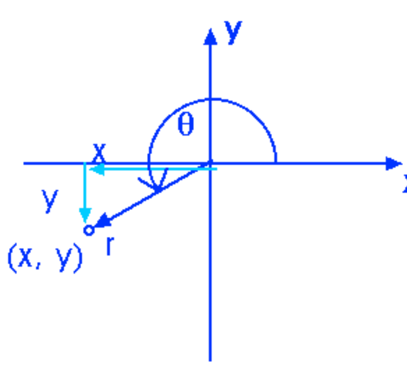
$y = r \sin \theta = (10)(\sin 150^o) = (10)(0.500) = 5.00$



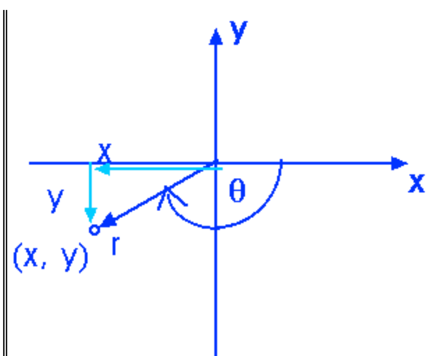
notice the signs and compare them with the diagram.

x = - 8.66 is located to the left and y = 5.00 is located up .

من الممكن ان نقيس الزاوية لتحديد الاتجاه من محور x مع عقارب عكس عقارب الساعة أو مع عقارب الساعة ولكن يجب ان نميز ذلك باشارة الزاوية.



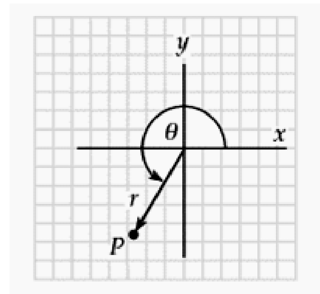
r = 10, θ = 210º



r = 10, θ = -150º

Example 2

The polar coordinates of a point are r = 5.5m and θ=240°. What are the Cartesian coordinates of this point?



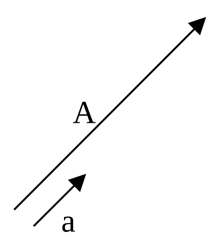
- **Solution**

$x = r \cos \theta = 5.5\times\cos 240^{o} = -2.75$ m

$y = r \sin \theta = 5.5\times\sin 240^{o} = -4.76$ m

The unit vector

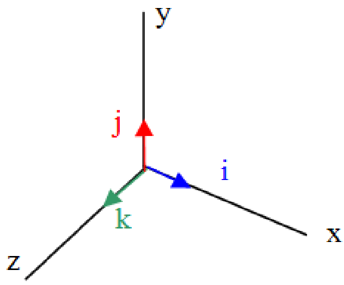
A unit vector is a vector having a magnitude of unity and its used to describe a direction in space.



المتجه ***A*** يمكن تمثيله بمقدار المتجه ضرب متجه الوحدة ***a*** كالتالي:

A= *a* *A*

كذلك يمكن تمثيل متجهات وحدة (i, j, k) لمحاور الإسناد المتعامدة (x, y, z) كما في الشكل التالي:-



i ≡ a unit vector along the *x*-axis

j ≡ a unit vector along the *y*-axis

k ≡ a unit vector along the *z*-axis

Instead of explicitly writing $A_x = 5$, $A_y = 0$; $B_x = 5$, $B_y = 5$; $C_x = -10$, $C_y = 0$; and $D_x = -5$, $D_y = 5$, we can write this same information in a different form. We can write

$$A = 5\,i + 0j$$

$$B = 5i + 5j$$

$$C = -10i + 0j$$

$$D = -5i + 5j$$

$$R = A + B + C + D$$

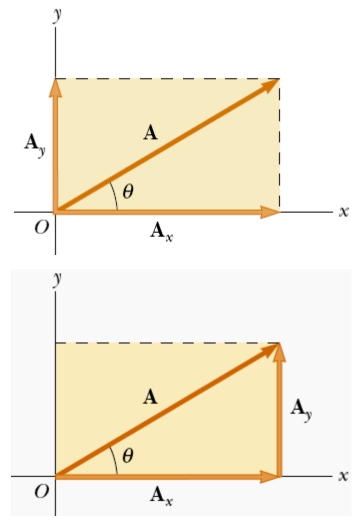
$$R = (5\,i + 0j) + (5i + 5j) + (-10i + 0j) + (-5i + 5j)$$

$$R = (5+5-10-5)i + (0+5+0+5)j$$

$$R = -5i + 10j$$

Components of a vector

Any vector lying in *xy* plane can be resolved into two components one in the *x*-direction and the other in the *y*-direction as shown in Figure



$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

عند التعامل مع عدة متجهات فإننا نحتاج إلى تحليل كل متجه إلى مركباته بالنسبة إلى محاور الإسناد (x,y) مما يسهل إيجاد المحصلة بدلاً من استخدام الطريقة البيانية لإيجاد المحصلة.

The magnitude of the vector **A**

$$A = \sqrt{A_x^2 + A_y^2}$$

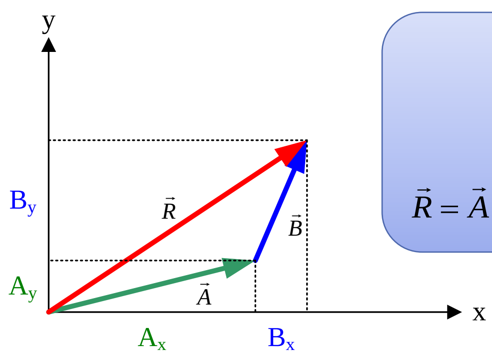
The direction of the vector to the *x*-axis

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

A vector **A** lying in the *xy* plane, having rectangular components A_x and A_y can be expressed in a unit vector notation

$$\mathbf{A} = A_x\mathrm{i} + A_y\mathrm{j}$$

ملاحظة: يمكن استخدام طريقة تحليل المتجهات في جمع متجهين A و B كما في الشكل التالي:



$$\vec{A} = A_x i + A_y j$$

$$\vec{B} = B_x i + B_y j$$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)i + (A_y + B_y)j$$

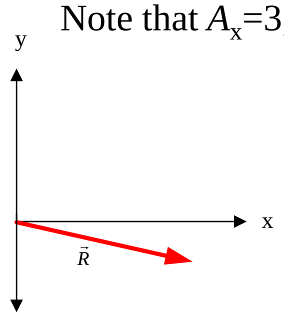
Example 3

Find the sum of two vectors A and B and given by

$$\vec{A} = 3i + 4j \text{ and } \vec{B} = 2i - 5j$$

Solutions

Note that A_x=3, A_y=4, B_x=2, and B_y=-5



$$\vec{R} = \vec{A} + \vec{B} = (3+2)i + (4-5)j = 5i - j$$

The magnitude of vector R is

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{25+1} = \sqrt{26} = 5.1$$

The direction of R with respect to *x*-axis is

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-1}{5} = -11^o$$

Example 4

Two vectors are given by $\vec{A} = 3i - 2j$ and $\vec{B} = -i - 4j$. Calculate (a) $\vec{A}+\vec{B}$, (b) $\vec{A}$ - $\vec{B}$, (c) $|\vec{A}+\vec{B}|$, (d) $|\vec{A}-\vec{B}|$, and (e) the direction of $\vec{A}+\vec{B}$ and $|\vec{A}-\vec{B}|$.

Solutions

(a) $\vec{A}+\vec{B} = (3i-2j)+(-i-4j) = 2i-6j$

(b) $\vec{A} - \vec{B} = (3i-2j)-(-i-4j) = 4i+2j$

(c) $|\vec{A}+\vec{B}| = \sqrt{2^2+(-6)^2} = 6.32$

(d) $|\vec{A}-\vec{B}| = \sqrt{4^2+2^2} = 4.47$

(e) For $\vec{A}+\vec{B}$, $\theta = \tan^{-1}(-6/2) = -71.6^\circ = 288^\circ$

For $\vec{A} - \vec{B}$, $\theta = \tan^{-1}(2/4) = 26.6^\circ$

Example 5

A particle moves from a point in the *xy* plane having rectangular coordinates (-3,-5)m to a point with coordinates (-1,8)m. (a) Write vector expressions for the position vectors in unit vector form for these two points. (b) What is the displacement vector?

Solution

(a)

$$\vec{R}_1 = x_1 i + y_1 j = (-3i - 5j)m$$

$$\vec{R}_2 = x_2 i + y_2 j = (-i + 8j)m$$

$$\Delta\vec{R} = \vec{R}_2 - \vec{R}_1$$

(b) Displacement =

$$\Delta\vec{R} = (x_2 - x_1)i + (y_2 - y_1)j = -i - (-3i) + 8j - (-5j) = (2i + 13j)m$$

EXAMPLE 6

A vector A has a negative *x* component 3 units in length and positive *y* component 2 units in length.

(a) Determine an expression for A in unit vector notation.

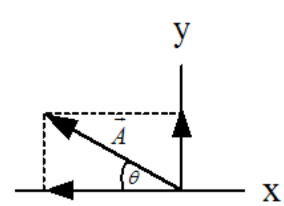
(b) Determine the magnitude and direction of A.

(c) What vector B when added to A gives a resultant vector with no *x* component and negative *y* component 4 units in length?

Solution

A_x = -3 units & A_y = 2 units

(a) A= A_xi+A_yj=-3i+2j units



(b) $|\vec{A}| = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{(-3)^2 + (2)^2} = 3.61\ units$

$\theta = \tan^{-1}(2/-3) = 33.7^o$ (relative to the –x axis)

(c) What vector B when added to A gives a resultant vector with no *x* component and negative *y* component 4 units in length?

$R_x = 0$ & $R_y = -4$; since $\vec{R} = \vec{A} + \vec{B}$, $\vec{B} = \vec{R} - \vec{A}$

$B_x = R_x - A_x = 0 - (-3) = 3$

$B_y = R_y - A_y = -4 - 2 = -6$

Therefore

$$\vec{B} = B_x i + B_y j = (3i - 6j)\ units$$

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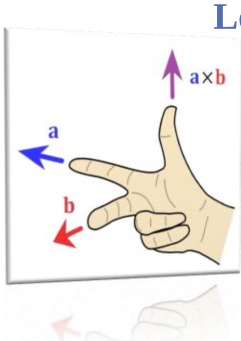
General Physics I

Mechanics: Principles & Applications

Lecture (3): Units, Dimensions and Vectors



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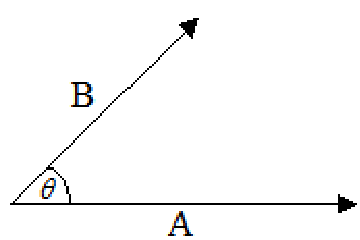
Product of vectors

There are two kinds of vector product the first one is called scalar product or dot product because the result of the product is a scalar quantity. The second is called vector product or cross product because the result is a vector perpendicular to the plane of the two vectors.



The scalar product

يعرف الضرب القياسي scalar product بالضرب النقطي dot product وتكون نتيجة الضرب القياسي لمتجهين كمية قياسية، وتكون هذه القيمة موجبة إذا كانت الزاوية المحصورة بين المتجهين بين 0 و 90 درجة وتكون النتيجة سالبة إذا كانت الزاوية المحصورة بين المتجهين بين 90 و 180 درجة وتساوي صفراً إذا كانت الزاوية 90.

A.B = +ve when $0 \le \theta < 90^o$

A.B = -ve when $90^o < \theta \le 180^o$

A.B = zero when $\theta = 90$

يعرف الضرب القياسي لمتجهين بحاصل ضرب مقدار المتجه الأول في مقدار المتجه الثاني في جيب تمام الزاوية المحصورة بينهما.

$$\vec{A}.\vec{B} = |A||B| \cos\theta$$

يمكن إيجاد قيمة الضرب القياسي لمتجهين باستخدام مركبات كل متجه كما يلي:

$$\vec{A} = A_x i + A_y j + A_z k$$

$$\vec{B} = B_x i + B_y j + B_z k$$

The scalar product is

$$\vec{A}.\vec{B} = (A_x i + A_y j + A_z k).(B_x i + B_y j + B_z k)$$

بضرب مركبات المتجه A في مركبات المتجه B ينتج التالي:

$$\vec{A}.\vec{B} = (A_x i.B_x i + A_x i.B_y j + A_x i.B_z k$$
$$+ A_y j.B_x i + A_y j.B_y j + A_y j.B_z k$$
$$+ A_z k.B_x i + A_z k.B_y j + A_z k.B_z k)$$

Therefore

$$\therefore \vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

The angle between the two vectors is

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|} = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|}$$

Example 1

Find the angle between the two vectors

$$\vec{A} = 2i + 3j + 4k \quad \vec{B} = i - 2j + 3k$$

Solution

$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|}$$

$$A_x B_x + A_y B_y + A_z B_z = (2)(1)+(3)(-2)+(4)(3)=8$$

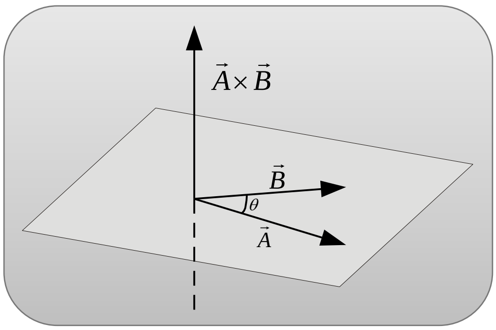
$$|A| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$|B| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$$

$$\cos\theta = \frac{8}{\sqrt{29}\sqrt{14}} = 0.397 \Rightarrow \theta = 66.6^o$$

The vector product

يعرف الضرب الاتجاهي *vector product* بـ *cross product* وتكون نتيجة الضرب الاتجاهي لمتجهين كمية متجهة. قيمة هذا المتجه $\vec{C} = \vec{A} \times \vec{B}$ واتجاهه عمودي على كل من المتجهين **A** و **B** وفي اتجاه دوران بريمة من المتجه **A** إلى المتجه **B** كما في الشكل التالي:



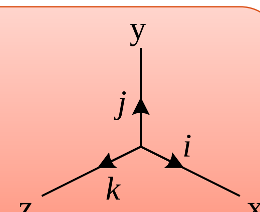
$$\vec{A} \times \vec{B} = AB \sin\theta$$

$$\vec{A} \times \vec{B} = AB \sin\theta$$

$$\vec{A} \times \vec{B} = (A_x i + A_y j + A_z k) \times (B_x i + B_y j + B_z k)$$

To evaluate this product we use the fact that the angle between the unit vectors *i, j , k* is 90º.

$$i \times i = 0 \qquad i \times j = k \qquad i \times k = -j$$
$$j \times j = 0 \qquad j \times k = i \qquad j \times i = -k$$
$$k \times k = 0 \qquad k \times i = j \qquad k \times j = -i$$



$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) i + (A_z B_x - A_x B_z) j + (A_x B_y - A_y B_x) k$$

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k}$$

$$= (a_2\times b_3 - a_3\times b_2)\mathbf{i} - (a_1\times b_3 - a_3\times b_1)\mathbf{j} + (a_1\times b_2 - a_2\times b_1)$$

If $\vec{C} = \vec{A}\times\vec{B}$, the components of C are given by

$$C_x = A_yB_z - A_zB_y$$

$$C_y = A_zB_x - A_xB_z$$

$$C_z = A_xB_y - A_yB_x$$

Example 2

If $\vec{C} = \vec{A}\times\vec{B}$ **, where** $\vec{A} = 3i - 4j$ **, and** $\vec{B} = -2i + 3k$ **, what is** $\vec{C}$ **?**

Solution

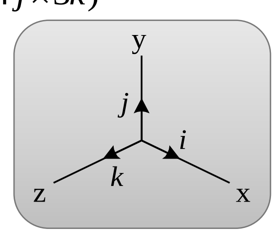
$$\vec{C} = \vec{A}\times\vec{B} = (3i - 4j)\times(-2i + 3k)$$

which, by distributive law, becomes

$$\vec{C} = -(3i\times 2i) + (3i\times 3k) + (4j\times 2i) - (4j\times 3k)$$

Using equation $\vec{A}\times\vec{B} = AB\sin\theta$ **to evaluate each term in the equation above we get**

$$\vec{C} = 0 - 9j - 8k - 12i = -12i - 9j - 8k$$



The vector C is perpendicular to both vectors A and B.

Example 3

Two vectors lying in the plane are given by the equations A = 2*i* + 3 *j* and B = -*i* + 2 *j* .

Find A×B and verify that A×B = -B× A

Solution

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ -1 & 2 & 0 \end{vmatrix} = i\begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ -1 & 0 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix}$$

$$\mathbf{A}\times\mathbf{B} = k\left(4-(-3)\right) = 7k$$

A = 2*i* + 3 *j* and **B** = -*i* + 2 *j*

$$\mathbf{B}\times\mathbf{A} = \begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ 2 & 3 & 0 \end{vmatrix} = i\begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix} - j\begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} + k\begin{vmatrix} -1 & 2 \\ 2 & 3 \end{vmatrix}$$

$$\mathbf{A}\times\mathbf{B} = k\left(-3-(4)\right) = -7k$$

So,

$$\mathbf{A}\times\mathbf{B} = \text{-}\mathbf{B}\times\mathbf{A}$$

Example 4

For the vectors *A* = -3*i* + 7 *j* - 4*k* and *B* = 6*i* -10 *j* + 9*k*, evaluate the expressions

$$\text{(a)}\ \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}||\mathbf{B}|}\right)\ \text{and}\ \text{(b)}\ \sin^{-1}\left(\frac{|\mathbf{A}\times\mathbf{B}|}{|\mathbf{A}||\mathbf{B}|}\right)$$

$$|\mathbf{A}| = \sqrt{9+49+16} = \sqrt{74}\quad \textit{and}\quad |\mathbf{B}| = \sqrt{36+100+81} = \sqrt{217}$$

$$|\mathbf{A}||\mathbf{B}| = \sqrt{74}\sqrt{217} = 126.720$$

$$\mathbf{A}\cdot\mathbf{B} = (-3i+7j-4k)\cdot(6i-10j+9k) = -18+(-70)+(-36) = -124$$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} i & j & k \\ -3 & 7 & -4 \\ 6 & -10 & 9 \end{vmatrix} = i(63-40) - j(-27+24) + k(30-42)$$

$$\mathbf{A}\times\mathbf{B} = 23i + 3j - 12k \qquad |\mathbf{A}\times\mathbf{B}| = \sqrt{23^2+3^2+12^2} = 26.115$$

$$\cos^{-1}\left(\frac{-124}{126.720}\right) = \cos^{-1}(-0.9785) \cong 168^\circ$$

$$\sin^{-1}\left(\frac{26.115}{126.720}\right) = \sin^{-1}(0.0.2061) \cong 12^\circ$$

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General Physics I

Mechanics: Principles & Applications

Lecture (4): Motion Kinematics

Displacement – Velocity – Acceleration

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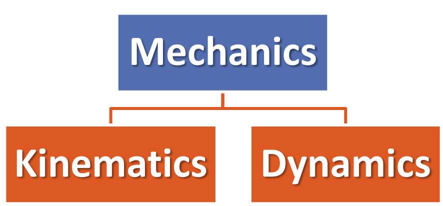
Contents of Unit 2: Mechanics

- The position vector and the displacement vector
- The average and Instantaneous velocity
- The average and Instantaneous acceleration
- One-dimensional
  - Free Fall
- Motion in two dimensions
  - Projectile motion
- Uniform Circular Motion
- Problems

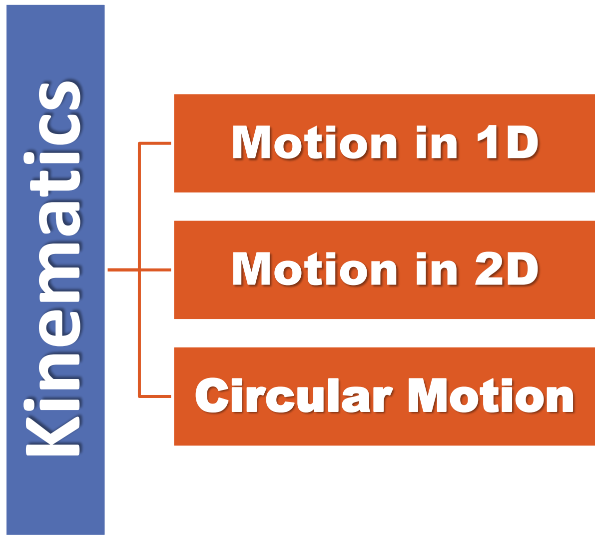
About Mechanics

علم الميكانيكا من العلوم الواسعة التي تهتم بدراسة حركة الأجسام ومسبباتها، ويتفرع من هذا العلم فروع أخرى مثل الكينماتيكا ***Kinematics*** والديناميكا ***Dynamics***.

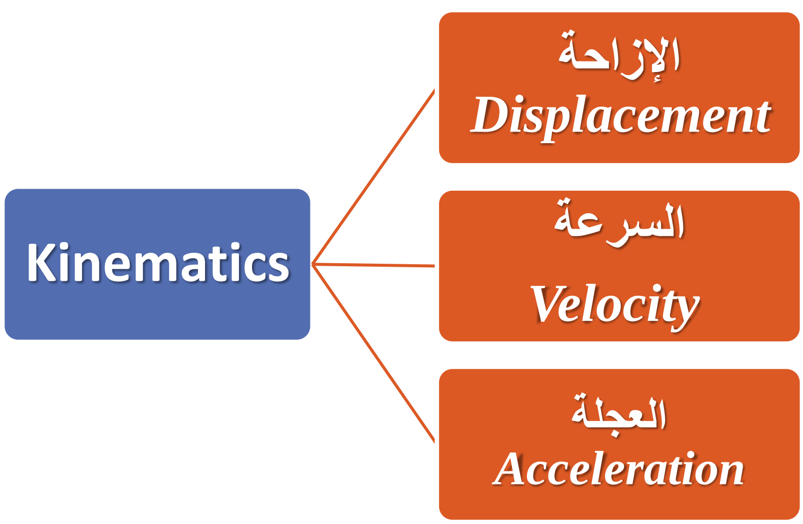


وعلم الكينماتيكا ***Kinematics*** يهتم بوصف حركة الأجسام دون النظر إلى مسبباتها، أما علم الديناميكا ***Dynamics*** فهو يدرس حركة الأجسام ومسبباتها مثل القوة والكتلة.

في البداية سنقوم بدراسة حركة الأجسام وعلاقتها بكل من الإحداثيات المكانية والزمنية. ثم بعد ذلك سوف ندرس الفرع الثاني وهو علم الديناميكا.



أساسيات دراسة علم وصف الحركة *Kinematics* للأجسام المادية



The Position Vector and the Displacement Vector

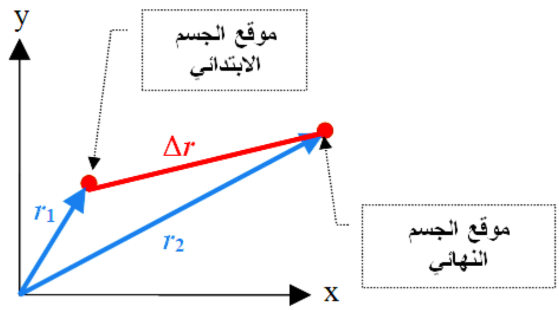
نحتاج هنا إلى اعتماد محاور إسناد لتحديد موضع الجسم المتحرك عند أزمنة مختلفة ومن المناسب اعتماد محاور الإسناد الكارتيزية أو ما سميت بـ *rectangular coordinate (x,y,z)*.

يمكن اعتبار متجه الموضع *Position vector* هو المتجه الواصل من مركز إسناد معين إلى مكان الجسم المراد تحديده.

$$\vec{r}_1 = x_1 i + y_1 j$$

$$\vec{r}_2 = x_2 i + y_2 j$$

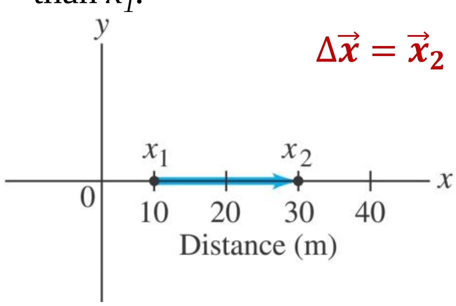
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

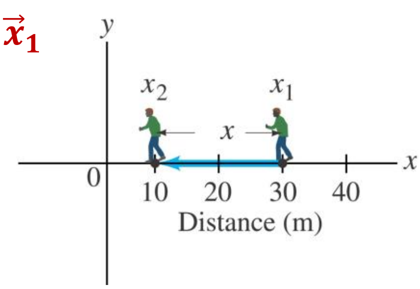


Δr **is the displacement vector which represent the change in the position vector.**

Δx is +ve if x_2 *is* greater than x_1; and –ve if x_2 *is* less than x_1.

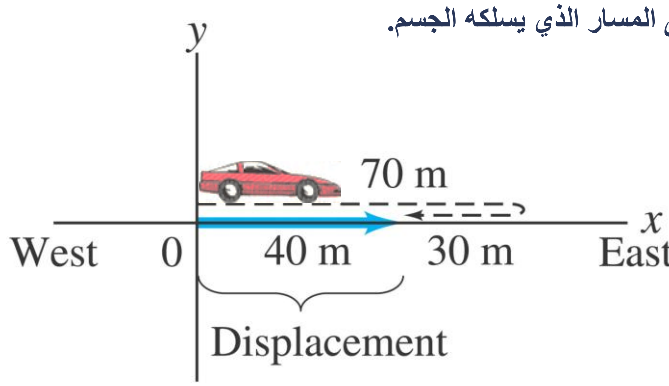
$$\Delta\vec{x} = \vec{x}_2 - \vec{x}_1$$





Displacement is positive. **Displacement is negative.**

لاحظ أن الإزاحة *displacement* تعتمد على المسافة بين نقطتي البداية والنهاية فقط ولا تعتمد على المسار الذي يسلكه الجسم.



Displacement has both a magnitude and a direction, it's a vector quantity, but **Distance** has a magnitude only, it's a scalar quantity.

Distance vs. Displacement

- **Assume a player moves from one end of the court to the other and back.**
- **Distance is twice the length of the court.**
- **Distance is always positive.**

- **Displacement is zero**

$$\Delta x = x_f - x_i = 0 \text{ since } x_f = x_i$$

Example 1

Write the position vector for a particle in the rectangular coordinate (x, y, z) for the points (5, -6, 0), (5, -4), and (-1, 3, 6).

Solution

For the point (5, -6, 0) the position vector is $\vec{r} = 5i - 6j$

For the point (5, -4) the position vector is $\vec{r} = 5i - 4j$

For the point (-1, 3, 6) the position vector is $\vec{r} = -i + 3j + 6k$

Example 2

Calculate the displacement vector for a particle moved from the point (4, 3, 2) to a point (8, 3, 6).

Solution

The position vector for the first point is $\vec{r}_1 = 4i + 3j + 2k$

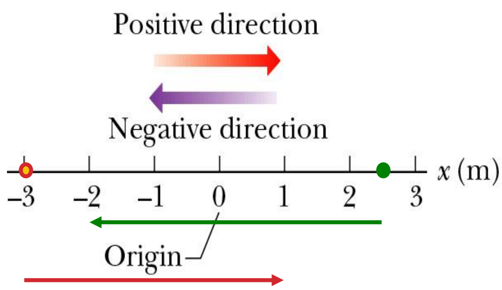
The position vector for the second point is $\vec{r}_2 = 8i + 3j + 6k$

The displacement vector

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\therefore \Delta\vec{r} = 4i + 4k$$

Example 3



$x_1\,(t_1)$ = + 2.5 m
$x_2\,(t_2)$ = - 2.0 m
Δx = -2.0 m - 2.5 m = -4.5 m

$x_1\,(t_1)$ = - 3.0 m
$x_2\,(t_2)$ = + 1.0 m
Δx = +1.0 m + 3.0 m = +4.0 m

Example 4

If the position of a particle is given as a function of time according to the equation

$$\vec{r}(t) = 3t^2 i + (3t - 2)j$$

where t in seconds. Find the displacement vector for t_1=1 and t_2=8

Solution

First we must find the position vector for the time t_1 and t_2

For t_1 =1s $\vec{r}_1\,(t_1) = 3i + j$

For t_2 =8s $\vec{r}_2\,(t_2) = 192i + 22j$

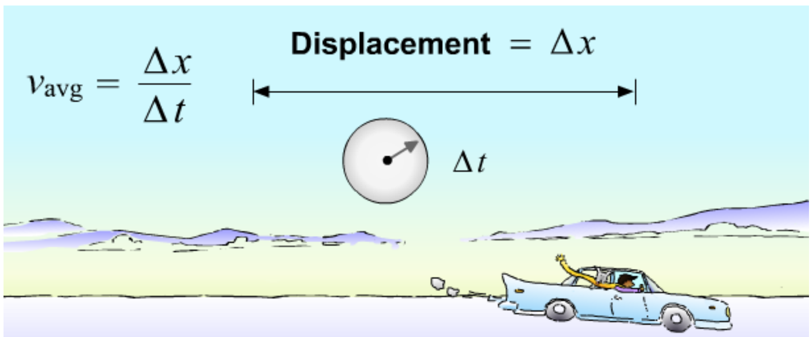
The displacement vector

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = 192i + 22j - 3i - j$$

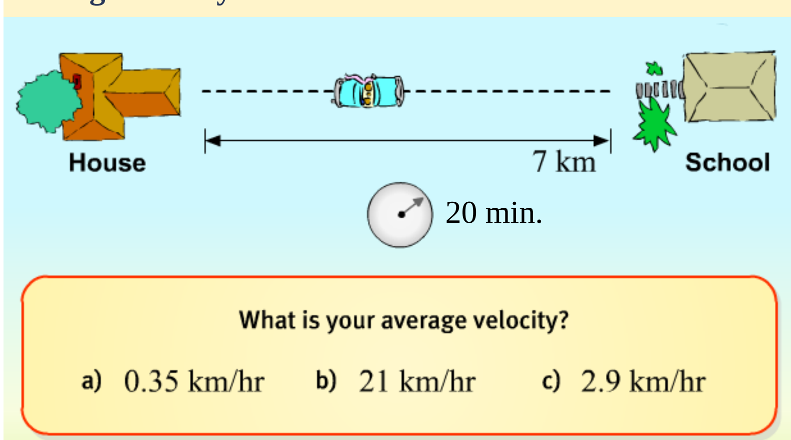
$$\Delta\vec{r} = 189i + 21j$$

The Average Velocity and Instantaneous Velocity

عند انتقال الجسم من موضع البداية عند الزمن t_1 إلى موضع النهاية عند الزمن t_2 فإن حاصل قسمة الإزاحة Δx على فرق الزمن t_2-t_1 يعرف بالسرعة ***Velocity*** وحيث أن الجسم يقطع المسافة بسرعات مختلفة فإن السرعة المحسوبة تسمى بمتوسط السرعة ***Average velocity***. ويمكن تعريف السرعة عند أية لحظة بالسرعة اللحظية ***Instantaneous velocity***.

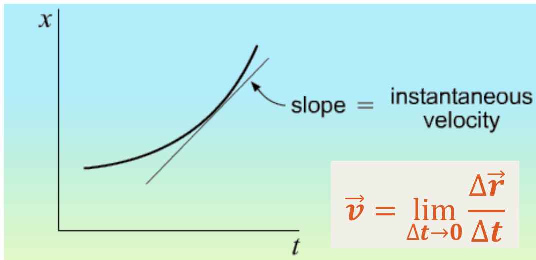


If it takes you 20 minutes to travel 7 Km, what is your average velocity?

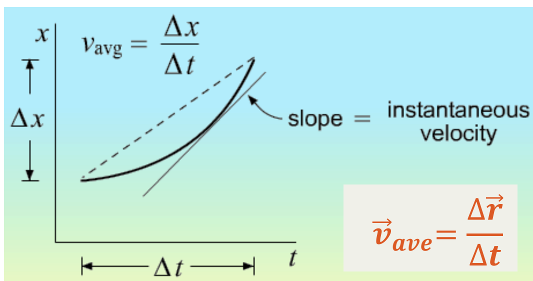


$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{7\ \text{km}}{20\ \text{min}} = \frac{7\ \text{km}}{0.33\ \text{hr}} = 21\ \text{km/hr}$$

The Average Velocity and Instantaneous Velocity



If you make a plot of displacement versus time, the slope of the curve gives the instantaneous velocity at any time.



While the average velocity is given over some finite time interval.

The *average velocity* of a particle is defined as the ratio of the displacement to the time interval.

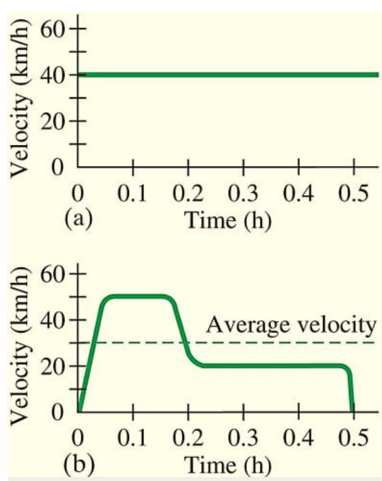
$$\vec{v}_{ave} = \frac{\Delta \vec{r}}{\Delta t}$$ *average velocity*

The *instantaneous velocity* of a particle is defined as the limit of the average velocity as the time interval approaches zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

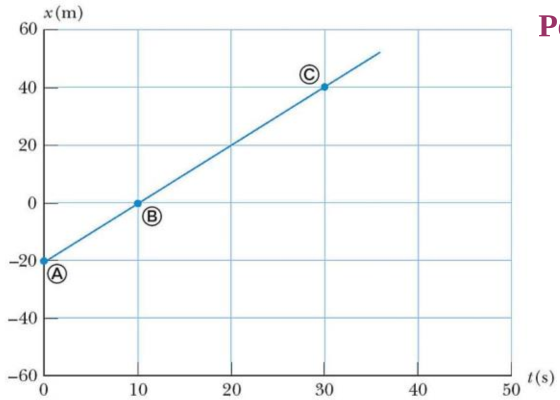
$$\therefore \vec{v} = \frac{d\vec{r}}{dt}$$ *instantaneous velocity*

The unit of the velocity is (m/s)



These graphs show (a) constant velocity and (b) varying velocity.

Position-Time Graph

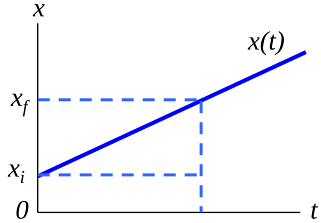


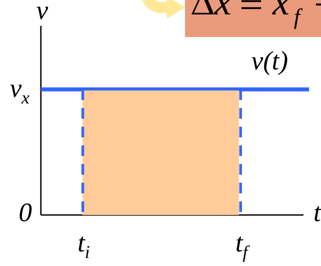
- ❑ Average velocity equals the slope of the line joining the initial and final positions. It is a vector quantity.
- ❑ An object moving with a constant velocity will have a graph that is a straight line.

Uniform Velocity

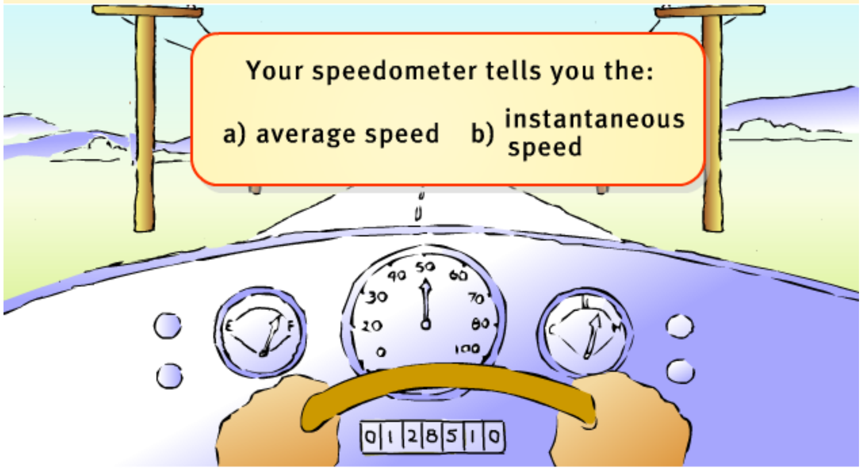
- ❑ Uniform velocity is constant velocity.
- ❑ The instantaneous velocities are always the same, all the instantaneous velocities will also equal the average velocity.
- ❑ Begin with $v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$ then $x_f = x_i + v_x \Delta t$

$$\Delta x = x_f - x_i = v_x \Delta t$$





As you are driving, the value given on your speedometer is which, the average speed or the instantaneous speed?



The Average Acceleration and Instantaneous Acceleration

عند انتقال الجسم من موضع البداية عند الزمن t_1 إلى موضع النهاية t_2 بسرعة ابتدائية v_1 وعند النهاية كانت السرعة v_2 فإن معدل تغير السرعة بالنسبة إلى الزمن يعرف باسم التسارع ***Acceleration*** أو متوسط التسارع ***Average Acceleration***، ويكون التسارع اللحظي ***Instantaneous acceleration*** هو حاصلة قسمة السرعة اللحظية على الزمن.

تنطلق السيارة من السكون لتصل إلى سرعة v_1، التغير في السرعة بالنسبة للزمن يسمى بالتسارع

The *average acceleration* of a particle is defined as the ratio of the change in the instantaneous velocity to the time interval.

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t}$$ *average acceleration*

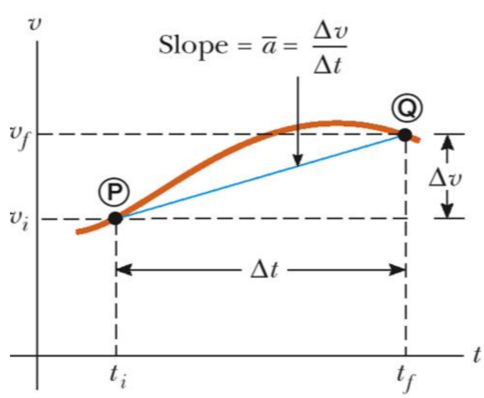
The *instantaneous acceleration* is defined as the limiting value of the ratio of the average velocity to the time interval as the time approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

$$\therefore \vec{a} = \frac{d\vec{v}}{dt}$$ *instantaneous acceleration*

$$\vec{a} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

The unit of the acceleration is (m/s^2)



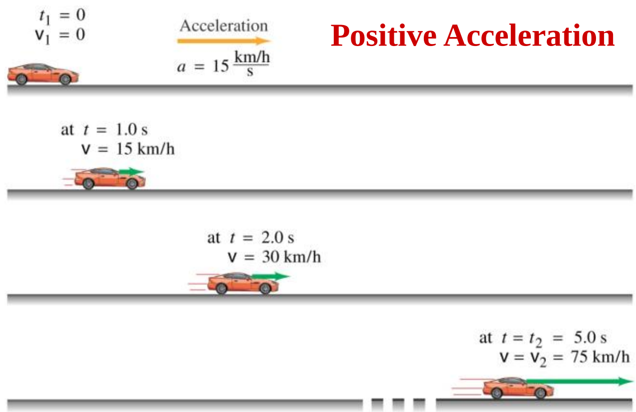
Velocity-Time Graph

- **Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph**
- **Velocity as a function of time**

$$v_f(t) = v_i + a_{avg}\,\Delta t \qquad \Delta v_{avg} = v_f - v_i = a_{avg}\,\Delta t$$

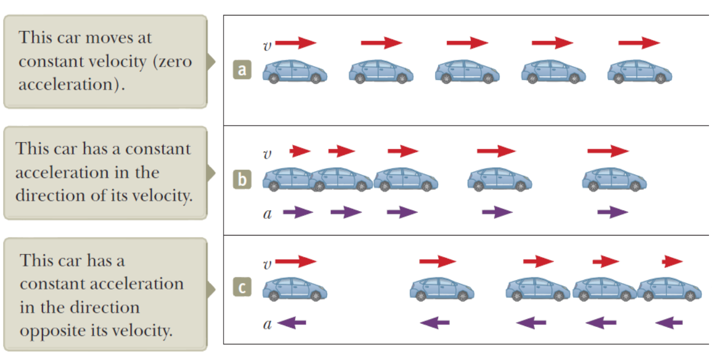
Positive Acceleration

لنفترض سيارة تبدأ الحركة من السكون أي $v_1 = 0$ عند زمن $t_1 = 0$. وبعد فترة زمنية قدرها $t_2 = 5s$ تصل السيارة إلى سرعة 75 km/h فإن متوسط التسارع للسيارة هو 15 km/h/s



يوضح الشكل أعلاه تأثير التسارع على زيادة سرعة السيارة بعد مرور 5 ثواني من انطلاقها حيث تكون السرعة بعد زمن قدره ثانية يساوي 15 km/h وبعد زمن ثانيتين تصل السرعة إلى 30 km/h وهكذا ....

Relation between velocity and acceleration



Zero acceleration
Constant velocity

Positive acceleration
Positive Velocity

Negative acceleration
Positive Velocity

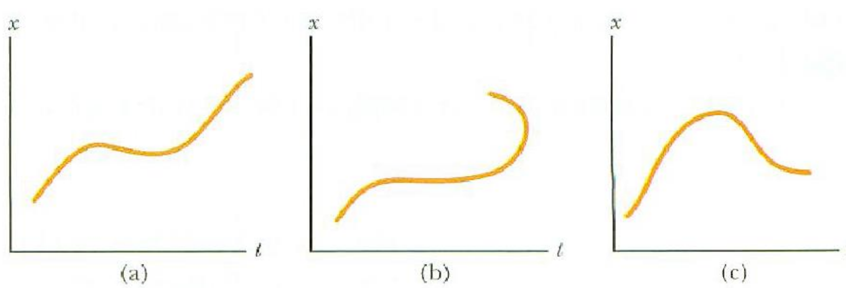
Three sets of strobe photographs of cars moving along a straight roadway in a single direction, from left to right. The time intervals between flashes of the stroboscope are equal in each part of the diagram.

Relation between velocity and acceleration

v_i	a	Motion
+	+	speeding up
−	−	speeding up
+	−	slowing down
−	+	slowing down
− or +	0	constant velocity
0	− or +	speeding up from rest
0	0	remaining at rest

Quick Quiz

The three graphs in the Figure below represent the position vs. time for objects moving along the x-axis. Which, if any, of these graphs is not physically possible?



Example 5

The coordinate of a particle moving along the x-axis depends on time according to the expression

$$x = 5t^2 - 2t^3$$

where x is in meters and t is in seconds.

1. Find the velocity and acceleration of the particle as a function of time.
2. Find the displacement during the first 2 seconds.
3. Find the velocity and acceleration of the particle after 2 seconds.

Solution

(a) The velocity and acceleration can be obtained as follow

$$v = \frac{dx}{dt} = 10t - 6t^2 \qquad x = 5t^2 - 2t^3$$

$$a = \frac{dv}{dt} = 10 - 12t$$

(b) using the equation $x = 5t^2 - 2t^3$ substitute for $t = 2$s

$x = 4$m

(c) using the result in part (a)

$v = -4$ m/s

$a = -14$ m/s^2

Example 6

A man swims the length of a 50m pool in 20s and makes the return trip to the starting position in 22s. Determine his average velocity in (a) the first half of the swim, (b) the second half of the swim, and (c) the round trip.

Solution

$$\text{(a) } v_1 = \frac{d}{t_1} = \frac{50}{20} = 2.5 \text{ m/s}$$

$$\text{(b) } v_2 = \frac{d}{t_2} = \frac{-50}{20} = -2.27 \text{ m/s}$$

(c) Since the displacement is zero for the round trip, $v_{ave} = 0$

Example 7

A car makes a 200km trip at an average speed of 40 km/h. A second car starting 1h later arrives at their mutual destination at the same time. What was the average speed of the second car?

Solution

$$t_1 = \frac{d}{v_1} = \frac{200}{40} = 5\text{h} \quad \text{for car 1}$$

$$t_2 = t_1 - 1 = 4\text{h} \quad \text{for car 2}$$

$$v_2 = \frac{d}{t_2} = \frac{200}{4} = 50\text{km/h}$$

Problems to be solved by yourself

1. A particle moves along the x-axis according to the equation $x=2t+3t^2$, where x is in m and t is in second. Calculate the instantaneous velocity and instantaneous acceleration at t=3s.
2. When struck by a club, a golf ball initially at rest acquires a speed of 31.0 m/s. If the ball is in contact with the club for 1.17 ms, what is the magnitude of the average acceleration of the ball?
3. A car makes a 200km trip at an average speed of 40km/h. A second car starting 1h later arrives at their mutual destination at the same time. What was the average speed of the second car?
4. A particle is moving with a velocity v_o=60m/s at t=0. Between t=0 and t=15s the velocity decreases uniformly to zero. What was the average acceleration during this 15s interval? What is the significance of the sign of your answer?

المحاضرة القادمة

One-dimensional motion with *constant acceleration*

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General Physics I

Mechanics: Principles & Applications

Lecture (5): Motion Kinematics
One-Dimensional Motion with Constant Acceleration

Dr. Hazem Falah Sakeek
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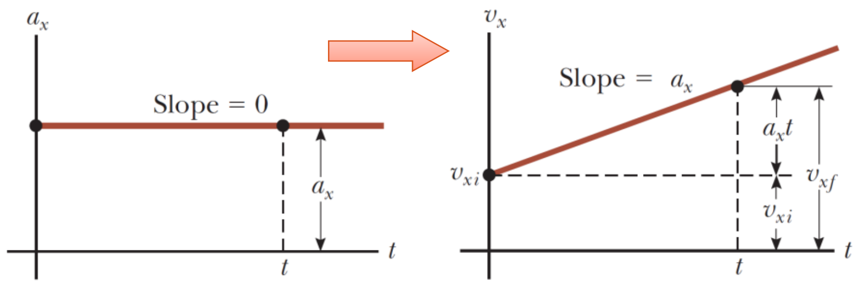
Contents of Unit 2: Mechanics

- The position vector and the displacement vector
- The average and Instantaneous velocity
- The average and Instantaneous acceleration
- One-dimensional
  - Free Fall
- Motion in two dimensions
  - Projectile motion
- Uniform Circular Motion
- Problems

One-dimensional Motion With *Constant Acceleration*

سندرس الآن الحركة في بعد واحد وذلك فقط عندما يكون فيها التسارع ثابتا constant acceleration. وفي هذه الحالة يكون التسارع اللحظي Instantaneous acceleration يساوى متوسط التسارع Average acceleration.

Instantaneous acceleration = Average acceleration



نتيجة لذلك فإن السرعة إما أن تتزايد أو تتناقص بمعدلات متساوية خلال الحركة. ويعبر عن ذلك رياضياً على النحو التالي:-

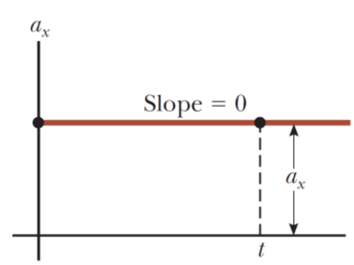
Kinematic Equations at Constant Acceleration

$$a = a_{ave} = \frac{v - v_o}{t - t_o}$$

Let $t_o = 0$ then the acceleration

$$a = \frac{v - v_o}{t}$$

$$v = v_0 + at$$



من هذه المعادلة يمكن إيجاد السرعة v عند أي زمن t إذا عرفنا السرعة الابتدائية v_o والتسارع الثابت a الذي يتحرك به الجسم. وإذا كان التسارع يساوي صفراً فإن السرعة لا تعتمد على الزمن، وهذا يعني أن السرعة النهائية تساوي السرعة الابتدائية. لاحظ أيضاً أن كل حد من حدود المعادلة السابقة له بعد سرعة (m/s).

Kinematic Equations at Constant Acceleration

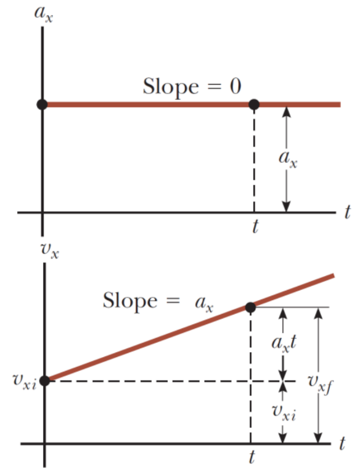
Since the velocity varies linearly (خطي) with time we can express the average velocity as

$$v_{ave} = \frac{v + v_\circ}{2}$$

To find the displacement Δx (x-x_o) as a function of time

$$\Delta x = v_{ave} \Delta t = \left(\frac{v + v_\circ}{2} \right) t$$

$$x = x_o + \frac{1}{2}(v + v_\circ)t$$



This equation provides the final position of the particle at time *t* in terms of the initial and final velocities.

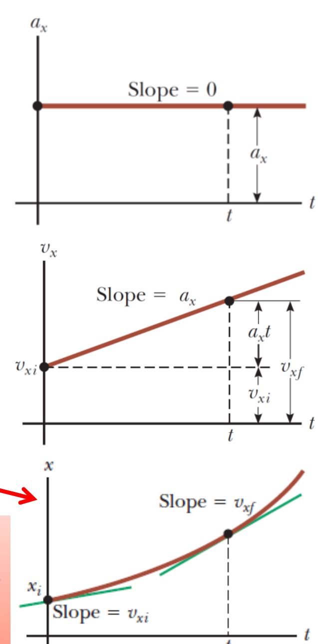
We can obtain another useful expression for the position of a particle under constant acceleration

$$x = x_o + \frac{1}{2}(v + v_\circ)t$$

$$v = v_0 + at$$

$$x = x_o + \frac{1}{2}(v_o + at + v_\circ)t$$

$$x = x_o + v_o t + \tfrac{1}{2} a t^2$$



This equation provides the final position of the particle at time *t* in terms of the initial position, the initial velocity, and the constant acceleration.

We can obtain an expression for the final velocity that does not contain time as a variable

$$x = x_o + \frac{1}{2}(v + v_o)t$$

$$v = v_0 + at \longrightarrow \therefore t = \frac{v - v_o}{a}$$

$$x = x_o + \frac{1}{2}(v + v_o)\frac{v - v_o}{a} \longrightarrow x = x_o + \frac{v^2 - v_o{}^2}{2a}$$

$$v^2 = v_0{}^2 + 2a(x - x_o)$$

This equation provides the final velocity in terms of the initial velocity, the constant acceleration, and the position of the particle

Kinematic Equations at Constant Acceleration

$v = v_0 + at$	**Velocity as a function of time**
$x = x_o + \frac{1}{2}(v + v_o)t$	**Position as a function of velocity and time**
$x = x_o + v_o t + \frac{1}{2}at^2$	**Position as a function of time**
$v^2 = v_0{}^2 + 2a(x - x_o)$	**Velocity as a function of position**

General Problem solving Strategy

1. Read Problem
2. Draw Diagram
3. Label physical quantities
4. Choose Equation (s)
5. Solve Equation (s)
6. Substitute known values
7. Chcck Answer

Example 1

A body moving with uniform acceleration has a velocity of 12cm/s when its *x* coordinate is 3cm. If its *x* coordinate 2s later is -5cm, what is the magnitude of its acceleration?

Solution

$$x = x_o + v_o t + \tfrac{1}{2} a t^2$$

$$-5 = 3 + 12\times 2 + 0.5\, a\, (2)^2$$

$$a = -16 \text{ cm/s}^2$$

Example 2

A car moving at constant speed of 30m/s suddenly stalls at the bottom of a hill. The car undergoes a constant acceleration of -2m/s^2 while ascending the hill.

1. Write equations for the position and the velocity as a function of time, taking x=0 at the bottom of the hill where v_o = 30m/s.

Solution

$$x = x_o + v_o t + \tfrac{1}{2} a t^2$$

$$x = 0 + 30\,t - t^2$$

$$x = 30\,t - t^2 \text{ m}$$

$$v = v_o + at$$

$$v = 30 - 2t \text{ m/s}$$

Example 2 continue

2. Determine the maximum distance traveled by the car up the hill after stalling.

Solution

x reaches a maximum when v = 0 then,

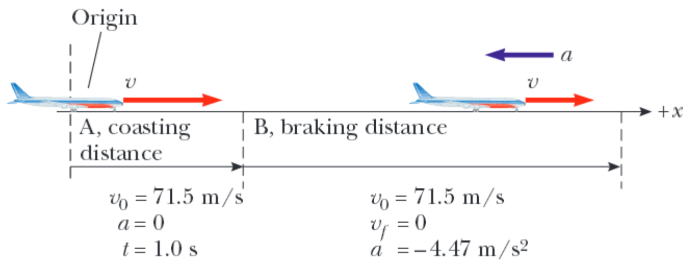
$$v = 30 - 2t = 0 \qquad \text{therefore} \qquad t = 15 \text{ s}$$

$$x_{max} = 30\,t - t^2$$

$$x = 30\,t - t^2 = 30\,(15) - (15)^2 = 225\text{m}$$

Example 3

A typical jetliner lands at a speed of 71.5m/s and decelerates at the rate of 4.47m/s^2. If the plane travels at a constant speed of 71.5m/s for 1.0 s after landing before applying the brakes, **what is the total displacement of the aircraft between touchdown on the runway and coming to rest?**



Solution

the displacement while the plane is coasting:

$$\Delta x_{\text{coasting}} = v_0 t + \tfrac{1}{2}at^2 = (71.5 \text{ m/s})(1.00 \text{ s}) + 0 = 71.5 \text{ m}$$

the displacement while the plane is braking:

$$v^2 = v_0^2 + 2a\,\Delta x_{\text{braking}}$$

$$\Delta x_{\text{braking}} = \frac{v^2 - v_0^2}{2a} = \frac{0 - (71.5 \text{ m/s})^2}{2.00(-4.47 \text{ m/s}^2)} = 572 \text{ m}$$

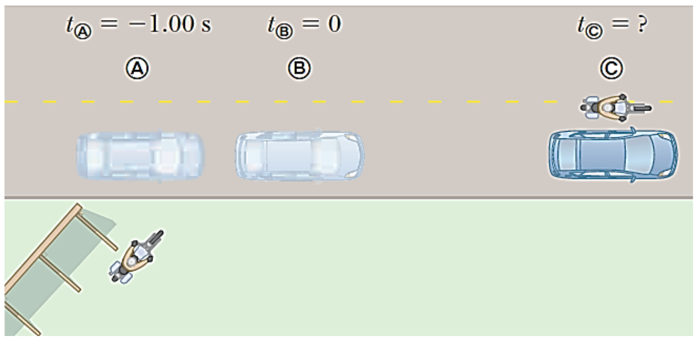
Sum the two results to find the total displacement:

$$\Delta x_{\text{coasting}} + \Delta x_{\text{braking}} = 72 \text{ m} + 572 \text{ m} = 644 \text{ m}$$

Exercise: A jet lands at 80.0 *m/s,* the pilot applying the brakes 2.0 s after landing. Find the acceleration needed to stop the jet within 5.00 × 10^2 m. Ans. a = - 9.41 m/s^2.

Example 4

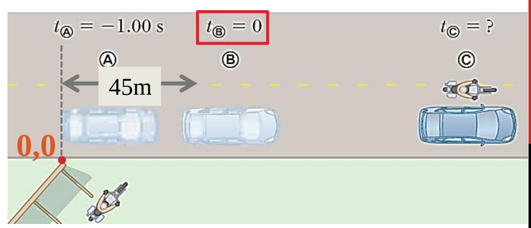
A car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.00 m/s^2. **(a) How long does it take her to overtake the car? (b) How fast is the trooper going at that time?**



Solution

(a) How long does it take her to overtake the car?

The position of each vehicle as a function of time.



$x_{car} = x_{Ⓑ} + v_{x\,car} t$ **Fro the car** (constant velocity)

$$x_f = x_i + v_{xi} t + \tfrac{1}{2} a_x t^2$$
$$x_{trooper} = 0 + (0)t + \tfrac{1}{2} a_x t^2 = \tfrac{1}{2} a_x t^2$$

Fro the trooper (constant acceleration)

$$x_{trooper} = x_{car}$$
$$\tfrac{1}{2} a_x t^2 = x_{Ⓑ} + v_{x\,car} t$$
$$\tfrac{1}{2} a_x t^2 - v_{x\,car} t - x_{Ⓑ} = 0$$

The positions of the car and trooper equal at position ©

$$t = \frac{v_{x\,car} \pm \sqrt{v_{x\,car}^2 + 2 a_x x_{Ⓑ}}}{a_x}$$

Solve the quadratic equation for the time at which the trooper catches the car

Solution continue

$$t = \frac{v_{x\,car} \pm \sqrt{v_{x\,car}^2 + 2a_x x_{Ⓑ}}}{a_x}$$

$$t = \frac{v_{x\,car}}{a_x} \pm \sqrt{\frac{v_{x\,car}^2}{a_x{}^2} + \frac{2x_{Ⓑ}}{a_x}}$$

Evaluate the solution, choosing the positive root because that is the only choice consistent with a time $t > 0$

$$t = \frac{45.0\ \text{m/s}}{3.00\ \text{m/s}^2} + \sqrt{\frac{(45.0\ \text{m/s})^2}{(3.00\ \text{m/s}^2)^2} + \frac{2(45.0\ \text{m})}{3.00\ \text{m/s}^2}} = \boxed{31.0\ \text{s}}$$

(b) How fast is the trooper going at that time?

$$v_{trooper} = v_o + a_{trooper}t = 0 + (3.0m/s^2)(31.0\ s)$$
$$= 93\ m/s$$

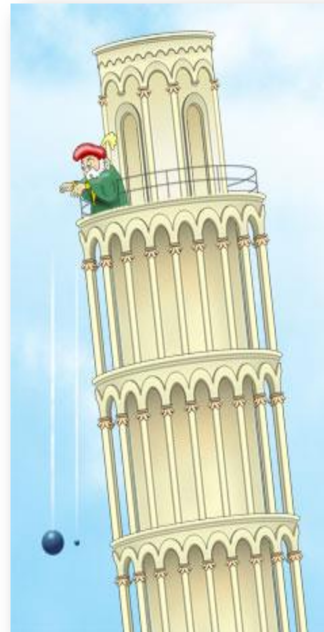
لماذا لم نقم باختيار t=0 عندما تجاوزت السيارة الموتورسيكل؟ اذا قمنا بذلك فاننا لا نستطيع استخدام معادلات الحركة عند ثبات التسارع للموتورسيكل لان تسارعه كان يساوي صفر خلال الثانية الأولى وبعد ذلك اصبح التسارع $3\ m/s^2$. وبالتالي باعتبار الزمن t=0 عندما بدأ الموتورسيكل الحركة فاننا نستطيع استخدام معادلات الحركة عند ثبات العجلة.

Problems to be solved by yourself

1. A car traveling initially at a speed of 60m/s is accelerated uniformaly to a speed 85m/s in 12s. How far does the car travel during the 12s interval?
2. A body moving with uniform acceleration has a velocity of 12cm/s when its coordinate is 3cm. If its x coordinate 2s later is -5cm, what is the magnitude of its acceleration?
3. The initial speed of a body is 5.2m/s. What is its speed after 2.5s if it (a) accelerates uniformaly at $3m/s^2$ and (b) accelerates uniformaly at $-3m/s^2$.

المحاضرة القادمة

Free Fall Acceleration



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General Physics I

Mechanics: Principles & Applications

Lecture (6): Motion Kinematics

One-Dimensional Motion
Free Fall Acceleration

Dr. Hazem Falah Sakeek
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Contents of Unit 2: Mechanics

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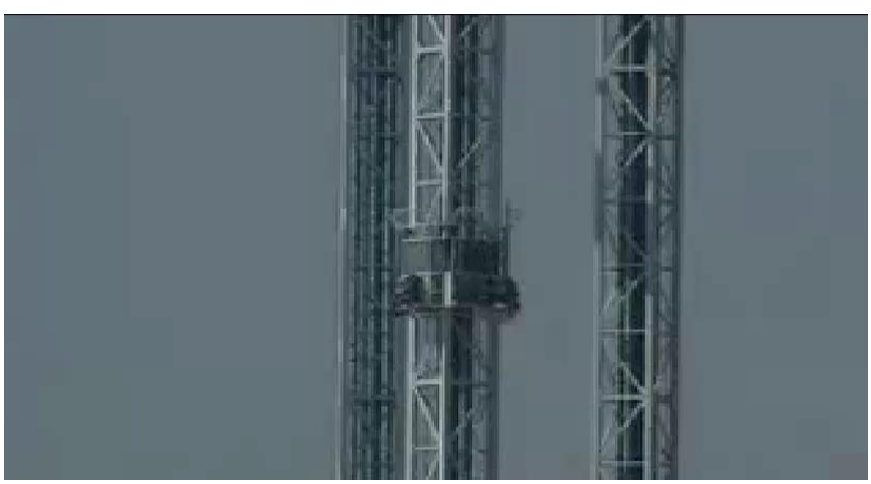
Falling Objects

Near the surface of the Earth, all objects experience approximately the same acceleration due to gravity.


(a) (b)

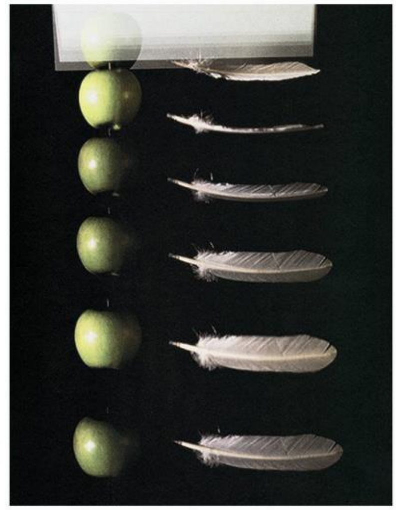
In the absence of air resistance, all objects fall with the same acceleration, although this may be hard to tell by testing in an environment where there is air resistance.

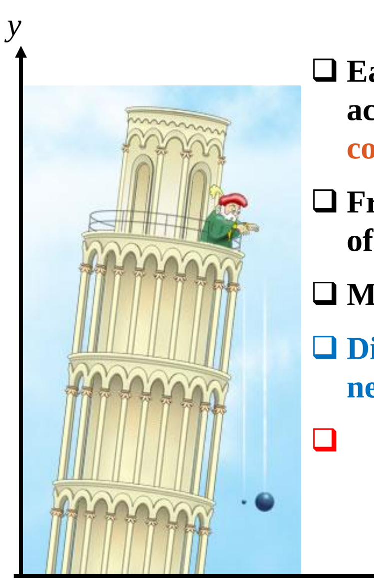


Free Fall Acceleration

One important example of constant acceleration is the "free fall" of an object under the influence of the Earth's gravity. The picture shows an apple and a feather falling in vacuum with identical motions.

The magnitude of this acceleration, designated as *g*, has the approximate value of $a = g = 9.81$ m/s^2





- Earth gravity provides a constant acceleration. Most important case of constant acceleration.
- Free-fall acceleration is independent of mass.
- Magnitude: $|a| = g = 9.8$ m/s^2
- Direction: always downward, so a_g is negative if define "up" as positive,
- $a = -g = -9.8$ m/s^2

Free Fall Kinematics Equations

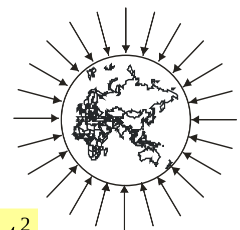
حيث أن عجلة الجاذبية الأرضية ثابتة نسبياً على ارتفاعات محدودة من سطح الأرض واتجاهها دائما في اتجاه مركز الأرض، وبالتالي يمكن استخدام المعادلات الأربع السابقة مع تغيير الرمز **x** بالرمز **y** وكذلك التعويض عن العجلة **a** بعجلة الجاذبية الأرضية بإشارة سالبة **-g** وذلك لأن عجلة الجاذبة الأرضية دائماً في اتجاه مركز الأرض وهذا يعبر عنه من خلال المحور **y** السالب.

$v = v_0 + at \quad \longrightarrow \quad v = v_0 - gt$

$x = x_o + \frac{1}{2}(v + v_o)t \quad \longrightarrow \quad y = y_o + \frac{1}{2}(v + v_o)t$

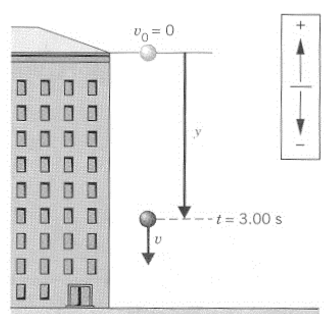
$x = x_o + v_o t + \frac{1}{2}at^2 \quad \longrightarrow \quad y = y_o + v_o t - \frac{1}{2}gt^2$

$v^2 = v_0{}^2 + 2a(x - x_o) \quad \longrightarrow \quad v^2 = v_0{}^2 - 2g(y - y_o)$



Example 1

A stone is dropped from rest from the top of a building, as shown in the Figure. After 3s of free fall, what is the displacement *y* of the stone?



Solution

From equation

$$y = y_o + v_o t - \tfrac{1}{2}gt^2$$

$$y = 0 + 0 - \tfrac{1}{2} \times 9.8 \times (3)^2$$

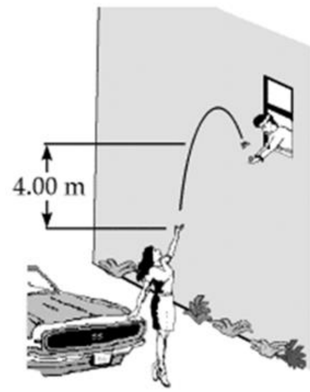
$$y = -44.1m$$

Example 2

A student throws a set of keys vertically upward to another student in a window 4m above as shown in the Figure. The keys are caught 1.5s later by the student.

(a) With what initial velocity were the keys thrown?

(b) What was the velocity of the keys just before they were caught?



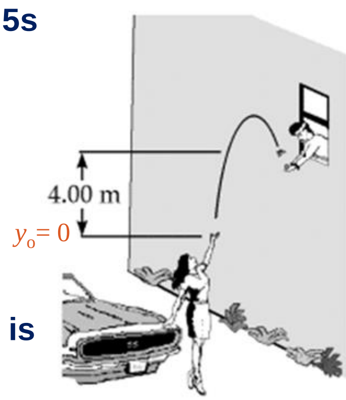
Solution

(a) Let $y_o = 0$ and y = 4m at t = 1.5s then we find

$$y = y_o + v_o t - \tfrac{1}{2} g t^2$$

$4 = 0 + 1.5\, v_o - 4.9\,(1.5)^2$

$v_o = 10$ m/s



(b) The velocity at any time $t > 0$ is given by

$$v = v_0 - gt$$

$v = 10 - 9.8\,(1.5) = -4.68$ m/s

Example 3

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The stone is launched 50.0 m above the ground, and the stone just misses the edge of the roof on its way down as shown in the Figure.

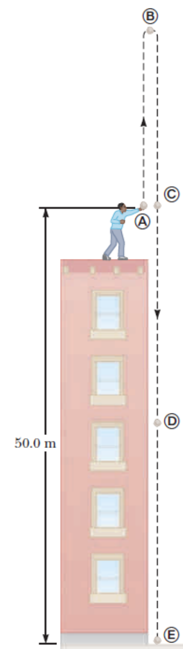
(A) Using $t_A = 0$ as the time the stone leaves the thrower's hand at position A, determine the time at which the stone reaches its maximum height.

(B) Find the maximum height of the stone.

(C) Determine the velocity of the stone when it returns to the height from which it was thrown.

(D) Find the velocity and position of the stone at $t = 5.00$ s.

(E) Find the time required for the stone to reach the ground



Solution

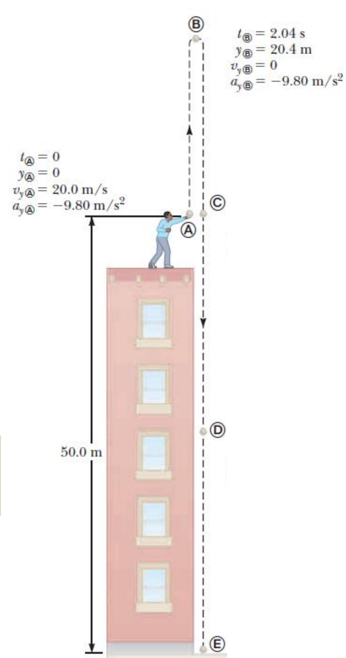
(A) determine the time at which the stone reaches its maximum height.

$$v = v_0 - gt$$

لاحظ ان السرعة عند اقصى
ارتفاع تساوي صفر

$$\therefore t = \frac{v - v_0}{-g}$$

$$t = t_{Ⓑ} = \frac{0 - 20.0\ \text{m/s}}{-9.80\ \text{m/s}^2} = 2.04\ \text{s}$$



Solution

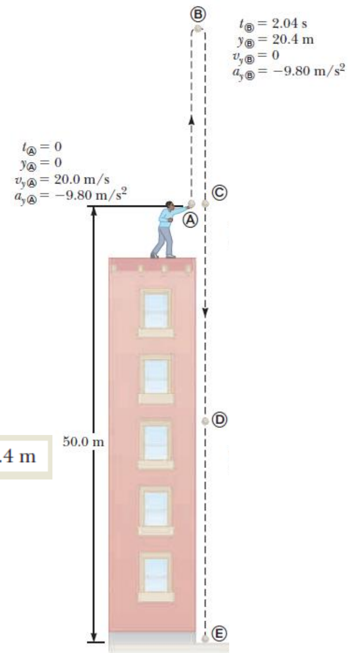
(B) Find the maximum height of the stone.

نعمل الان الزمن المستغرق ليصل الجسم إلى اقصى ارتفاع وبالتالي نستطيع حساب اقصى مسافة من خلال المعادلة:

$$y = y_o + v_o t - \tfrac{1}{2} g t^2$$

$$y_{max} = y_{Ⓑ} = y_{Ⓐ} + v_{xⒶ} t + \tfrac{1}{2} a_y t^2$$

$$y_{Ⓑ} = 0 + (20.0\ \text{m/s})(2.04\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(2.04\ \text{s})^2 = 20.4\ \text{m}$$



Solution

(C) Determine the velocity of the stone when it returns to the height from which it was thrown.

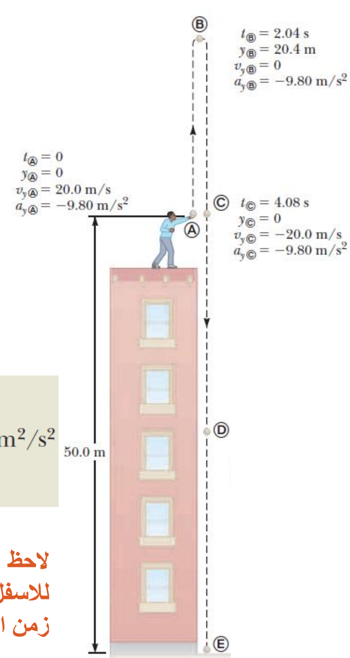
$$v^2 = v_0^{\ 2} - 2g(y - y_o)$$

$$v_{yⒸ}^{\ 2} = v_{yⒶ}^{\ 2} + 2a_y(y_Ⓒ - y_Ⓐ)$$

$$v_{yⒸ}^{\ 2} = (20.0\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(0 - 0) = 400\ \text{m}^2/\text{s}^2$$

$$v_{yⒸ} = -20.0\ \text{m/s}$$

لاحظ ان السرعة هنا تساوي قيمة السرعة الابتدائية واتجاهها للاسفل وذلك لان الحركة تمت في عجلة الجاذبية الأرضية وبالتالي زمن الصعود يساوي زمن الهبوط.



Solution

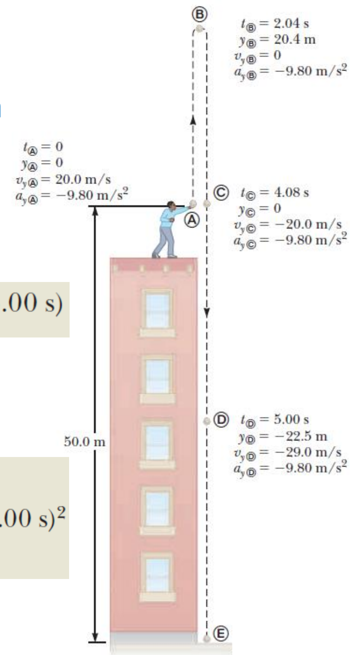
(D) Find the velocity and position of the stone at t = 5.00 s.

$$v = v_0 - gt$$

$$v_{y\text{D}} = v_{y\text{A}} + a_y t = 20.0\text{ m/s} + (-9.80\text{ m/s}^2)(5.00\text{ s})$$

v_{yD}=-29.0 m/s

$$y_{\text{D}} = y_{\text{A}} + v_{y\text{A}} t + \tfrac{1}{2} a_y t^2$$
$$= 0 + (20.0\text{ m/s})(5.00\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(5.00\text{ s})^2$$
$$= -22.5\text{ m}$$



Solution

(E) Find the time required for the stone to reach the ground

$$y = y_o + v_o t - \tfrac{1}{2} g t^2$$

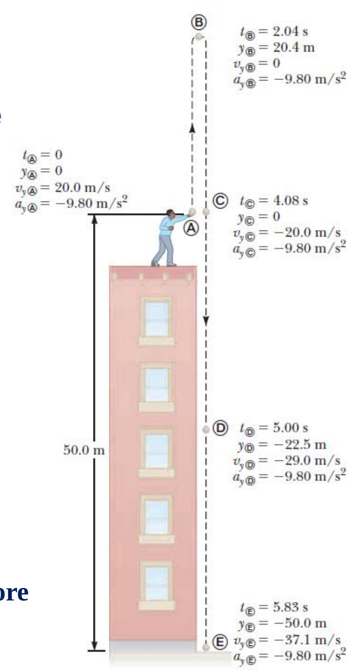
بالتعويض عن y=-50m وعن y_o=0

$$-50.0\text{ m} = (20.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$$

Apply the quadratic formula and take the positive root:

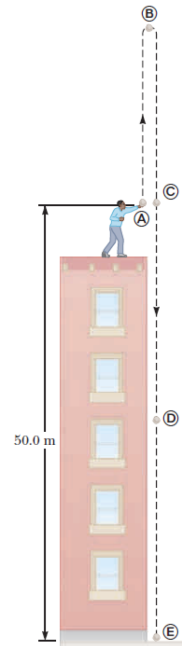
$$t = 5.83\text{ s}$$

Exercise: Find the velocity of the stone just before it hit the ground



Example 4

Use symbolic manipulation to find (a) the maximum time t_{max} it takes the ball to reach its maximum height and (b) an expression for the maximum height that doesn't depend on time. Answers should be expressed in terms of the quantities v_o, g and y_o only.



(a) the maximum time t_{max}

$$v = v_0 - gt$$

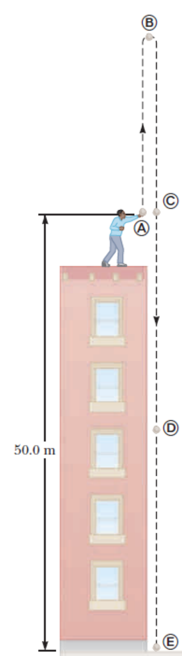
$$v - v_0 = -gt \quad \longrightarrow \quad \frac{v - v_0}{-g} = t$$

نعوض عن v=0 عند اقصى ارتفاع نحصل على:

$$\frac{0 - v_0}{-g} = t_{max} \quad \longrightarrow \quad \therefore t_{max} = \frac{v_0}{g}$$

وهي المعادلة التي تعطي زمن الوصول إلى اقصى ارتفاع

(b) an expression for the maximum height



$$y = y_o + v_o t - \tfrac{1}{2} g t^2$$

بالتعويض عن t_{max} عند اقصى ارتفاع y_{max} من المعادلة

$$\therefore t_{max} = \frac{v_0}{g}$$

نحصل على ما يلي:

$$y_{max} = y_o + v_o\left(\frac{v_o}{g}\right) - \tfrac{1}{2} g \left(\frac{v_o}{g}\right)^2$$

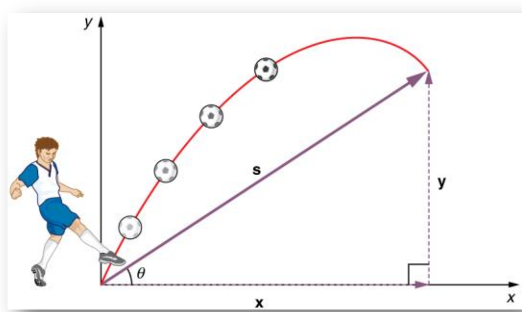
$$y_{max} = y_o + \left(\frac{v_o^{\,2}}{g}\right) - \tfrac{1}{2}\left(\frac{v_o^{\,2}}{g}\right)$$

$$\therefore y_{max} = y_o + \frac{v_o^{\,2}}{2g}$$

وهي المعادلة التي تعطي اقصى ارتفاع

Problems to be solved by yourself

1. A hot air balloon is traveling vertically upward at a constant speed of 5m/s. When it is 21m above the ground, a package is released from the balloon. (a) How long after being released is the package in the air? (b) What is the velocity of the package just before impact with the ground? (c) Repeat (a) and (b) for the case of the balloon descending at 5m/s.
2. A ball thrown vertically upward is caught by the thrower after 20s. Find (a) the initial velocity of the ball and (b) the maximum height it reaches.
3. A stone falls from rest from the top of a high cliff. A second stone is thrown downward from the same height 2s later with an initial speed of 30m/s. If both stones hit the ground below simultaneously, how high is the cliff?



المحاضرة القادمة

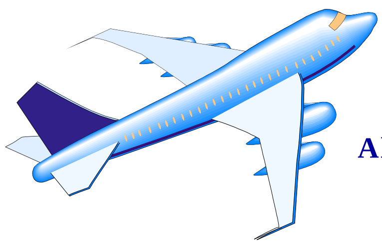
Two-dimensional motion with *constant acceleration*

Physics Academy
www.physicsacademy.org

General Physics I

Mechanics: Principles & Applications

Lecture (7): Motion Kinematics
Two-Dimensional Motion



Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Contents of Unit 2: Mechanics

- The position vector and the displacement vector
- The average and Instantaneous velocity
- The average and Instantaneous acceleration
- One-dimensional Motion with constant acceleration
  - Free Fall
- Motion in two dimensions
  - Projectile motion
- Uniform Circular Motion
- Problems

The Kinematics of a Particle Moving in Two Dimensions

Motion in Two Dimensions

درسنا في الفصل السابق الحركة في بعد واحد أي عندما يتحرك الجسم في خط مستقيم على محور x أو أن يسقط الجسم سقوطاً حراً في محور y، سندرس الآن حركة جسم في بعدين أي في كل من x,y مثل حركة المقذوفات حيث يكون للإزاحة والسرعة مركبتان في اتجاه المحور x والمحور y.

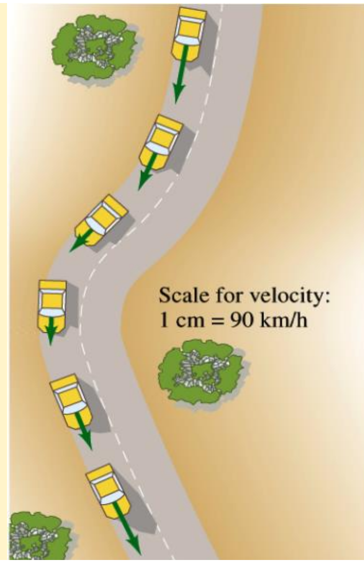
Motion in two dimensions like the motion of projectiles and satellites and the motion of charged particles in electric fields. Here we shall treat the motion in plane with constant acceleration and uniform circular motion.

Motion in Two Dimension

In previous lectures, we found that the motion of a particle along a straight line such as the *x* axis is completely known if its position is known as a function of time. Let us now extend this idea to two-dimensional motion of a particle in the *xy* plane.

In order to study motion in two dimensions, we need to use the concepts of vectors.

Position, velocity, and acceleration in two dimensions are determined by not only specifying their magnitude, but also their direction.



The Displacement Vector in 2D

Assume that the magnitude and direction of the acceleration remain unchanged during the motion.

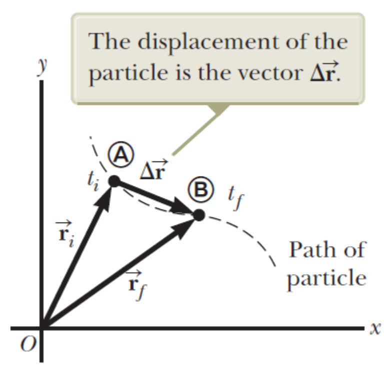
The position vector for a particle moving in two dimensions (xy plane) can be written as

$$\vec{r} = x_i + y_j$$

where x, y, and r change with time as the particle moves

The displacement vector $\Delta\vec{r}$ for a particle

$$\Delta\vec{r} = r_f - r_i$$



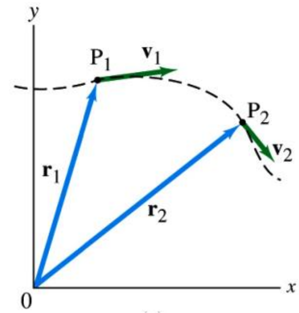
يتحرك جسم على المسار المنحني في البعدين *x,y*. يحدد المتجه r_1 موضع الجسم عند الزمن t_1 ويحدد المتجه r_2 موضعه عند الزمن t_2 وتكون الازاحة ممثلة بالمتجه $\Delta\vec{r}$

The Velocity Vector in 2D

The direction of the instantaneous velocity $\vec{v}$ of a particle is tangent to the path at the particle's position.

The velocity of the particle as a function of time is given by

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}i + \frac{dy}{dt}j \Longrightarrow \vec{v} = v_x i + v_y j$$



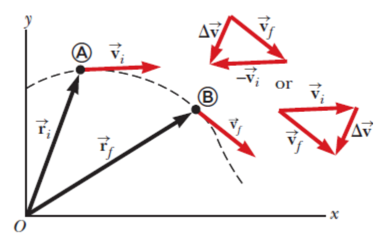
We define the **average velocity** $\vec{v}_{ave}$ of a particle during the time interval Δt as the displacement of the particle divided by the time interval:

$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t}$$

Because displacement is a vector quantity and the time interval is a positive scalar quantity, we conclude that the average velocity is a vector quantity directed along $\Delta\vec{r}$

The Acceleration Vector in 2D

As a particle moves from one point to another along some path, its instantaneous velocity vector changes from $\vec{v}_i$ at time t_i to $\vec{v}_f$ at time t_f . Knowing the velocity at these points allows us to determine the average acceleration of the particle.



The average acceleration $\vec{a}_{ave}$ of a particle is defined as the change in its instantaneous velocity vector $\Delta\vec{v}$ divided by the time interval Δt during which that change occurs:

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

The acceleration of the particle as a function of time is given by

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}i + \frac{dv_y}{dt}j \Longrightarrow \vec{a} = a_x i + a_y j$$

Two-dimensional kinematics is similar to one-dimensional kinematics, but we must now use full vector notation rather than positive and negative signs to indicate the direction of motion.

One Dimension

Position: ***x***

Displacement: ***Δx***

Velocity: **displacement per unit time. Sign is equal to the sign of the displacement *Δ x***

Acceleration: **change in velocity *Δ v* per unit time. Sign is equal to the sign of the velocity difference *Δ v*.**

Two Dimensions

Position vector: ***r***

Displacement vector: ***Δ r***

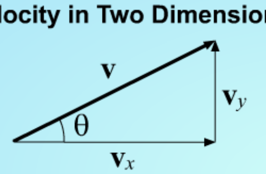
Velocity vector: **change in the position vector per unit time. The direction is equal to the direction of the displacement vector *Δ r*.**

Acceleration vector: **change in the velocity vector per unit time. The direction is equal to the direction of the velocity difference vector *Δ v*.**

Motion in Two Dimensions

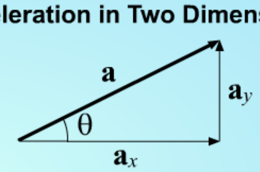
The motion in two dimensions can be modeled as two *independent* motions in each of the two perpendicular directions associated with the *x* and *y* axes. That is, any influence in the *y* direction does not affect the motion in the *x* direction and vice versa.

Velocity in Two Dimensions



$$v_x = v \cos\theta$$
$$v_y = v \sin\theta$$
$$|v| = \sqrt{v_x^2 + v_y^2}$$
$$\tan\theta = \frac{v_y}{v_x}$$

Acceleration in Two Dimensions



$$a_x = a \cos\theta$$
$$a_y = a \sin\theta$$
$$|a| = \sqrt{a_x^2 + a_y^2}$$
$$\tan\theta = \frac{a_y}{a_x}$$

Kinematics Equations with Constant Acceleration

When the acceleration is constant then we can substitute

$$v_x = v_{xo} + a_x t \qquad v_y = v_{yo} + a_y t$$

In

$$\vec{v} = v_x i + v_y j$$

We get

$$v = (v_{xo} + a_x t)i + (v_{yo} + a_y t)j$$

$$v = (v_{xo} i + v_{yo} j) + (a_x i + a_y j)t$$

then

$$v = v_o + at$$

من المعادلة نستنتج أن سرعة جسم عند زمن محدد *t* يساوى الجمع الاتجاهى للسرعة الابتدائية والسرعة الناتجة من العجلة المنتظمة.

Kinematics Equations with Constant Acceleration

Since our particle moves in two dimension *x* and *y* with constant acceleration then

$$x = x_o + v_{xo} t + \tfrac{1}{2} a_x t^2 \qquad y = y_o + v_{yo} t + \tfrac{1}{2} a_y t^2$$

but

$$r = x_i + y_j$$

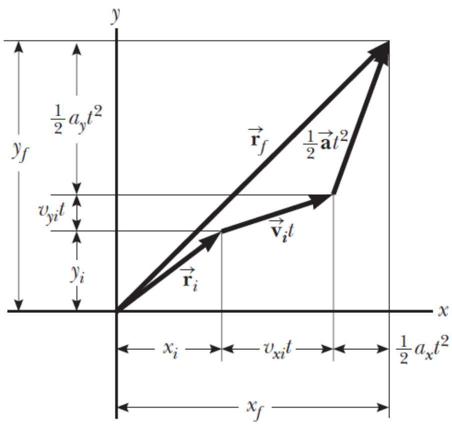
$$r = \left(x_o + v_{xo} t + \frac{1}{2} a_x t^2\right) i + \left(y_o + v_{yo} t + \frac{1}{2} a_y t^2\right) j$$

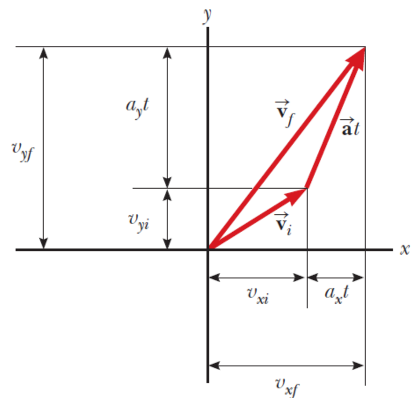
$$r = (x_o i + y_o j) + (v_{xo} i + v_{yo} j)t + \frac{1}{2}(a_x i + a_y j)t^2$$

$$r = r_o + v_o t + \frac{1}{2} a t^2$$

من المعادلة نستنتج أن متجه الإزاحة $r\text{-}r_o$ هو عبارة عن الجمع الإتجاهى لمتجه الإزاحة الناتج عن السرعة الابتدائية $v_o t$ والإزاحة الناتجة عن العجلة المنتظمة $\frac{1}{2} at^2$.

Vector representations and components of the velocity and the position of a particle moving with a constant acceleration $\vec{a}$.





$$\boldsymbol{r} = \boldsymbol{r_o} + \boldsymbol{v_o}\boldsymbol{t} + \frac{1}{2}\boldsymbol{a}\boldsymbol{t}^2$$

$$\boldsymbol{v} = \boldsymbol{v_o} + \boldsymbol{a}\boldsymbol{t}$$

Kinematics Equations with Constant Acceleration

One Dimension

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad v = v_0 + at$$

Two Dimensions

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 \qquad \mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \qquad v_x = v_{0x} + a_x t$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \qquad v_y = v_{0y} + a_y t$$

Quiz

[1] Which of the following objects can't be accelerating?

(a) An object moving with a constant speed;

(b) An object moving with a constant velocity;

(c) An object moving along a curve.

[2] Consider the following controls in an automobile: gas pedal, brake, steering wheel. The controls in this list that cause an acceleration of the car are

(a) all three controls,

(b) the gas pedal and the brake,

(c) only the brake,

(d) only the gas pedal.

Example 1

A ball is projected horizontally with a velocity v_o of magnitude 5m/s. Find its position and velocity after 0.25s

Solution

The initial angle is 0. The initial vertical velocity component is therefore 0.

The horizontal velocity component equals the initial velocity and is constant.

$$x = v_o t = 5 \times 0.25 = 1.25 \text{ m}$$

$$y = -1/2\ g\ t^2 = -0.306 \text{ m}$$

The distance of the projectile is given by $r = \sqrt{x^2 + y^2} = 1.29m$

The component of velocity are

$$v_x = v_o = 5 \text{ m/s}$$

$$v_y = -g\ t = -2.45 \text{ m/s}^2$$

The resultant velocity is given by $v = \sqrt{v_x^2 + v_y^2} = 5.57 m/s$

The angle θ is given by $\theta = tan^{-1}\left(\frac{-2.45}{5}\right) = -26.1^o$

Example 2

A particle moves in the xy plane, starting from the origin at t = 0 with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle experiences an acceleration in the x direction, given by $a_x = 4.0\ m/s^2$.

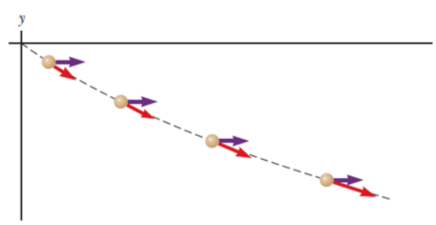
(A) Determine the total velocity vector at any time.

(B) Calculate the velocity and speed of the particle at t = 5.0 s and the angle the velocity vector makes with the x axis.

Solution (A)

نلاحظ من مركبات السرعة الابتدائية ان الجسم يتحرك في بعدين بحيث ان المركبة X للسرعة تتأثر بعجلة ثابتة ولذلك فهي تزداد مع الزمن، في حين ان المركبة Y تبقى ثابتة مع الزمن لعدم وجود عجلة في هذا الاتجاه.

نلاحظ من الشكل الموضح ان متجه السرعة يزداد مع الزمن.



$v_{xi} = 20$ m/s, $v_{yi} = -15$ m/s, $a_x = 4.0\ m/s^2$, and $a_y = 0$.

$$v = v_o + at$$

$$v = (v_{xo} + a_x t)i + (v_{yo} + a_y t)j$$

$$v = (20 + 4.0t)i + (-15 + 0t)j$$

$$v = (20 + 4.0t)i - 15j$$

Notice that the x component of velocity increases in time while the y component remains constant

Solution (B)

لايجاد متجه سرعة الجسم بعد مرور 5 ثواني نعوض في المعادلة:

$$\boldsymbol{v} = (\mathbf{20} + \mathbf{4.0}\boldsymbol{t})\boldsymbol{i} - \mathbf{15}\boldsymbol{j}$$

$$\boldsymbol{v} = (\mathbf{20} + \mathbf{4.0} \times \mathbf{5.0})\boldsymbol{i} - \mathbf{15}\boldsymbol{j} = (\mathbf{40}\boldsymbol{i} - \mathbf{15}\boldsymbol{j})\ \mathbf{m/s}$$

لايجاد مقدار واتجاه سرعة الجسم بعد مرور 5 ثواني:

$$|v| = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(40)^2 + (-15)^2} = 43\ m/s$$

$$\theta = tan^{-1}\left(\frac{-15}{40}\right) = -21^o$$

The negative sign for the angle θ indicates that the velocity vector is directed at an angle of 21° below the positive *x* axis.

Problems to be solved by yourself

1. At t=0, a particle moving in the xy plane with constant acceleration has a velocity of v_o=3i - 2j at the origin. At t=3s, its velocity is v=9i+7j. Find (a) the acceleration of the particle and (b) its coordinates at any time t.
2. The position vector of a particles varies in time according to the expressions r=(3i-6t^2j) m. (a) Find expressions for the velocity and acceleration as a function of time. (b) Determine the particle's position and velocity at t=1s.
3. A particle initially located at the origin has an acceleration of a = 3j m/s^2 and an initial velocity of v_o = 5i m/s. Find (a) the vector position and velocity at any time t and (b) the coordinates and speed of the particle at t = 2 s.
4. A fish swimming horizontally has velocity v_o = (4i + j) m/s at a point in the ocean whose distance from a certain rock is r_o = (10i - 4j) m. After swimming with constant acceleration for 20.0 s, its velocity is v = (20i - 5j) m/s. (a) what are the components of the acceleration? (b) what is the direction of the acceleration with respect to unit vector i? (c) where is the fish at t = 25 s and in what direction is it moving?

المحاضرة القادمة

Projectile motion

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General Physics I

Mechanics: Principles & Applications

Lecture (8): Motion Kinematics
Two-Dimensional Motion
Projectile motion

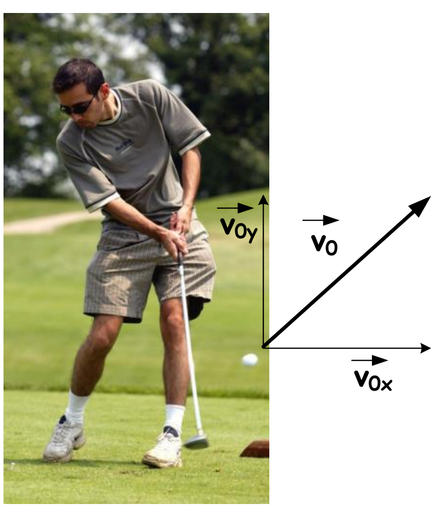
Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Contents of Unit 2: Mechanics

- The position vector and the displacement vector
- The average and Instantaneous velocity
- The average and Instantaneous acceleration
- One-dimensional Motion with constant acceleration
  - Free Fall
- Motion in two dimensions
  - Projectile motion
- Uniform Circular Motion
- Problems

Projectile Motion

Projectile Motion



تعتبر حركة المقذوفات **Projectile motion** من الأمثلة على الحركة في بعدين، وسوف نقوم بإيجاد معادلات الحركة للمقذوفات لتحديد الإزاحة الأفقية والرأسية والسرعة والعجلة من خلال العديد من الأمثلة.

إذا تحرك جسم في الفضاء تحت تأثير الجاذبية فقط فإننا نقول إنه مقذوف، ككرة قدم تطير في الهواء، أو ماء يندفع من نافورة. وتتم حركة المقذوف عادة في نفس المستوي إذا لم يكن هناك رياح أو قوى غير الجاذبية مؤثرة عليه.

Characteristics of Projectile Motion

Horizontal

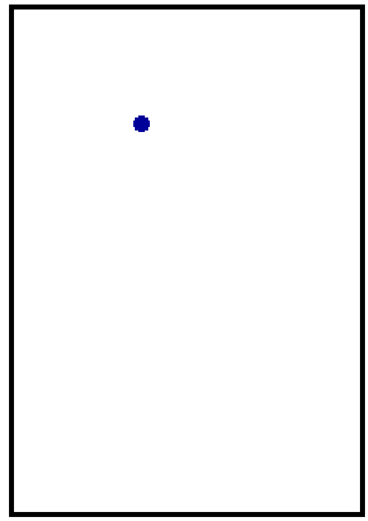
- Motion of a ball rolling freely along a level surface.
- Horizontal velocity is ALWAYS constant.

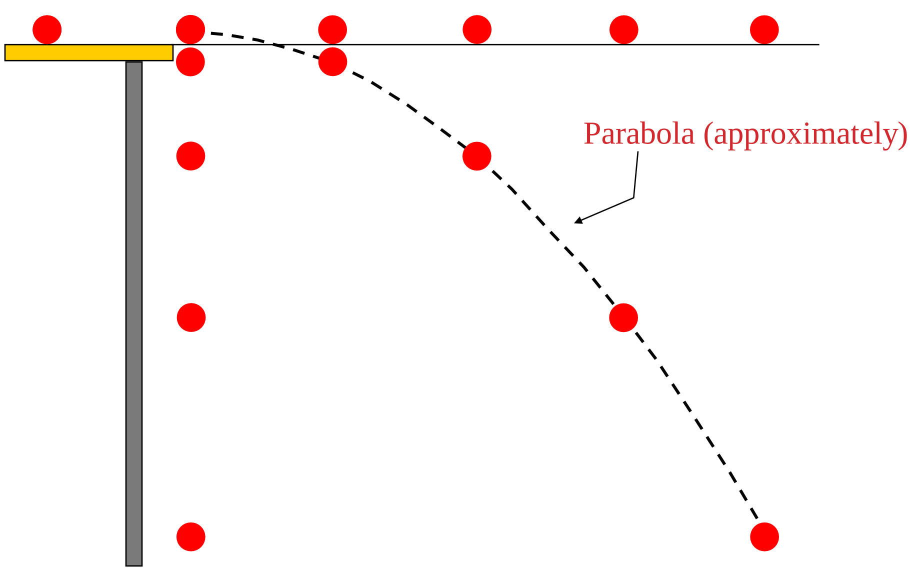
Vertical

- Motion of a freely falling object.
- Force due to gravity.
- Vertical component of velocity changes with time.

Parabolic

- Path traced by an object accelerating only in the vertical direction while moving at constant horizontal velocity.



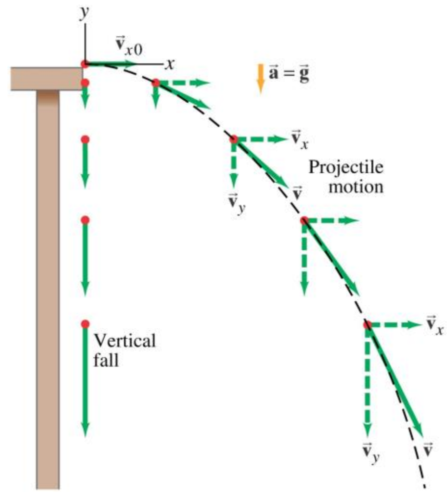


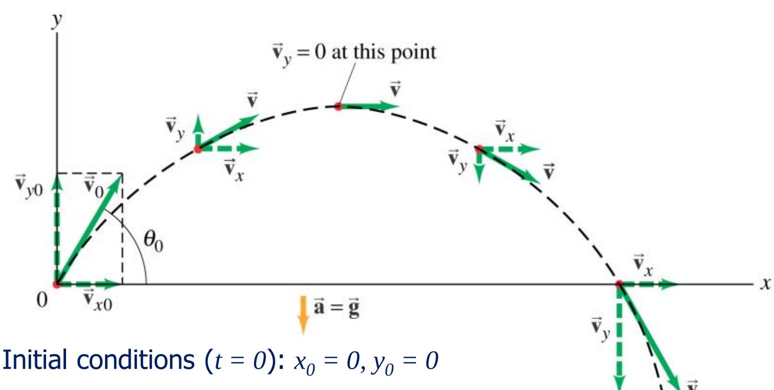
Important facts about Projectile Motion

The most important experimental fact about projectile motion in two dimensions is that the horizontal and vertical motions are completely independent of each other. This means that motion in one direction has no effect on motion in the other direction.

In general, the equations of constant acceleration developed in previous lectures follow separately for both the x-direction and the y-direction. An important difference is that **the initial velocity now has two components**.

We assume that at $t = 0$ the projectile leaves the origin with an initial velocity v_o. If the velocity vector makes an angle θ^o with the horizontal, where θ^o is called the ***projection* angle**.





Initial conditions ($t = 0$): $x_0 = 0, y_0 = 0$

$v_{xo} = v_o\cos\theta_o$ & $v_{yo} = v_o\sin\theta_o$

Velocity as a function of time

$v_x = v_{xo} = v_o\cos\theta_o$ = constant

$v_y = v_{yo} - gt = v_o\sin\theta_o - gt$

$$\vec{v} = v_x i + v_y j$$

Position as a function of time

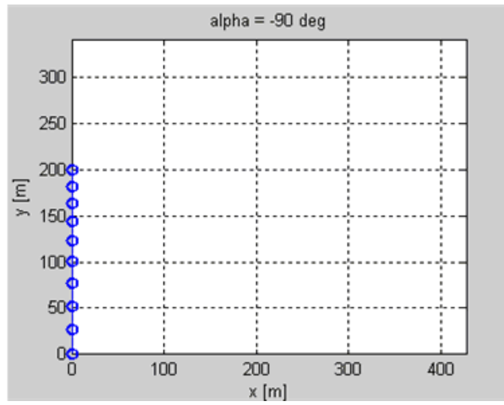
$x = v_{xo}\, t = (v_o\cos\theta_o)t$

$y = v_{yo}\, t - ½\, g\, t^2 = (v_o\sin\theta_o)t - ½\, g\, t^2$

$$\vec{r} = xi + yj$$

Trajectory of Projectile Motion

- Initial conditions ($t = 0$): $x_o = 0, y_o = 0$
  $v_{xo} = v_o \cos\theta_o$ and $v_{yo} = v_o \sin\theta_o$



- **Horizontal motion:**

$$x = 0 + v_{xo}t \Rightarrow t = \frac{x}{v_{xo}}$$

- **Vertical motion:**

$$y = 0 + v_{yo}t - \tfrac{1}{2}gt^2$$

$$y = v_{yo}\left(\frac{x}{v_{xo}}\right) - \frac{g}{2}\left(\frac{x}{v_{xo}}\right)^2$$

$$y = x\tan\theta_0 - \frac{g}{2{v_0}^2\cos^2\theta_0}x^2$$

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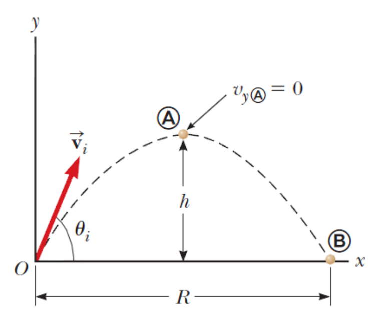
Horizontal Range and Maximum Height of a Projectile

The peak point A, which has Cartesian coordinates ($R/2$, h), and the point B, which has coordinates (R, 0). The distance R is called the *horizontal range* of the projectile, and the distance h is its *maximum height*.

Let us find h and R mathematically in terms of v_i, θ_i, and g.



We can determine h by noting that at the peak $v_y = 0$. Therefore, we can use the y component of Equation below to determine the time t at which the projectile reaches the peak:

$$v_y = v_{yo} - gt$$

$$0 = v_o \sin\theta_o - gt$$

$$t_A = \frac{v_o \sin\theta_o}{g}$$

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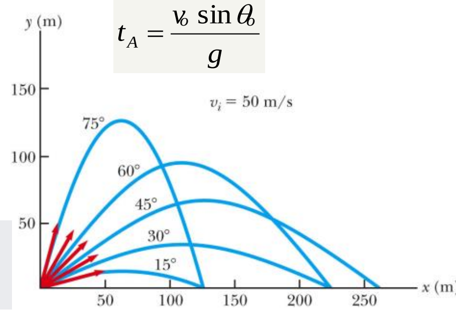
To find the maximum height h we use the equation

$$y = (v_o sin\theta_o)t - \frac{1}{2}gt^2$$

by substituting for the time t_A in the above equation



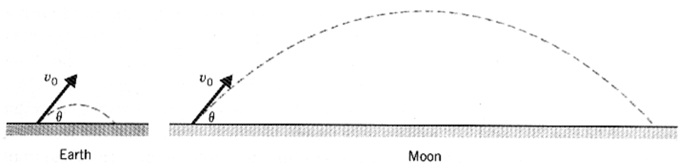
$$t_A = \frac{v_0 \sin\theta_0}{g}$$

$$h = (v_o sin\theta_o)\frac{v_o sin\theta_o}{g} - \frac{1}{2}\left(\frac{v_o sin\theta_o}{g}\right)^2$$

$$h = \frac{{v_o}^2 sin^2\theta_o}{2g}$$

اقصى ارتفاع للمقذوف

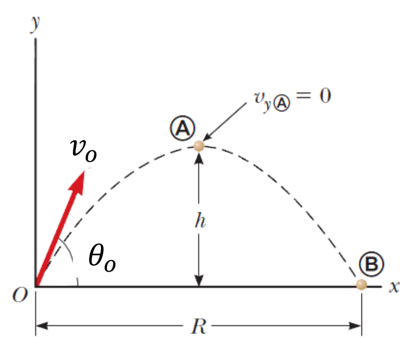
نلاحظ أقصى ارتفاع يصل إليه الجسم المتحرك في بعدين كحركة المقذوفات يعتمد على عجلة الجاذبية، وعليه فإن المقذوفات على سطح القمر تأخذ مساراً ذو مدى وارتفاع أكبر منه على سطح الأرض.



The range R is the horizontal position of the projectile at a time that is twice the time at which it reaches its peak, that is, at time $t_B = 2t_A$. Using the x component of Equation,

$$x = v_{xo}t = (v_o cos\theta_o)t$$

noting that $v_{xi} = v_{xB} = v_i \cos\theta_i$, and setting $x_B = R$ at $t = 2t_A$, we find that



$$t_A = \frac{v_0 \sin\theta_0}{g}$$

$$R = v_{xo}t_B = (v_o cos\theta_o)2t_A$$

$$R = (v_o cos\theta_o)\frac{2v_o sin\theta_o}{g} = \frac{2{v_o}^2 sin\theta_o cos\theta_o}{g}$$

Using the identity sin2θ = 2sinθ cosθ, we can write R in the more compact form

$$R = \frac{{v_o}^2 sin2\theta_o}{g}$$

المسافة التي يقطعها المقذوف افقيا

Maximum Range of the Projectile

The maximum value of R from Equation is

$$R = \frac{v_o{}^2 \sin 2\theta_o}{g}$$

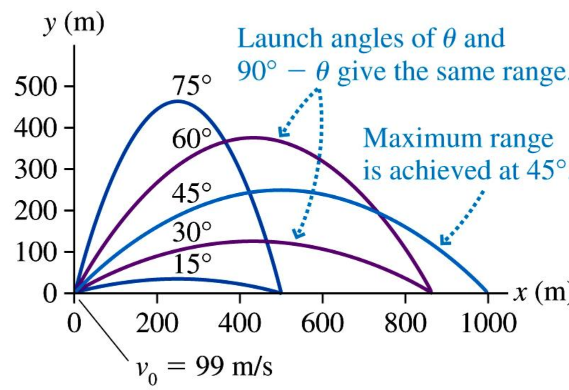
$$R_{max} = \frac{v_o{}^2}{g}$$

This result makes sense because the maximum value of $\sin 2\theta_o$ is 1, which occurs when

$2\theta_o = 90°$.

Therefore, R is a maximum when

$\theta_o = 45°$.



Example 1

A long-jumper leaves the ground at an angle of 20° to the horizontal and at a speed of 11 m/s. (a) How far does he jump? (b) The maximum height reached?

سنقوم بحل هذا المثال بالطريقة المستخدمة في اشتقاق اقصى ارتفاع والمدى ومن ثم سنتحقق من الحل بالتعويض مباشرة في المعادلتين

$$h = \frac{v_o{}^2 \sin^2\theta_o}{2g}$$

$$R = \frac{v_o{}^2 \sin 2\theta_o}{g}$$

Solution

(a) How far does he jump?

$x = (v_o\cos\theta_o)t = (11 \times \cos20)\, t$

x can be found if t is known, from the equation

$v_y = v_o\sin\theta_o - gt$

$0 = 11\sin20 - 9.8\, t_1$

$t_1 = 0.384$ s

where t_1 is the time required to reach the top then $t = 2t_1$

$t = 0.768$ s

therefore

$x = 7.94$ m

حل اخر

$$R = \frac{v_o{}^2 sin2\theta_o}{g} = \frac{(11)^2 sin2(20)}{9.8}$$

$$R = 7.94\ m$$

(b) The maximum height reached?

The maximum height reached is found using the value of t_1=0.384s

$y_{max} = (v_o\sin\theta_o)t_1 - 1/2g\, t_1^2$

$y_{max} = 0.722$ m

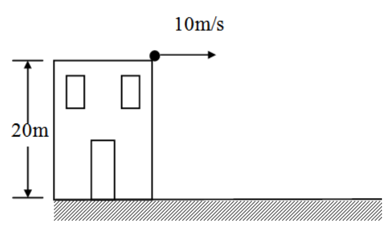
حل اخر

$$h = \frac{v_o{}^2 sin^2\theta_o}{2g} = \frac{(11)^2(sin20)^2}{2(9.8)}$$

$$h = 0.722\ \text{m}$$

Example 2

An object is thrown horizontally with a velocity of 10m/s from the top of a 20m high building as shown in the Figure. Where does the object strike the ground?



Solution

Consider the vertical motion: $v_{yo} = 0$, $a_y = 9.8$ m/s^2, $y = 20$ m, Then

$y = v_{yo} - 1/2\, g\, t^2$

$t = 2.02$ s

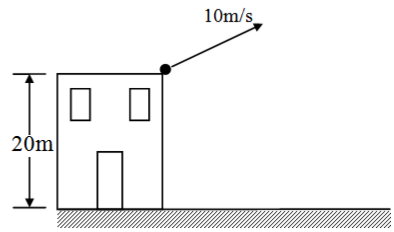
Consider the horizontal motion: $v_{xo} = v_x = 10$ m/s

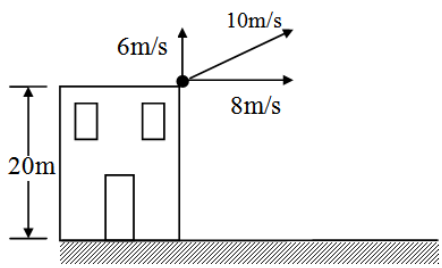
$x = v_x\, t$

$x = 20.2$ m

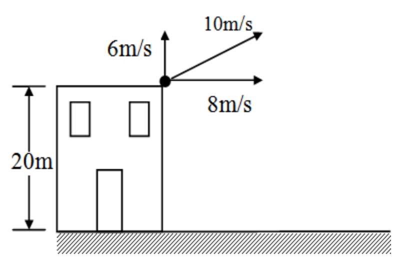
Example 3

Suppose that in the example above the object had been thrown upward at an angle of 37º to the horizontal with a velocity of 10m/s. Where would it land?





EXAMPLE 4



Solution

Consider the vertical motion

v_{yo} = 6 m/s

a_y = -9.8m/s^2 y = 20m

To find the time of flight we can use

$y = v_{yo}\,t - \frac{1}{2} g t^2$

since we take the top of the building is the origin the we substitute for

y = -20m

$-20 = 6\,t - \frac{1}{2}\, 9.8\, t^2$

t = 2.73s

Consider the horizontal motion

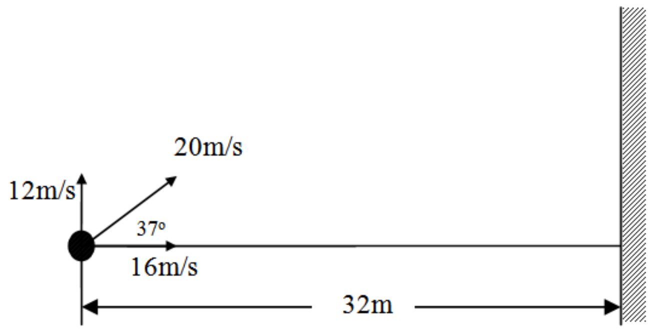
$v_x = v_{xo}$ = 8m/s

Then the value of x is given by

$x = v_x\, t$ = 22m

EXAMPLE 4

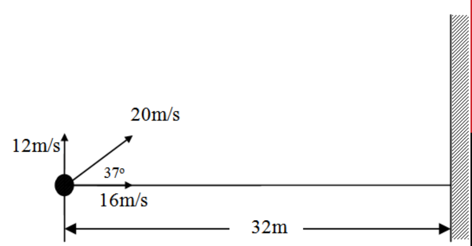
In the Figure shown below where will the ball hit the wall



Solution

We have

$v_x = v_{xo} = 16\text{m/s}$ **and** $x = 32\text{m}$



Then the time of flight is given by

$x = vt$

$t = 2\text{s}$

To find the vertical height after 2s we use the relation

$y = v_{yo}\, t - \frac{1}{2} g t^2$

Where $v_{yo} = 12\text{m/s}, \quad t = 2\text{s}$

$y = 4.4\text{m}$

Since *y* is positive value, therefore the ball hit the wall at 4.4m from the ground.

To determine whether the ball is going up of down we estimate the velocity and from its direction we can know

$v_y = v_{yo} - gt$

$v_y = -7.6\text{m/s}$

Since the final velocity is negative then the ball must be going down.

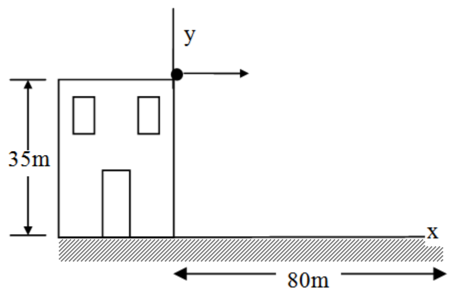
Example 5

A ball is thrown horizontally from the top of a building 35 m high. The ball strikes the ground at a point 80 m from the base of the building. Find

(a) the time the ball is in flight,

(b) its initial *v*elocity, and

(c) the *x* and *y* components velocity just before the ball strikes the ground.



Solution

$x_o=0$, $y_o=35m$, $v_{xo}=v_o$, $a_x=0$, $v_{yo}=0$, $a_y=9.8m/s^2$

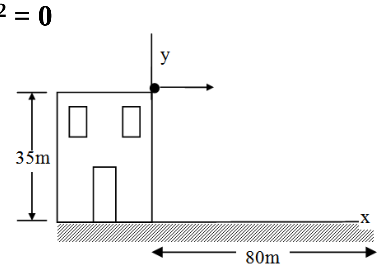
when the ball reaches the ground, $x = 80m$ and $y = 0$

(a) To find the time it takes to reach the ground,

$$y = y_o + v_{yo}\, t - \tfrac{1}{2}\, a_y\, t^2 = 35 - 4.9t^2 = 0$$

thus

$$t = 2.67s$$



(b) The initial velocity

Using $x = x_o+v_{xo}t = v_o t$ with $t = 2.67s$

$$80 = v_o(2.67)$$

$$v_o=29.9m/s$$

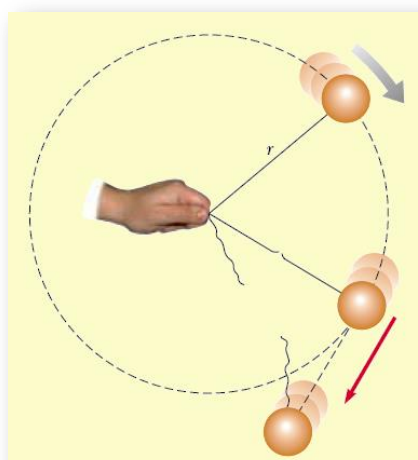
(c) The components of velocity just before the ball strikes the ground.

$$v_x = v_{xo}= 29.9\ m/s$$

$$v_y = v_{yo} - gt = 0 - 9.8\ (2.67) = -26.2\ m/s$$

Problems to be solved by yourself

1. A student stands at the edge of a cliff and throws a stone horizontally over the edge with speed of 18m/s. The cliff is 50m above the ground. How long after being released does the stone strike the beach below the cliff? With what speed and angle of impact does it land?
2. A football kicked at an angle of 50^{o} to the horizontal, travels a horizontal distance of 20m before hitting the ground. Find (a) the initial speed of the football, (b) the time it is in the air, and (c) the maximum height it reaches.
3. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?



المحاضرة القادمة

Uniform Circular Motion

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General Physics I

Mechanics: Principles & Applications

Lecture (9): Motion Kinematics
Uniform Circular Motion

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

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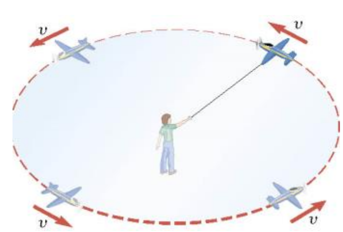
Circular Motion

Circular Motion Terms

The point or line that is the *center* of the circle is the *axis of rotation*.

If the axis of rotation is *inside* the object, the object is *rotating (spinning)*.

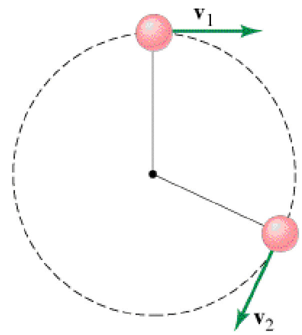
If the axis of rotation is *outside* the object, the object is *revolving*.



Circular motion

At constant speed v

Uniform circular motion



Uniform circular motion

Constant speed, or,
constant magnitude of velocity

Motion along a circle:
Changing direction of velocity

It is often surprising to students to find that even though an object moves at a constant speed in a circular path, it still has an acceleration.

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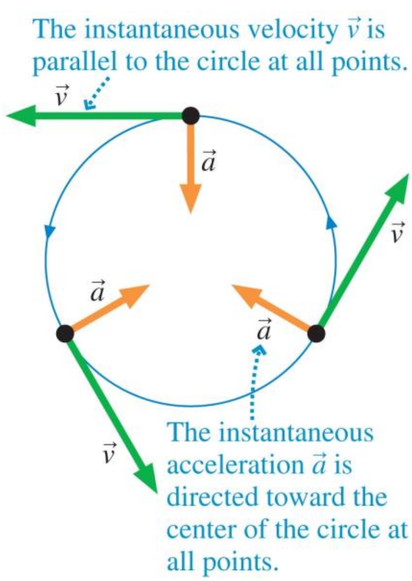
Circular Motion



من الممكن أن يتحرك جسم على مسار دائري بسرعة ثابتة *constant speed*. قد يخطر لنا الآن أن العجلة في هذه الحالة تساوى صفراً، وذلك لأن السرعة ثابتة، وهذا غير صحيح لأن الجسم يتحرك على مسار دائري لذا توجد عجلة. ولشرح ذلك نحن نعلم أن السرعة كمية متجه، والعجلة هي عبارة عن كمية متجه لأنها تساوى معدل التغير في السرعة بالنسبة للزمن، والتغير في السرعة قد يكون في المقدار أو في الاتجاه. وفي حالة حركة الجسم على مسار دائري منتظم فإن العجلة لا تؤثر على مقدار السرعة إنما تغير من اتجاه السرعة، ولهذا فإن الجسم يتحرك على مسار دائري وبسرعة ثابتة.

يكون متجه السرعة دائما عمودياً على نصف القطر وفى اتجاه المماس عند أية نقطة على المسار الدائري.

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Magnitude of the Acceleration of the Particle

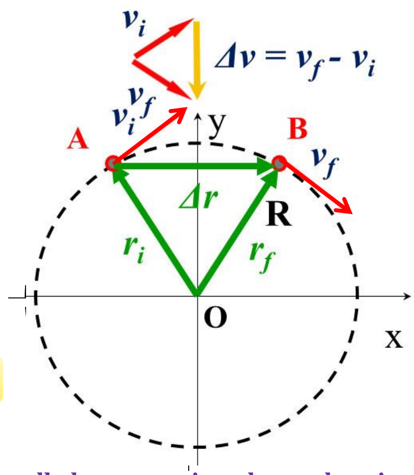
- **Magnitude:**

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i \qquad \vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \quad \text{so,} \quad \Delta v = \frac{v\Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta r}{\Delta t}\frac{v}{r} = \frac{v^2}{r}$$

$$a_c = \frac{v^2}{r}$$ Centripetal Acceleration



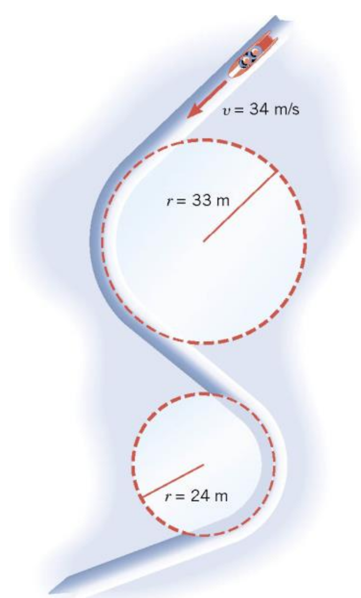
- **Direction: Centripetal**

An acceleration of this nature is called a centripetal acceleration (centripetal means center-seeking). The subscript on the acceleration symbol reminds us that the acceleration is centripetal.

EXAMPLE 1

The Effect of Radius on Centripetal Acceleration

The race track contains turns with radii of 33 m and 24 m. Find the centripetal acceleration at each turn for a speed of 34 m/s.
Express answers as multiples of $g=9.8 m/s^2$

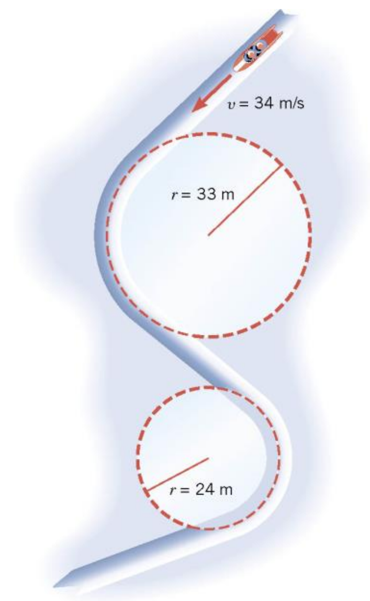


Solution

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(34\,\mathrm{m/s})^2}{33\,\mathrm{m}} = 35\,\mathrm{m/s^2} = 3.6g$$

$$a_c = \frac{(34\,\mathrm{m/s})^2}{24\,\mathrm{m}} = 48\,\mathrm{m/s^2} = 4.9g$$

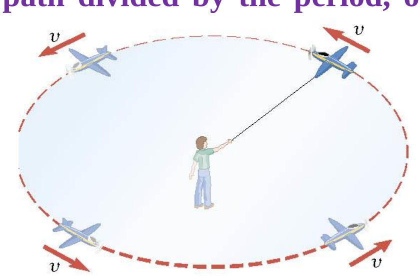


Period of Uniform Circular Motion

In many situations, it is convenient to describe the motion of a particle moving with constant speed in a circle of radius r in terms of the period T, which is defined as the time interval required for one complete revolution of the particle.

In the time interval T, the particle moves a distance of 2πr, which is equal to the circumference of the particle's circular path. Therefore, because its speed is equal to the circumference of the circular path divided by the period, or $v = \frac{2\pi r}{T}$, it follows that

$$\therefore T = \frac{2\pi r}{v}$$



Summary Uniform Circular Motion

Velocity:

- Magnitude: constant v
- The direction of the velocity is tangent to the circle

Acceleration:

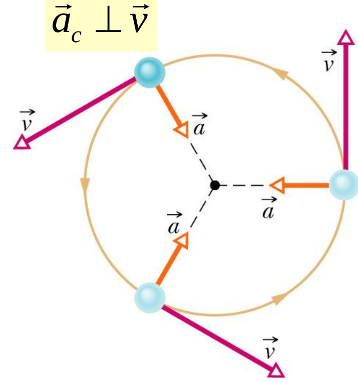
- Magnitude: $a_c = \frac{v^2}{r}$
- directed toward the center of the circle of motion

Period:

- time interval required for one complete revolution of the particle

$$T = \frac{2\pi r}{v}$$

$$\vec{a}_c \perp \vec{v}$$



Example 2

A particle moves in a circular path 0.4m in radius with constant speed. If the particle makes five revolution in each second of its motion, find (a) the speed of the particle and (b) its acceleration.

Solution

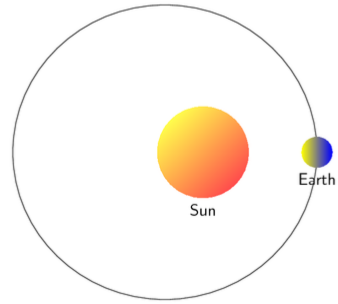
(a) Since r=0.4m, the particle travels a distance 0f $2\pi r$ = 2.51m in each revolution. Therefore, it travels a distance of 12.6m in each second (since it makes 5 rev. in the second).

$$v = 12.6 \text{ m/s}$$

(b) $$a_c = \frac{v^2}{r} = \frac{(12.6)^2}{0.4} = 397\ m/s^2$$

Example 3

What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun? (the radius of the Earth's orbit around the Sun, which is 1.496×10^{11} m).



Solution

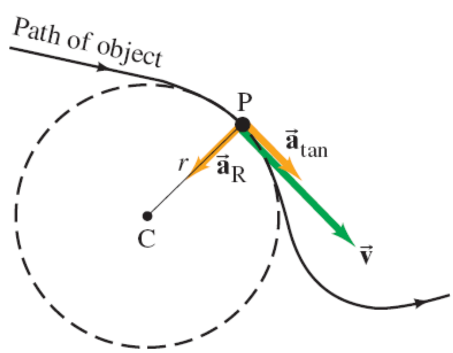
$$a_c = \frac{v^2}{r} \Rightarrow v = \frac{2\pi r}{T} \Rightarrow a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} \Rightarrow a_c = \frac{4\pi^2 r^2}{T^2 r}$$

$$a_c = \frac{4\pi^2 r^2}{T^2}$$

$$a_c = \frac{4\pi^2(1.496 \times 10^{11}\ \text{m})}{(1\ \text{yr})^2}\left(\frac{1\ \text{yr}}{3.156 \times 10^7\ \text{s}}\right)^2 = 5.93 \times 10^{-3}\ \text{m/s}^2$$

Non-uniform Circular Motion

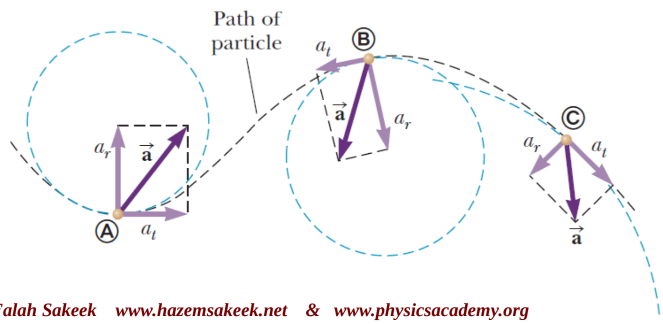
If an object is moving in a circular path but at varying speeds, it must have a tangential component to its acceleration as well as the radial one.



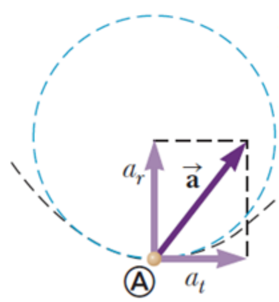
Tangential and Radial Acceleration

A particle moves to the right along a curved path, and its velocity changes both in direction and in magnitude as described in the Figure.

The velocity vector is always tangent to the path; the acceleration vector $\vec{a}$, however, is at some angle to the path. At each of three points A, B, and C in the Figure, the dashed blue circles represent the curvature of the actual path at each point. **The radius of each circle is equal to the path's radius of curvature at each point.**



As the particle moves along the curved path, the direction of the total acceleration vector $\vec{a}$ changes from point to point. At any instant, this vector can be resolved into two components based on an origin at the center of the dashed circle corresponding to that instant: a radial component a_r along the radius of the circle and a tangential component a_t perpendicular to this radius. **The *total* acceleration vector $\vec{a}$ can be written as the vector sum of the component vectors:**



$$\vec{a} = \vec{a}_r + \vec{a}_t \qquad \textbf{Total acceleration}$$

The tangential acceleration component causes a change in the speed v of the particle. This component is parallel to the instantaneous velocity, and its magnitude is given by

$$a_t = \left|\frac{dv}{dt}\right| \qquad \textbf{Tangential acceleration}$$

The radial acceleration component arises from a change in direction of the velocity vector and is given by

$$a_r = -a_c = -\frac{v^2}{r} \quad \textbf{Radial acceleration}$$

The negative sign indicates that the direction of the centripetal acceleration is toward the center of the circle representing the radius of curvature.

Magnitude of vector $\vec{a}$

$$a = \sqrt{{a_r}^2 + {a_t}^2}$$

The direction of $\vec{a}_t$ is either in the same direction as $\vec{v}$ (if v is increasing) or opposite $\vec{v}$ (if v is decreasing, as at point).

Quiz

(1) A particle moves in a circular path of radius r with speed v. It then increases its speed to 2v while traveling along the same circular path.

(i) The centripetal acceleration of the particle has changed by what factor?

Choose one: (a) 0.25 (b) 0.5 (c) 2 (d) 4 (e) impossible to determine

(ii) From the same choices, by what factor has the period of the particle changed?

---

(2) A particle moves along a path, and its speed increases with time.

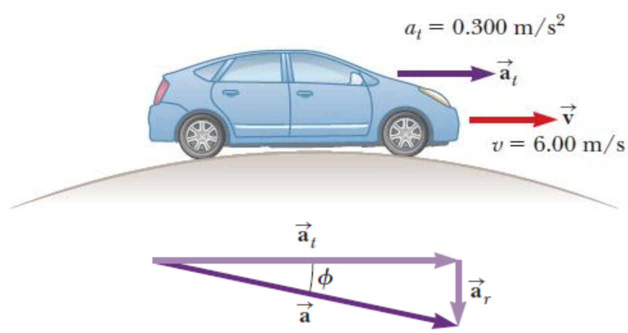
(i) In which of the following cases are its acceleration and velocity vectors parallel?

(a) when the path is circular (b) when the path is straight (c) when the path is a parabola (d) never

(ii) From the same choices, in which case are its acceleration and velocity vectors perpendicular everywhere along the path?

Example 4

A car leaves a stop sign and exhibits a constant acceleration of 0.300 m/s² parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What are the magnitude and direction of the total acceleration vector for the car at this instant?



Solution

v = 6.00 m/s and r = 500 m. The radial acceleration vector is directed straight downward, and the tangential acceleration vector has magnitude 0.300 m/s² and is horizontal.

the radial acceleration

$$a_r = -\frac{v^2}{r} = -\frac{(6.00\ \text{m/s})^2}{500\ \text{m}} = -0.072\,0\ \text{m/s}^2$$

magnitude of acceleration

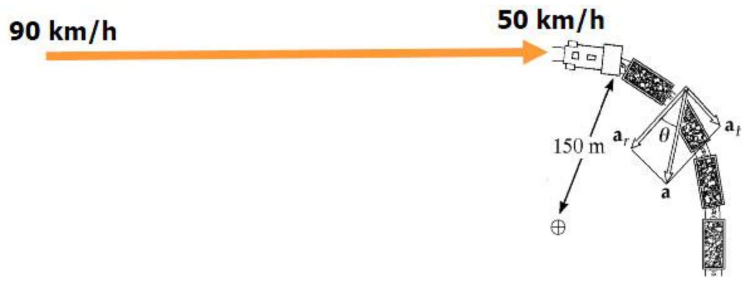
$$\sqrt{a_r^2 + a_t^2} = \sqrt{(-0.072\,0\ \text{m/s}^2)^2 + (0.300\ \text{m/s}^2)^2}$$
$$= 0.309\ \text{m/s}^2$$

Direction of acceleration

$$\phi = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\left(\frac{-0.072\,0\ \text{m/s}^2}{0.300\ \text{m/s}^2}\right) = -13.5°$$

Example 5

A train slows down as it rounds a sharp horizontal turn, slowing from 90km/h to 50km/h in the 15s that it takes to start to round the bend. The radius of the curve is 150m. Compute the acceleration at the train.

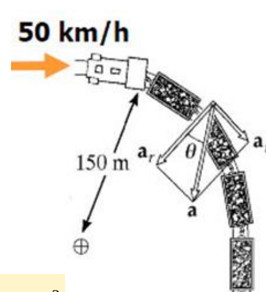


Solution

يجب تحويل السرعة من وحدة km/h إلى وحدة m/s كالتالي:-

$$50km/h = \left(50\frac{km}{h}\right)\left(10^3\frac{m}{km}\right)\left(\frac{1h}{3600s}\right) = 13.89m/s$$

$$90km/h = \left(90\frac{km}{h}\right)\left(10^3\frac{m}{km}\right)\left(\frac{1h}{3600s}\right) = 25m/s$$



$$a_r = \frac{v^2}{r} = \frac{13.89}{150} = 1.29\text{m/s}^2$$

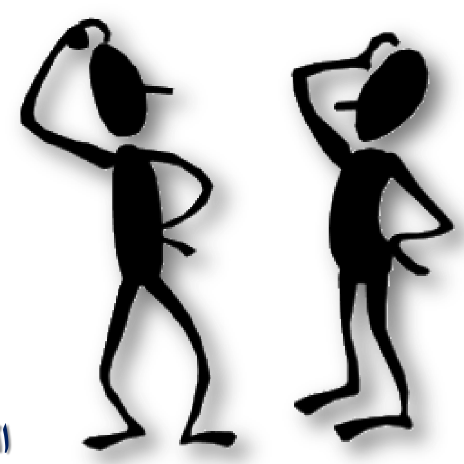
$$a_t = \frac{\Delta v}{\Delta t} = \frac{13.89-25}{15} = \text{-}0.741\text{m/s}^2$$

$$a = \sqrt{a_r{}^2 + a_t{}^2} = \sqrt{(1.29)^2 + (-0.741)^2} = 1.48m/s^2$$

$$\tan^{-1}\left(\frac{|a_t|}{|a_r|}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right) = 29.9^o$$

Problems to be solved by yourself

1. Find the acceleration of a particle moving with a constant speed of 8 m/s in a circle 2 m in radius.
2. The speed of a particle moving in a circle 2 m in radius increases at the constant rate of 3 m/s^2. At some instant, the magnitude of the total acceleration is 5 m/s^2. At this instant find (a) the centripetal acceleration of the particle and (b) its speed.
3. A student swings a ball attached to the end of a string 0.6 m in length in a vertical circle. The speed of the ball is 4.3 m/s at its highest point and 6.5 m/s at its lowest point. Find the acceleration of the ball at (a) its highest point and (b) its lowest point.



المحاضرة القادمة

Discussion

Physics Academy

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General Physics I

Mechanics: Principles & Applications

Lecture (10): Dynamics
The Law of Motion

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Mechanics: Dynamics
The Law of Motion

Introduction

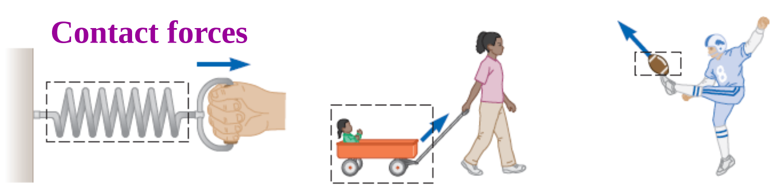
في الجزء السابق ركزنا على علم وصف الحركة من إزاحة وسرعة وعجلة دون النظر إلى مسبباتها وهذا العلم يسمى علم الكينماتيكا ***Kinematics***، وفي هذا الجزء من المقرر سوف ندرس مسبب الحركة وهو كمية فيزيائية هامة تدعى القوة ***Force*** والتي وضع العالم نيوتن ثلاث قوانين أساسية تعتمد على الملاحظات التجريبية التي أجراها منذ أكثر من ثلاث قرون. والعلم الذي يدرس العلاقة بين حركة الجسم والقوة المؤثرة عليه هو من علوم الميكانيكا الكلاسيكية ***Classical mechanics*** والتي تعرف باسم ديناميكا ***Dynamics***، وكلمة كلاسيك هنا تدل على أننا نتعامل فقط مع سرعات اقل بكثير من سرعة الضوء وأجسام أكبر بكثير من الذرة.

Isaac Newton
English physicist and mathematician
(1642–1727)
Isaac Newton's work represents one of the greatest contributions to science ever made by an individual.

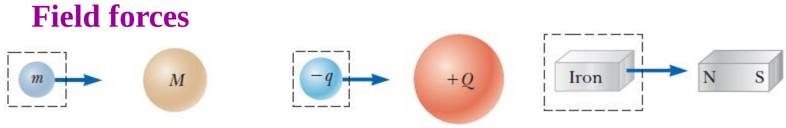
The Concept of Force

- ❖ The word *force* refers to an interaction with an object by means of muscular activity and some change in the object's velocity.
- ❖ Forces do not always cause motion, however. For example, when you are sitting, a gravitational force acts on your body and yet you remain stationary.

Contact forces



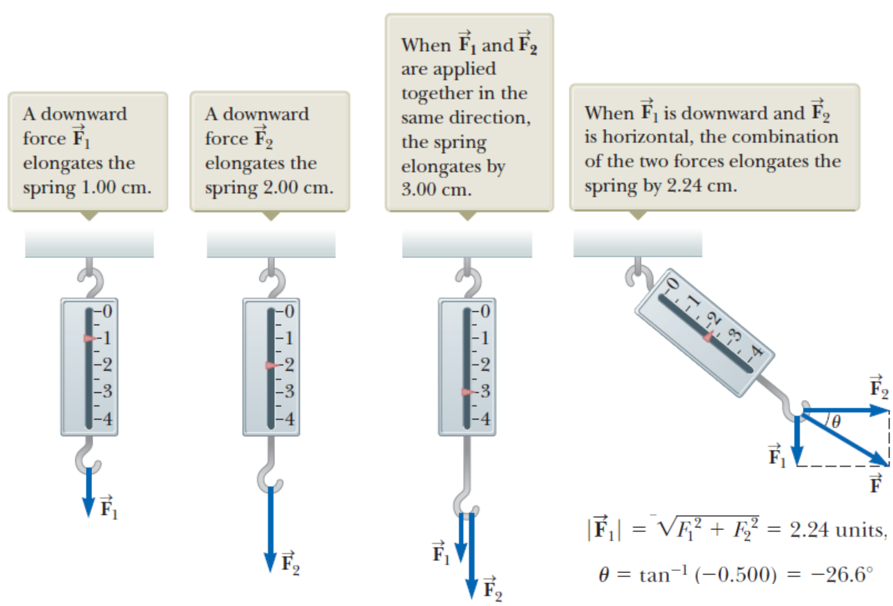
Field forces



Fundamental Forces in Nature

Types of force
- Gravitational forces
- Electromagnetic forces
- Strong forces
- Weak forces

The Vector Nature of Force



Newton's Laws of Motion

Newton's Laws

- *Newton's first law*
  The law of equilibrium
- *Newton's second law*
  The law of acceleration
- *Newton's third law*
  The law of action-reaction

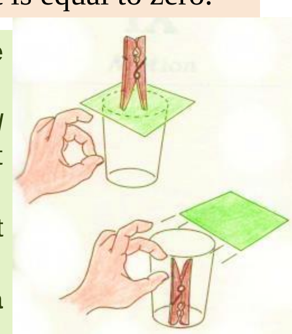
Newton's First Law

Newton's first law**, **the law of equilibrium, **states that an object at rest will remain at rest and an object in motion will remain in motion with a constant velocity unless acted on by a *net external force.***

$$\sum \vec{F} = 0$$

The **net force (***total force, resultant force or unbalanced force***)** on an object is equal to zero.

- ❑ **when no force acts on an object, the acceleration of the object is zero.**
- ❑ From the first law, we conclude that any *isolated object* is either at rest or moving with constant velocity.
- ❑ The tendency of an object to resist any attempt to change its velocity is called **inertia.**
- ❑ We can define **force** as **that which causes a change in motion of an object.**



Newton's Second Law

Newton's second law, the law of acceleration, states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\sum \vec{F} = m\vec{a}$$

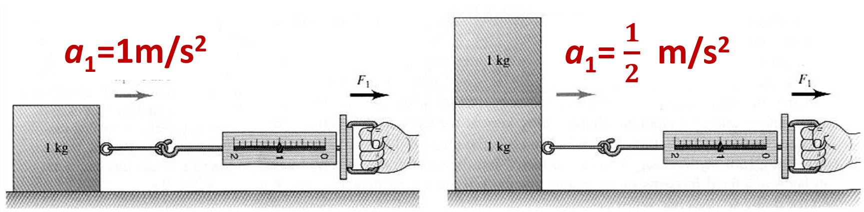
where m is the mass of the body and a is the acceleration of the body

The **net force (***total force, resultant force or unbalanced force***)** on an object is the vector sum of all forces acting on the object.

Then the unit of the force is (Kg.m/s^2) which is called **Newton (N)**

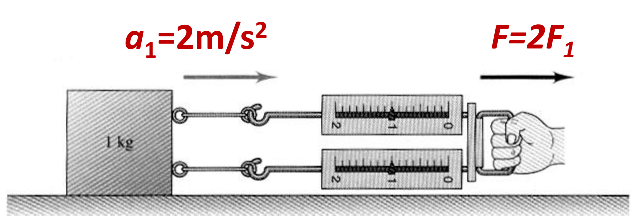
Components of force

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$



(a) (b)

إذا زادت الكتلة بمقدار الضعف مع ثبوت قوة الشد فإن العجلة تقل بمقدار النصف.



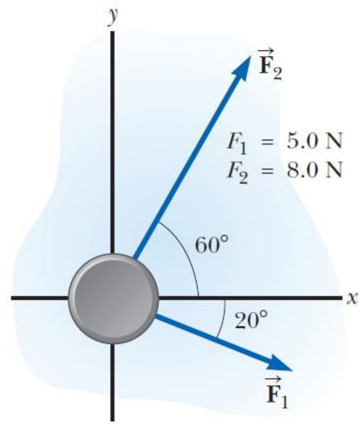
(c)

إذا تضاعفت قوة الشد فإن العجلة تزداد بمقدار الضعف.

Example 1

A hockey puck having a mass of 0.30 kg slides on the frictionless, horizontal surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in the Figure. The force $\mathbf{F}_1$ has a magnitude of 5.0 N, and the force $\mathbf{F}_2$ has a magnitude of 8.0 N.

Determine both the magnitude and the direction of the puck's acceleration.



Solution

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos(-20°) + F_2 \cos 60°$$
$$= (5.0\ \text{N})(0.940) + (8.0\ \text{N})(0.500) = 8.7\ \text{N}$$

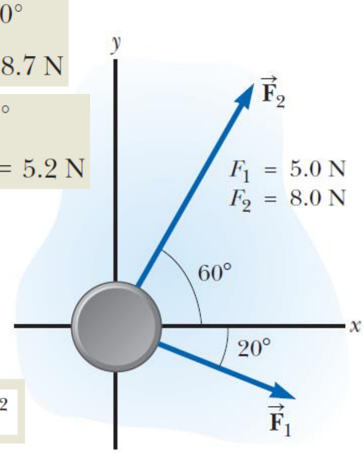
$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin(-20°) + F_2 \sin 60°$$
$$= (5.0\ \text{N})(-0.342) + (8.0\ \text{N})(0.866) = 5.2\ \text{N}$$

$$a_x = \frac{\sum F_x}{m} = \frac{8.7\ \text{N}}{0.30\ \text{kg}} = 29\ \text{m/s}^2$$

$$a_y = \frac{\sum F_y}{m} = \frac{5.2\ \text{N}}{0.30\ \text{kg}} = 17\ \text{m/s}^2$$

$$a = \sqrt{(29\ \text{m/s}^2)^2 + (17\ \text{m/s}^2)^2} = 34\ \text{m/s}^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 31°$$

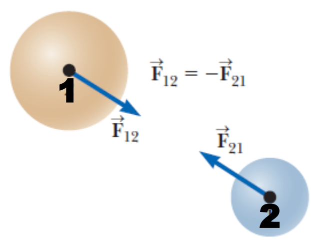


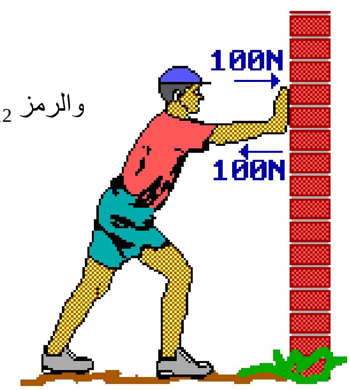
Newton's Third Law

Newton's third law, *the law of action-reaction*, **states that when two bodies interact, the force which body "A" exerts on body "B" (the action force) is equal in magnitude and opposite in direction to the force which body "B" exerts on body "A" (the reaction force).**

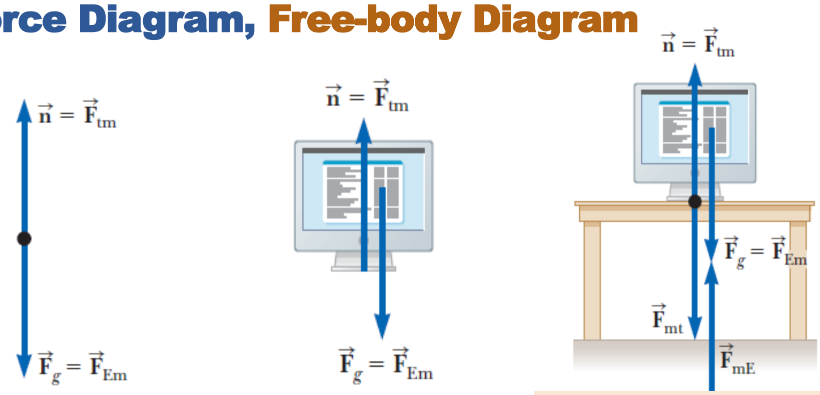
$$\vec{F}_{12} = -\vec{F}_{21}$$

والرمز F_{12} يعني القوة التي يتأثر بها الجسم الأول نتيجة للجسم الثاني.





Force Diagram, Free-body Diagram



free-body diagram

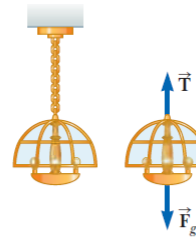
forces acting on *one object* (the monitor)

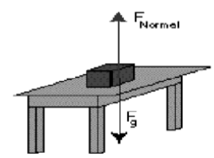
Force acting on the monitor, one acting on the table, and one acting on the Earth.

When analyzing an object subject to forces, we are interested in the net force acting on one object, which we will model as a particle. Therefore, a free-body diagram helps us isolate only those forces on the object and eliminate the other forces from our analysis.

Particular Forces

Particular Forces
- Gravitational Force
- Tension Force
- Normal Force
- Spring Force
- Friction Force





Gravitational Force and Weight

All objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the **gravitational force** $\vec{F}_g$. This force is directed toward the center of the Earth, and its magnitude is called the **weight** of the object.

We saw in previous lecture that a freely falling object experiences an acceleration $\vec{g}$ acting toward the center of the Earth. Applying Newton's second law $\sum \vec{F} = m\vec{a}$ to a freely falling object of mass *m*, with $\vec{a} = \vec{g}$ and $\sum \vec{F} = \vec{F}_g$

$$\vec{F}_g = m\vec{g}$$

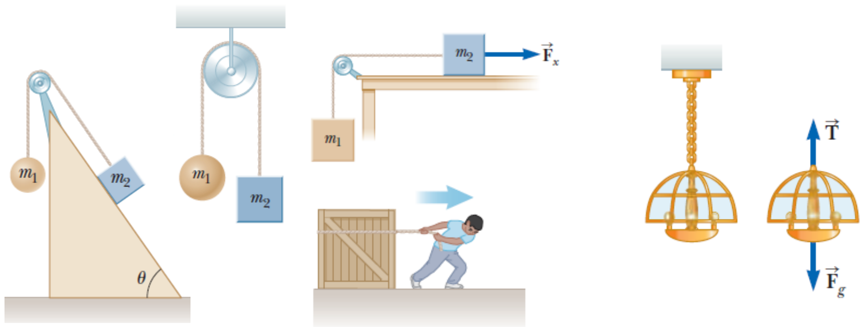
Therefore, the weight of an object, being defined as the magnitude of $\vec{F}_g$, is equal to *mg*:

$$F_g = mg = W$$

Because it depends on *g*, weight varies with geographic location. Because *g* decreases with increasing distance from the center of the Earth, objects weigh less at higher altitudes than at sea level.

Tension Force

عند سحب جسم بواسطة حبل فإن القوة المؤثرة على الجسم من خلال الحبل تدعى قوة الشد ***Tension*** ويرمز لها بالرمز ***T*** ووحدته **N**. ويظهر في الشكل صور مختلفة من قوة الشد وكيفية تحديدها على الشكل.



$$\sum \vec{F} = m\vec{a} \qquad \sum \vec{F} = 0$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

General Problem solving Strategy

1. Conceptualize. Draw a simple diagram of the system. The diagram helps establish the mental representation. Establish convenient coordinate axes for each object in the system.

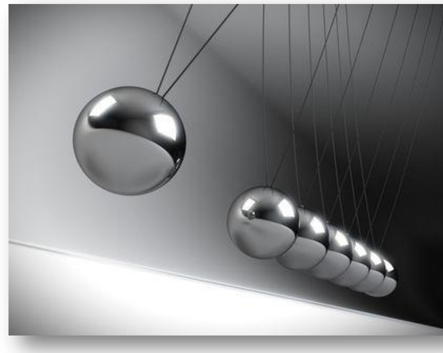
2. Categorize. If an acceleration component for an object is zero, the object is modeled as a particle in equilibrium in this direction and o $\sum F = 0$. If not, the object is modeled as a particle under a net force in this direction and o $\sum F = ma$.

3. Analyze. Isolate the object whose motion is being analyzed. Draw a free-body diagram for this object. For systems containing more than one object, draw *separate* free-body diagrams for each object. *Do not* include in the free-body diagram forces exerted by the object on its surroundings.

4. Find the components of the forces along the coordinate axes. Apply the appropriate model from the Categorize step for each direction. Check your dimensions to make sure that all terms have units of force.

5. Solve the component equations for the unknowns. Remember that you generally must have as many independent equations as you have unknowns to obtain a complete solution.

6. Finalize. Make sure your results are consistent with the free-body diagram. Also check the predictions of your solutions for extreme values of the variables.



المحاضرة القادمة

Analysis Models Using Newton's Second Law

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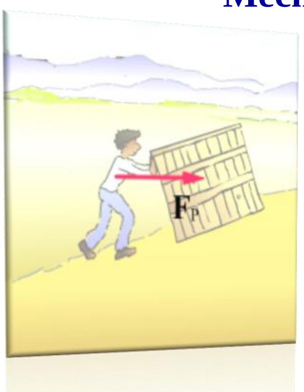
General Physics I

Mechanics: Principles & Applications



Lecture (11): Dynamics

Examples on

Newton's Law of Motion

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Mechanics: Dynamics
The Law of Motion

1 The law of motion

2 The concept of force

3 Newton's laws of motion

4 Weight and tension

5 Force of friction

6 Questions with solution

7 Problems

The Particle in Equilibrium

If the acceleration of an object is zero, the object is treated with the **particle in equilibrium** model. In this model, the net force on the object is zero:

$$\sum \vec{F} = 0$$

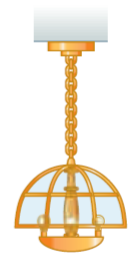
The forces acting on the lamp are the downward gravitational force $\vec{F}_g$

The upward force $\vec{T}$ exerted by the chain.

$$\sum F_x = 0 \quad \& \quad \sum F_y = 0$$

$$\sum F_y = T - F_g = 0$$

$$\therefore T = F_g$$



The Particle Under a Net Force

If an object experiences an acceleration, its motion can be analyzed with the **particle under a net force** model.

$$\sum \vec{F} = m\vec{a}$$

The horizontal force $\vec{T}$ being applied to the crate acts through the rope.

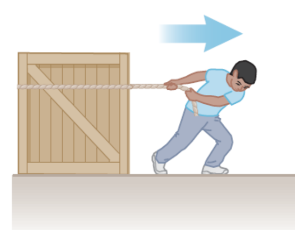
The vertical forces are the gravitational force $\vec{F}_g$ and the normal force $\vec{n}$.

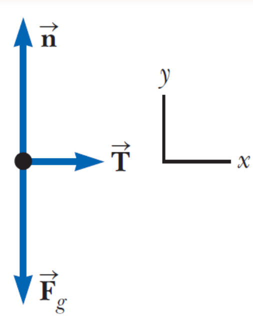
لاحظ هنا ان هناك تسارع في اتجاه محور x بينما لا يوجد تسارع على محور y لذا نحصل على

$$\sum F_x = ma_x \rightarrow T = ma_x \rightarrow \quad a_x = \frac{T}{m}$$

$$\sum F_y = ma_y \rightarrow n - F_g = 0 \rightarrow n = F_g$$

the normal force has the same magnitude as the gravitational force but acts in the opposite direction.



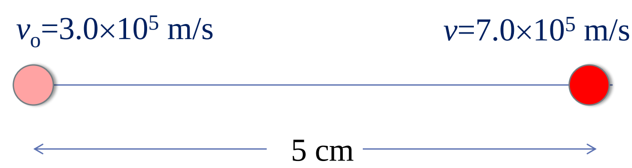


Example 1

An electron of mass 9.1×10^{-31} kg has an initial speed of 3.0×10^{5} m/s. It travels in a straight line, and its speed increases to 7.0×10^{5} m/s in a distance of 5.0cm. Assuming its acceleration is constant,

(a) determine the force on the electron and

(b) compare this force with the weight of the electron, which we neglected.



Solution

نحتاج الى حساب قيمة التسارع التي يتحرك بها الالكترون وهذا يتم من خلال تطبيق قوانين الحركة في بعد واحد لان الالكترون يتحرك على خط مستقيم، من المعادلة

$$v^2 = v_o^2 + 2a(x - x_o) \Rightarrow a = \frac{v^2 - v_o^2}{2x}$$

(a) لحساب القوة المؤثرة على الالكترون نستخدم قانون نيوتن الثاني

$$F = ma$$

$$F = \frac{m(v^2 - v_o^2)}{2x} = 3.6 \times 10^{-18} N$$

(b) لحساب وزن الالكترون نستخدم قانون نيوتن الثاني مع التعويض عن التسارع بعجلة الجاذبية الأرضية

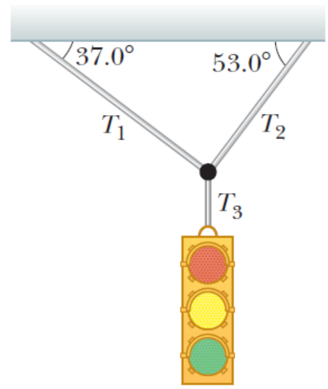
$$W = m\,g = (9.1 \times 10^{-31}\ \text{kg})\ (9.8\ \text{m/s}^2) = 8.9 \times 10^{-30}\ \text{N}$$

The accelerating force is approximately 10^{11} times the weight of the electron.

Example 2

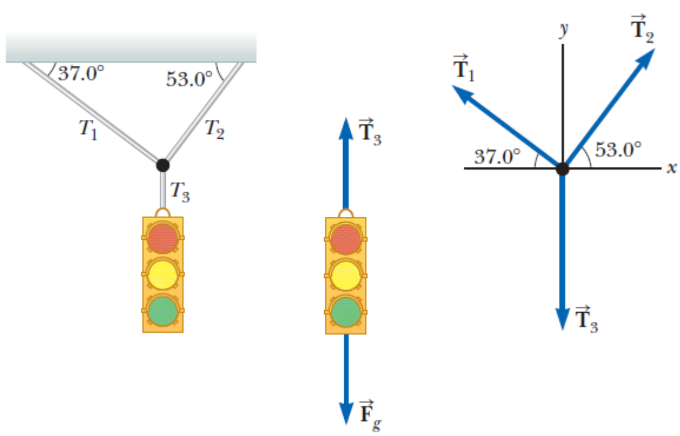
A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in the Figure. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N.

Does the traffic light remain hanging in this situation, or will one of the cables break?



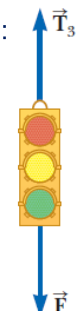
Solution

We construct a diagram of the forces acting on the traffic light, and a free-body diagram for the knot that holds the three cables together



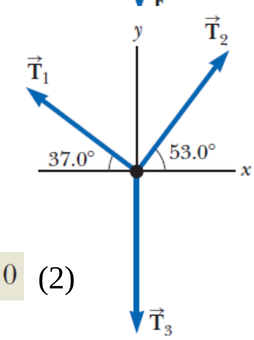
Apply Newton's second law for the traffic light in the y direction:

$$\sum F_y = 0 \rightarrow T_3 - F_g = 0$$
$$T_3 = F_g = 122 \text{ N}$$



Resolve the forces acting on the knot into their components:

Force	x Component	y Component
$\vec{T}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\vec{T}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\vec{T}_3$	0	-122 N



Apply Newton's second law to the knot:

$$\sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0 \quad (1)$$

$$\sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122 \text{ N}) = 0 \quad (2)$$

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Solve Equation (1) for T_2 in terms of T_1:

$$-T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

$$T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33 T_1$$

Substitute this value for T_2 into Equation (2):

$$T_1 \sin 37.0° + T_2 \sin 53.0° + (-122 \text{ N}) = 0$$

$$T_1 \sin 37.0° + (1.33 T_1)(\sin 53.0°) - 122 \text{ N} = 0$$
$$T_1 = 73.4 \text{ N}$$
$$T_2 = 1.33 T_1 = 97.4 \text{ N}$$

Both values are less than 100 N, so the cables will not break.

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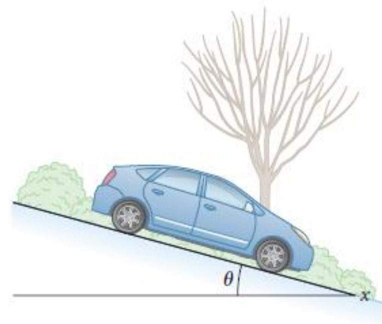
Example 3

A car of mass *m* is on an icy driveway inclined at an angle θ as in the Figure.

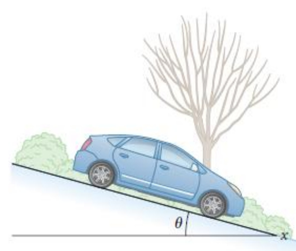


(A) Find the acceleration of the car, assuming the driveway is frictionless.

(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is *d*. **How** long does it take the front bumper to reach the bottom of the hill, and **what** is the car's speed as it arrives there?

Solution

(A) Find the acceleration



تتحرك السيارة في هذه الحالة تحت تأثير عجلة الجاذبية الأرضية بتطبيق قانون نيوتن الثاني

$$\sum \vec{F} = m\vec{a}$$

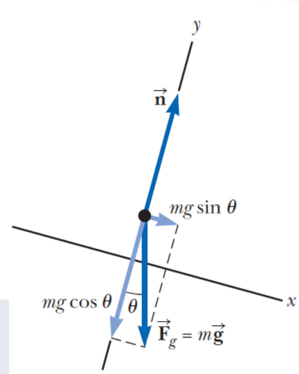
بتحليل القوى على مخطط free-body diagram لمركباته كما هو في الشكل نجد ان

$$\sum F_x = ma_x \qquad \& \qquad \sum F_y = ma_y = 0$$

$$\sum F_x = mg\sin\theta = ma_x \quad (1)$$

$$\sum F_y = n - mg\cos\theta = 0 \quad (2)$$

$$a_x = g\sin\theta \quad (3)$$



Note: that the acceleration component a_x is independent of the mass of the car! It depends only on the angle of inclination and on *g*.

Solution

(B) How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?

Equation (3) shows that the acceleration *ax* is constant. $a_x = g\sin\theta$

$x_i = 0$ and its final position as $x_f = d$, and $v_{xi} = 0$,

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \;\longrightarrow\; d = \tfrac{1}{2}a_x t^2$$

$$t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g\sin\theta}}$$

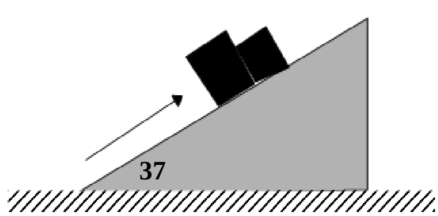
$$v_{xf}^2 = 2a_x d \;\longrightarrow\; v_{xf} = \sqrt{2a_x d} = \sqrt{2gd\sin\theta}$$

Note: the time t at which the car reaches the bottom and its final speed v_{xf} are independent of the car's mass, as was its acceleration.

Example 4

Two blocks having masses of 2 kg and 3 kg are in contact on a fixed smooth inclined plane as shown in the Figure.

Treating the two blocks as a composite system, calculate the force *F* that will accelerate the blocks up the incline with acceleration of 2m/s^2,

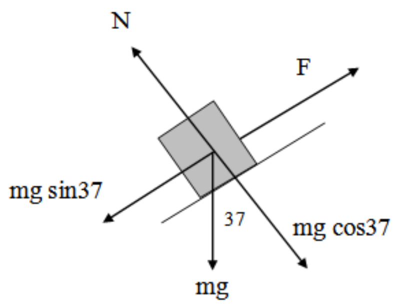


Solution

We can replace the two blocks by an equivalent 5 kg block.

لإيجاد القوة التي تدفع الكتلة لتتحرك بتسارع 2m/s2
نحلل القوى إلى مركباتها على محور x وy وحيث ان
انه لا يوجد تسارع على محور y فان

N = mg cos(37º)



وعلى محور x

$\sum F = F - mg \sin(37^o) = ma$

$F - 5*9.8*(0.6) = 5*2$

$F = 39.4$ N

Example 5

The parachute on a race car of weight 8820N opens at the end of a quarter-mile run when the car is travelling at 55 m/s. What is the total retarding force required to stop the car in a distance of 1000 m in the event of a brake failure?

Solution

$W = 8820$ N, $g = 9.8$ m/s^2 , $v_o = 55$ m/s, $v_f = 0$, $x_f - x_o = 1000$ m

$$m = \frac{W}{g} = 900kg$$

$$v_f^2 = v_o^2 + 2a(x - x_o),$$

$0 = 55^2 + 2a(1000)$, giving $a = -1.51$ m/s^2

$$\sum F = ma = (900 \text{ kg})(-1.51 \text{ m/s}^2) = -1.36 \times 10^3 \text{ N}$$

The minus sign means that the force is a retarding force.

Example 6

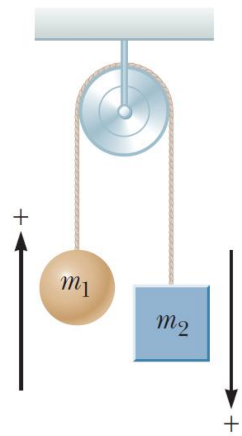
Two masses of 3 kg and 5 kg are connected by a light string that passes over a smooth pulley as shown in the Figure.

Determine

(a) the tension in the string,

(b) the acceleration of each mass, and

(c) the distance each mass moves in the first second of motion if they start from rest.



Solution

(a) $m_1 a = T - m_1 g$ **(1)**

$m_2 a = m_2 g - T$ **(2)**

Add (1) and (2)

$(m_1 + m_2)\, a = (m_2 - m_1)\, g$

$$a = \frac{m_2 - m_1}{(m_2 + m_1)g} = \frac{5-3}{(5+3)(9.8)} = 2.45 m/s^2$$

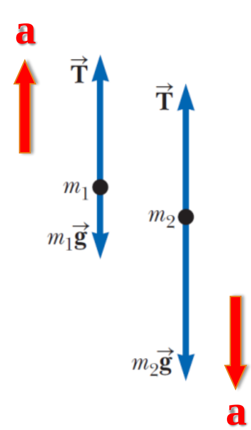
(b) $T = m_2\,(g - a) = 5(9.80 - 2.45) = 36.6$ **N**

(c) Substitute a into (1)

$$T = m_1\,(a + g) = \frac{2m_1 m_2 g}{m_1 + m_2}$$

$$s = \frac{at^2}{2} \quad (v_0 = 0)$$

At $t = 1$s, $\quad s = \frac{(2.45)(1^2)}{2} = 1.23$m

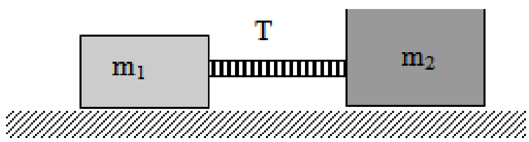


Example 7

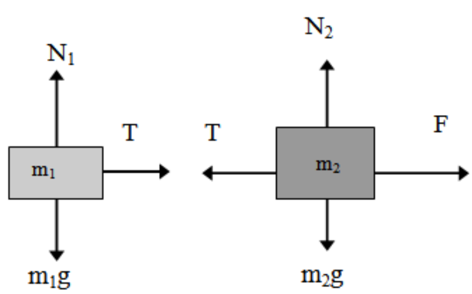
Two blocks connected by a light rope are being dragged by a horizontal force *F* as shown in the Figure. Suppose that *F* = 50 N, m_1 = 10 kg, m_2 = 20 kg,

(a) Draw a free-body diagram for each block.

(b) Determine the tension, *T*, and the acceleration of the system.



Solution



$\Sigma F_x(m_1) = T = m_1 a$ $\Sigma F_x(m_2) = 50 - T = m_2 a$

$\Sigma F_y(m_1) = N_1 - m_1 g = 0$ $\Sigma F_y(m_2) = N_2 - m_2 g = 0$

$T = 10\,a,$ $50 - T = 20\,a$

Adding the expression above gives

$50 = 30\,a,$

$a = 1.66\ \text{m/s}^2$

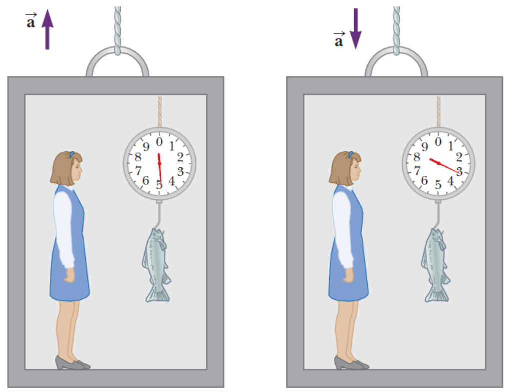
$T = 16.6\text{N}$

Example 8

A person weighs a fish of mass *m* on a spring scale attached to the ceiling of an elevator as illustrated in the Figure.

(A) Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

(B) Evaluate the scale readings for a 40.0-N fish if the elevator moves with an acceleration *ay* ±2.00 m/s^2.



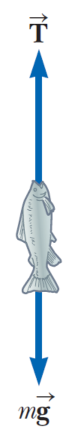
Solution

عندما يكون المصعد في حالة سكون او عندما يتحرك بسرعة ثابتة

If the elevator is either at rest or moving at constant velocity, the fish is a particle in equilibrium, so

$$\sum F_y = T - F_g = 0$$

$$T = F_g = mg$$



Remember that the scalar *mg* is the weight of the fish.

عندما يتحرك المصعد بتسارع

Now suppose the elevator is moving with an acceleration $\vec{a}$ relative to an observer standing outside the elevator in an inertial frame. The fish is now a particle under a net force.

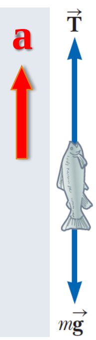
Apply Newton's second law to the fish:

$$\sum F_y = T - mg = ma_y$$

For upward acceleration

$$T = mg + ma_y$$

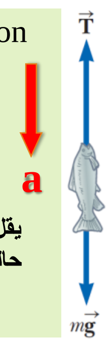
يزداد وزن السمكة الظاهري T في حالة تسارع المصعد إلى الأعلى



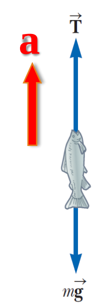
For downward acceleration

$$T = mg - ma_y$$

يقل وزن السمكة الظاهري T في حالة تسارع المصعد إلى الاسفل



(B) Evaluate the scale readings for a 40.0-N fish if the elevator moves with an acceleration *ay* ±2.00 m/s².

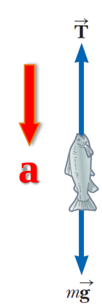


Evaluate the scale reading from Equation (1) if **a** is upward:

$$T = (40.0\ \text{N})\left(\frac{2.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} + 1\right) = \boxed{48.2\ \text{N}}$$

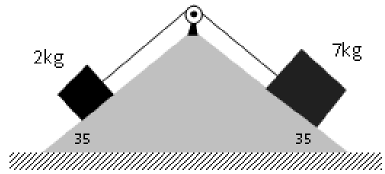
Evaluate the scale reading from Equation (1) if **a** is downward:

$$T = (40.0\ \text{N})\left(\frac{-2.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} + 1\right) = \boxed{31.8\ \text{N}}$$

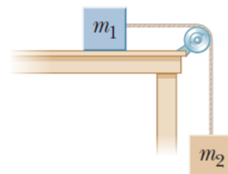


Problems to be solved by yourself

(1) Two blocks of mass 2kg and 7kg are connected by a light string that passes over a frictionless pulley as shown in. The inclines are smooth. Find (a) the acceleration of each block and (b) the tension in the string.



(2) An object of mass m_1=5kg placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging object of mass m_2=9kg. (a) Draw free-body diagrams of both objects. Find (b) the magnitude of the acceleration of the objects and (c) the tension in the string.



(3) A 72-kg man stands on a spring scale in an elevator starting from rest, the elevator ascends, attaining it maximum velocity of 1.2 m/s in 0.8 s. It travels wit this constant velocity for the next 5.0 s. The elevator then undergoes a uniform negative acceleration for 1.5s and comes to rest. What does the spring scale register (a) before the elevator starts to move (b) during the first 0.8s? (c) while the elevator is travelling at constant velocity? (d) during the negative acceleration period?

المحاضرة القادمة

Force of friction

Physics Academy

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General Physics I

Mechanics: Principles & Applications

Lecture (12): Dynamics
Force of Friction

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Mechanics: Dynamics
The Law of Motion

1 The law of motion

2 The concept of force

3 Newton's laws of motion

4 Weight and tension

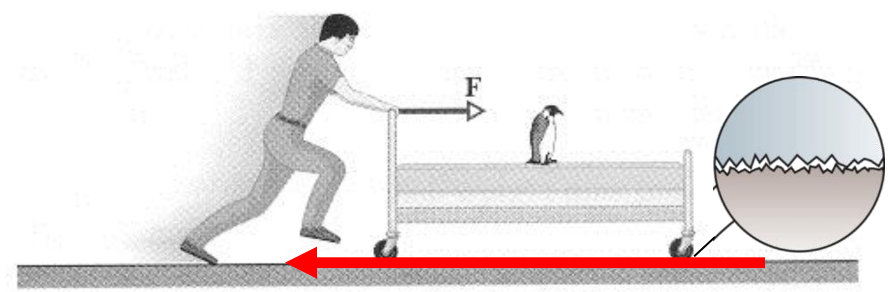
5 Force of friction

6 Questions with solution

7 Problems

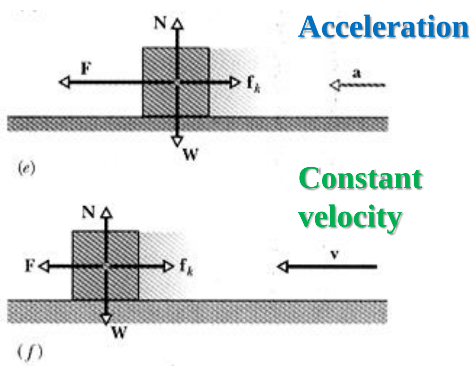
Force of Friction

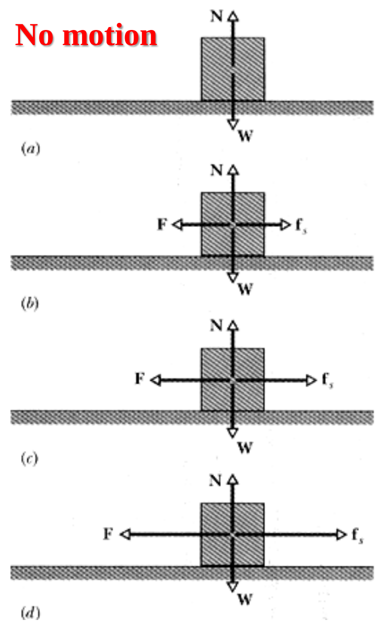
قوة الاحتكاك *force of friction* ويرمز لها بالرمز *f* واتجاه هذه القوة دائماً عكس اتجاه الحركة وهي ناتجة عن خشونة الأسطح المتحركة.



Force of Friction

- **When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion. This resistance is called the *force of friction***
- **This is due to the interactions between the object and its environment**
- **We will be concerned with two types of frictional force**
  - Force of static friction: f_s
  - Force of kinetic friction: f_k
- **Direction: opposite the direction of the intended motion**
  - **If moving:** in direction opposite the velocity
  - **If stationary:** in direction of the vector sum of other forces





من التجارب العملية لوحظ أن قوة الاحتكاك للأجسام الساكنة أكبر من قوة الاحتكاك للأجسام المتحركة .

static friction الاحتكاك السكوني
kinetic friction الاحتكاك الحركي

Static Friction

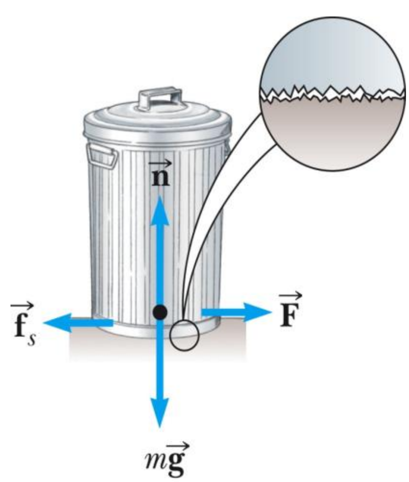
Static friction acts to keep the object from moving

If F increases, so does f_s

If F decreases, so does f_s

$$f_s \le \mu_s N$$

μ_s *Coefficient of static friction*

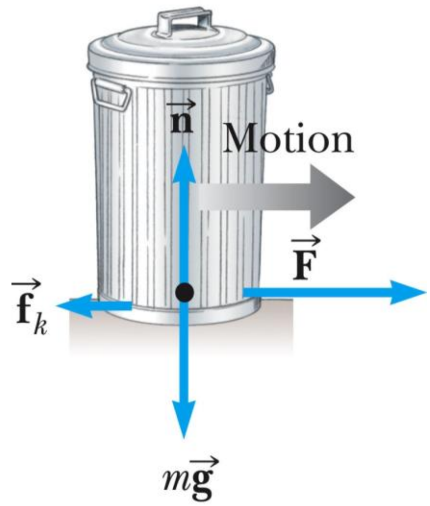


Kinetic Friction

The force of kinetic friction f_k acts when the object is in motion

$$f_k = \mu_k N$$

μ_k *Coefficient of kinetic friction*



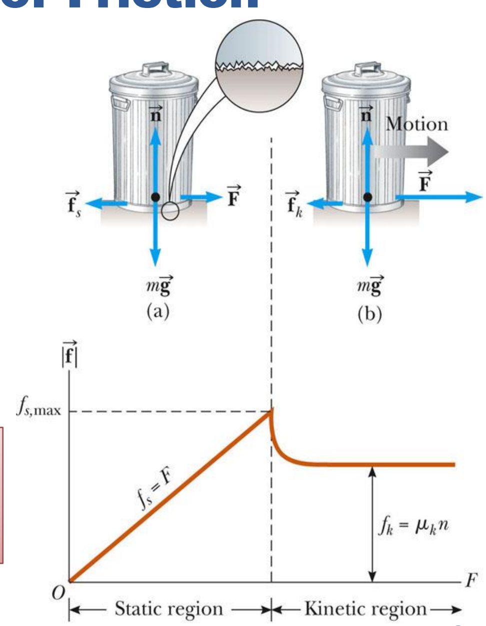
Explore Forces of Friction

Vary the applied force

Note the value of the frictional force

Note what happens when the can starts to move

معامل الاحتكاك الحركي يكون دائما أكبر من معامل الاحتكاك السكوني ومعامل الاحتكاك ليس له وحدة.



Summary

Force of friction

Static of friction	Kinetic of friction
$f_s = \mu_s N$	$f_k = \mu_k N$

❑ حيث μ تسمى معامل الاحتكاك

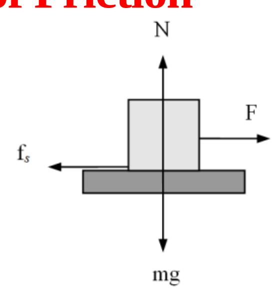
❑ في حالة الاحتكاك السكوني تسمى *Coefficient of static friction*، μ_s

❑ في حالة الاحتكاك الحركي تسمى μ_k, *Coefficient of kinetic friction*.

Evaluation of the Force of Friction

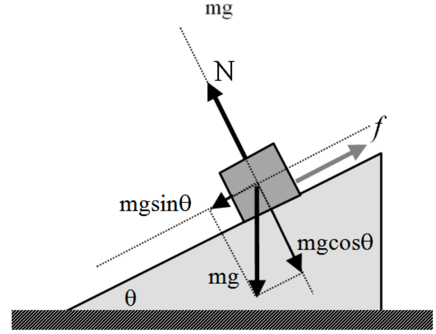
Case (1) when a body slides on a horizontal surface

$$f_k = \mu_k N$$
$$\text{Since} \quad N = mg$$
$$f_k = \mu_k mg$$



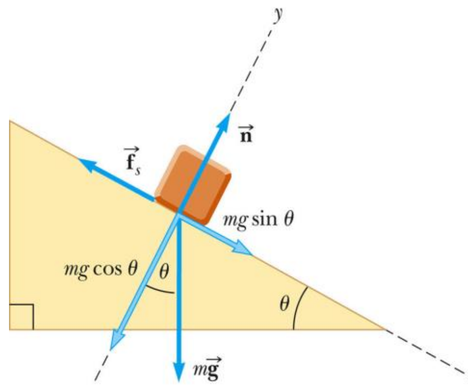
Case (2) when a body slides on an inclined surface

$$f_k = \mu_k N$$
$$\text{Since} \quad N = mg\cos\theta$$
$$f_k = \mu_k mg\cos\theta$$



Example 1

Suppose a block with a mass of 2.50 kg is resting on a ramp. If the coefficient of static friction between the block and ramp is 0.350, what maximum angle can the ramp make with the horizontal before the block starts to slip down?

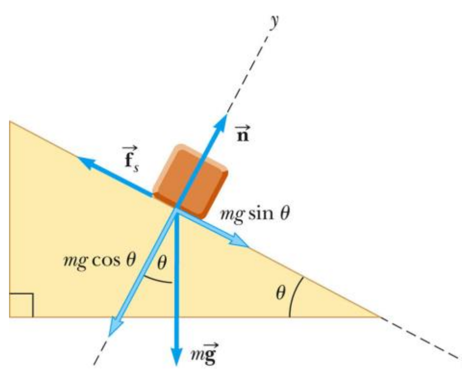


Solution

Newton 2nd law:

$$\sum F_x = mg\sin\theta - \mu_s N = 0$$

$$\sum F_y = N - mg\cos\theta = 0$$



Then $N = mg\cos\theta$

$$\sum F_x = mg\sin\theta - \mu_s mg\cos\theta = 0$$

$$mg\sin\theta = \mu_s mg\cos\theta$$

$$\tan\theta = \mu_s = 0.350$$

$$\theta = \tan^{-1}(0.350) = 19.3^\circ$$

Example 2

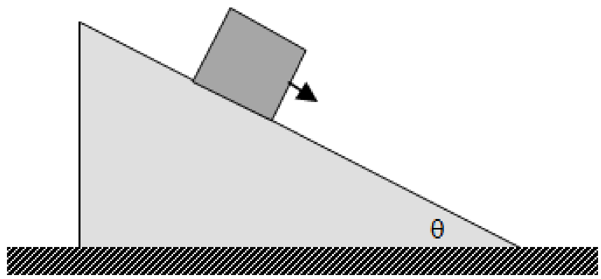
A 3kg block starts from rest at the top of 30º incline and slides a distance of 2m down the incline in 1.5s.

Find (a) the acceleration of the block,

(b) the coefficient of kinetic friction between the block and the plane,

(c) the friction force acting on the block, and

(d) the speed of the block after it has slid 2m.



Solution

Given m = 3kg, θ = 30º, x = 2m, t = 1.5s

$x = \frac{1}{2}at^2 \Rightarrow 2 = \frac{1}{2}a(1.5)^2 \Rightarrow a = 1.78\text{m/s}^2$

$mg\sin30 - f = ma \Rightarrow f = m(g\sin30 - a) \Rightarrow f = 9.37\text{N}$

$N - mg\cos30 = 0 \Rightarrow N = mg\cos30$

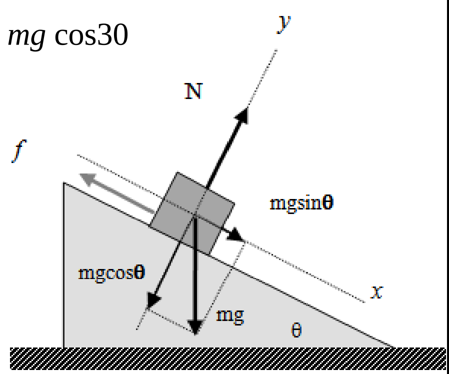
$f = 9.37\text{N}$

$\mu_k = \frac{f}{N} = 0.368$

$v^2 = v_o^2 + 2a(x - x_o)$

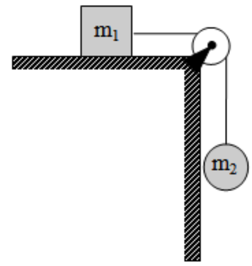
$v^2 = 0 + 2(1.78)(2) = 7.11\text{m}^2/\text{s}^2$

$\mathbf{v = 2.67m/s}$

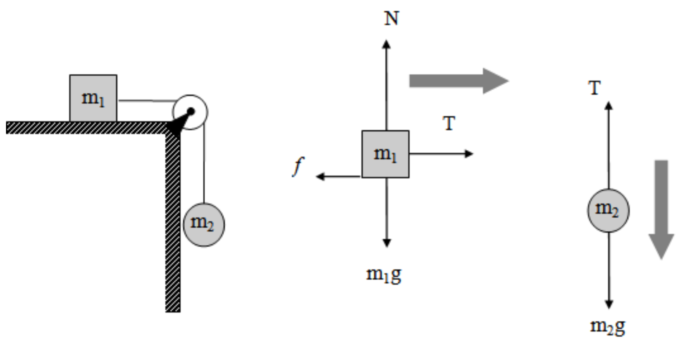


Example 3

Two blocks are connected by a light string over a frictionless pulley as shown in Figure. The coefficient of sliding friction between m_1 and the surface is μ. Find the acceleration of the two blocks and the tension in the string.



Solution

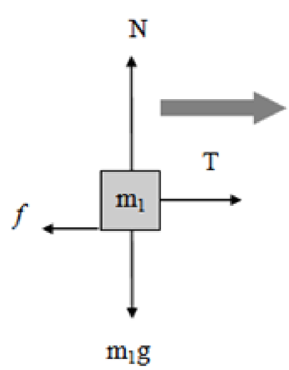


Consider the motion of m_1. Since its motion to the right, then $T>f$. If T were less than f, the blocks would remain stationary.

$$\sum F_x \text{ (on } m_1) = T - f = m_1 a$$

$$\sum F_y \text{ (on } m_1) = N - m_1 g = 0$$

$$N = m_1 g$$



Since $f = \mu N$,

$f = \mu\, m_1 g$

Then

$$T - \mu\, m_1 g = m_1 a$$

$$T = m_1 a + \mu\, m_1 g$$

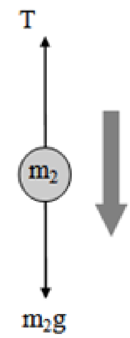
$$T = m_1(a + \mu g) \quad (1)$$

For m_2, the motion is downward, therefore $m_2g>T$. Note that T is uniform through the rope. That is the force which acts on the right is also the force which keeps m_2 from free falling. The equation of motion for m_2 is:

$\sum F_y$ (on m_2) = $T - m_2g = - m_2a$

$T = m_2(g-a)$ (2)

$T = m_1(a+\mu g)$ (1)



Equating equation (1) and (2)

$m_1(a+\mu g) = m_2(g-a)$

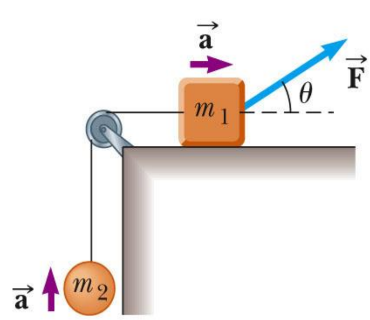
$$a = \left(\frac{m_2 - \mu m_1}{m_1 + m_2}\right)g$$

The tension T is

$$T = m_2\left(1 - \frac{m_2 - \mu m_1}{m_1 + m_2}\right)g \Rightarrow T = \frac{m_1 m_2 (1+\mu)g}{m_1 + m_2}$$

Example 4

A block of mass m_1 on a rough, horizontal surface is connected to a ball of mass m_2 by a lightweight cord over a lightweight, frictionless pulley as shown in figure. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k. Find the magnitude of acceleration of the two objects.

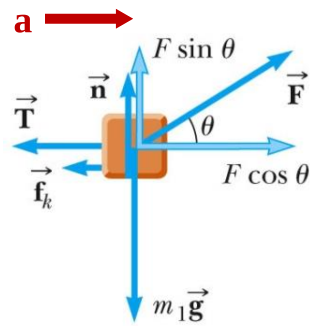


For m$_1$:

$$\sum F_x = F\cos\theta - f_k - T = m_1 a$$

$$\sum F_y = N + F\sin\theta - m_1 g = 0$$

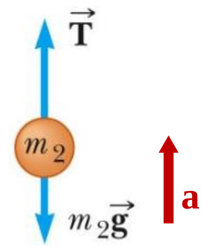
$$N = m_1 g - F\sin\theta$$



For m$_2$:

$$\sum F_y = T - m_2 g = m_2 a$$

$$T = m_2(a+g)$$



$$f_k = \mu_k N = \mu_k(m_1 g - F\sin\theta)$$

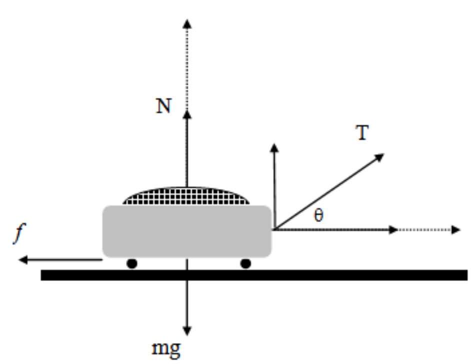
$$F\cos\theta - \mu_k(m_1 g - F\sin\theta) - m_2(a+g) = m_1 a$$

$$a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_2 + \mu_k m_1)g}{m_1 + m_2}$$

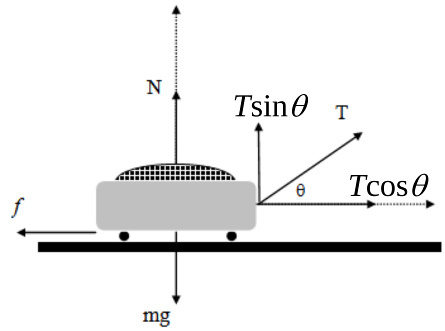
Example 5

A cart is loaded with bricks has a total mass of 18kg and is pulled at constant speed by a rope. The rope is inclined at 20^o above the horizontal and the cart moves on a horizontal plane. The coefficient of kinetic friction between the ground and the cart is 0.5

What is the tension in the rope? When the cart is moved 20m.



SOLUTION



$T\cos\theta - f = 0 \qquad (1)$

$N + T\sin\theta - mg = 0 \qquad (2)$

$f = \mu N = \mu(mg - T\sin\theta) \qquad (3)$

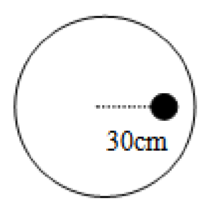
Substitute (3) in (1)

$$T\cos\theta - \mu(mg - T\sin\theta) = 0$$

$$\therefore T = \frac{\mu mg}{\cos\theta + \mu\sin\theta} = 79.4N$$

EXAMPLE

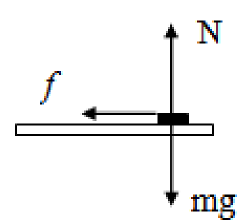
A coin is placed 30cm from the centre of a rotating, horizontal turntable. The coin is observed to slip when its speed is 50cm/s. What is the coefficient of static friction between the coin and the turntable?



Solution

$$N = mg$$

بتطبيق قانون نيوتن الثاني F = ma نحصل على



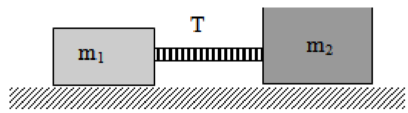
$$f = ma = m\frac{v^2}{r}$$

Since $f = \mu_s N = \mu_s mg$

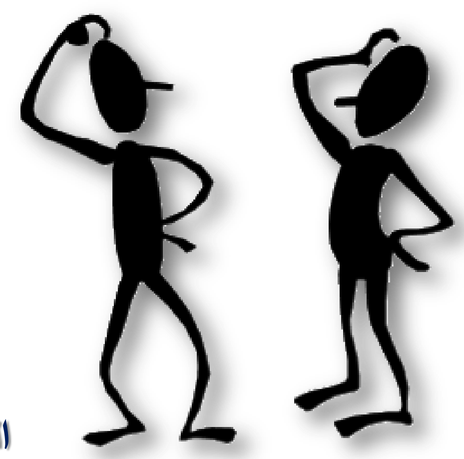
$$\mu_s mg = m\frac{v^2}{r} \Rightarrow \mu_s = \frac{v^2}{rg} = 0.085$$

Problems to be solved by yourself

1. A 25kg block is initially at rest on a rough, horizontal surface. A horizontal force of 75N is required to set the block in motion. After it is in motion, a horizontal force of 60N is required to keep the block moving with constant speed. Find the coefficient of static and kinetic friction from this information.
2. The coefficient of static friction between a 5kg block and horizontal surface is 0.4. What is the maximum horizontal force that can be applied to the block before it slips?
3. Two blocks connected by a light rope are being dragged by a horizontal force *F*. Suppose that *F* = 50 N, m_1 = 10 kg, m_2 = 20 kg, and the coefficient of kinetic friction between each block and the surface is 0.1.



   (a) Draw a free-body diagram for each block.
   (b) Determine the tension, *T*, and the acceleration of the system.



المحاضرة القادمة

Solving Problems and Discussion

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General Physics I

Mechanics: Principles & Applications

Unit4: Work and Energy

Lecture (13) Work Done by a Constant Force

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Unit4: Work and Energy

1 Introduction

2 Work done by a constant force

3 Work done by a varying force

4 Work done by a spring

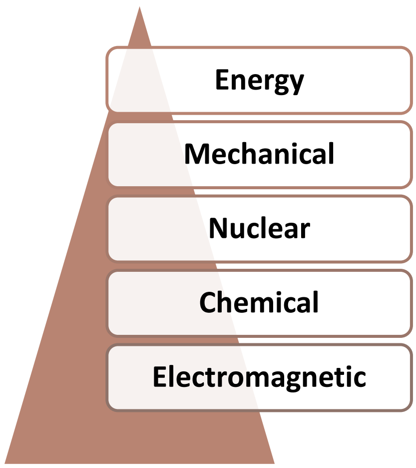
5 Work and kinetic energy

6 Power

7 Questions with solution

8 Problems

Introduction



إن مفهوم الشغل والطاقة مهم جداً في علم الفيزياء، حيث توجد الطاقة في الطبيعة في صور مختلفة مثل الطاقة الميكانيكية *Mechanical energy*، والطاقة الكهرومغناطيسية *Electromagnetic energy*، والطاقة الكيميائية *Chemical energy*، والطاقة الحرارية *Thermal energy*، والطاقة النووية *Nuclear energy*. إن الطاقة بصورها المختلفة تتحول من شكل إلى آخر ولكن في النهاية الطاقة الكلية ثابتة. فمثلا الطاقة الكيميائية المختزنة في بطارية تتحول إلي طاقة كهربية لتتحول بدورها إلى طاقة حركية. ودراسة تحولات الطاقة مهم جداً لجميع العلوم.

Work and Energy

وفى هذا الجزء من المقرر سوف نركز على ***Mechanical energy***. وذلك لأنه يعتمد على مفاهيم القوة التي وضعها نيوتن في القوانين الثلاثة، ويجدر الذكر هنا أن الشغل والطاقة كميات قياسية وبالتالي فإن التعامل معها سيكون أسهل من استخدام قوانين نيوتن للحركة، وذلك لأننا كنا نتعامل وبشكل مباشر مع القوة وهى كمية متجهة. وحيث أننا لم نجد أية صعوبة في تطبيق قوانين نيوتن وذلك لأن مقدار القوة المؤثرة على حركة الأجسام ثابت، ولكن إذا ما أصبحت القوة متغيرة وبالتالي فإن العجلة ستكون متغيرة وهنا يكون التعامل مع مفهوم الشغل والطاقة اسهل بكثير في مثل هذه الحالات.

والشغل قد يكون ناتجاً من قوة ثابتة ***constant force*** أو من قوة متغيرة ***varying force***. وسوف ندرس كلا النوعين في هذا الفصل.

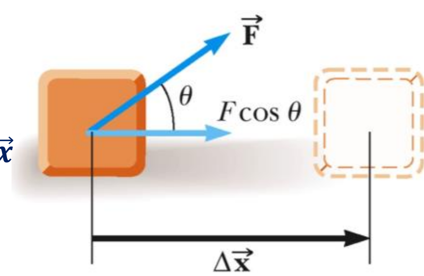
The Concept of Work

Work Done by a Constant Force

The work done by a constant force is defined as the distance moved multiplied by the component of the force in the direction of displacement:

$W \equiv (F\cos\theta)\Delta x$ $W = \vec{F}.\Delta\vec{x}$ **Work is a scalar quantity**

- **F is the magnitude of the force**
- **Δx is the magnitude of the**
- **object's displacement**
- **θ is the angle between $\vec{F}$ and $\Delta\vec{x}$**



$W \equiv F\Delta x$ When $\theta = 0$

SI Unit: Newton • meter = Joule
N • m = J

Example 1

Find the work done by a 45N force in pulling the luggage carrier at an angle $\theta = 50^o$ for a distance d = 75m.

Solution

$$W = (F\cos\theta)\, d$$

$$W = 45 \cos 50^o \times 75$$

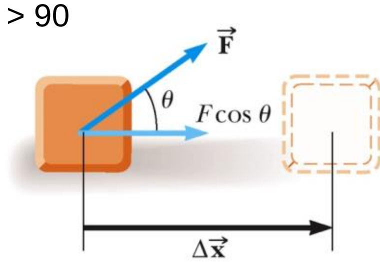
$$W = 2170 \text{ J}$$

Work can be Positive, Negative, or Zero

Work can be positive, negative, or zero. The sign of the work depends on the direction of the force relative to the displacement.

$$W = \vec{F}.\Delta\vec{x} \qquad W \equiv (F\cos\theta)\Delta x$$

- **Work positive:** W > 0 if 90 > θ > 0
- **Work negative:** W < 0 if 180 > θ > 90
- **Work zero:** W = 0 if θ = 90°
- **Work maximum if** θ = 0°
- **Work minimum if** θ = 180°



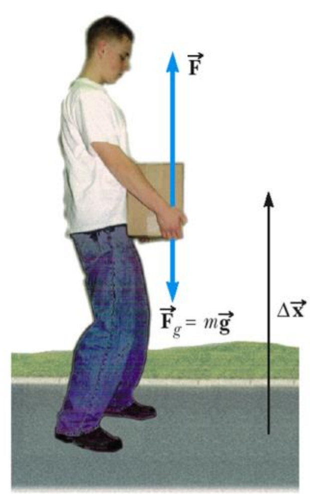
Example: When Work is Zero

- A man carries a bucket of wate horizontally at constant velocit
- The force does no work on the bucket
- Displacement is horizontal
- Force is vertical
- cos 90° = 0

$$W \equiv (F\cos\theta)\Delta x$$

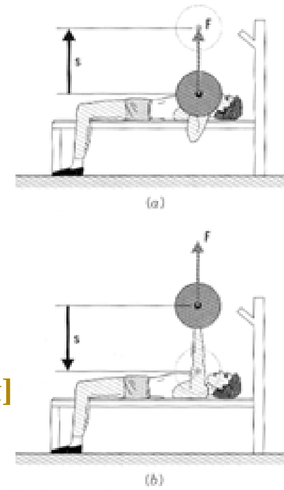
Example: Work can be Positive or Negative

- Man does positive work lifting box.
- Man does negative work lowering box.
- Gravity does positive work when box lowers.
- Gravity does negative work when box is raised.



Example 2

The weight lifter is bench-pressing a barbell whose weight is 710N. He raises the barbell a distance 0.65m above his chest and then lowers the barbell the same distance. Determine the work done on the barbell by the weight lifter during (a) the lifting phase and (b) the lowering phase.



Solution

(a) The work done during the lifting phase is

$$W = (F\cos\theta)\, d = 710 \cos 0^{o} \times 0.65 = 460\text{J}$$

[*Positive work*]

(b) The work done during the lowering phase is

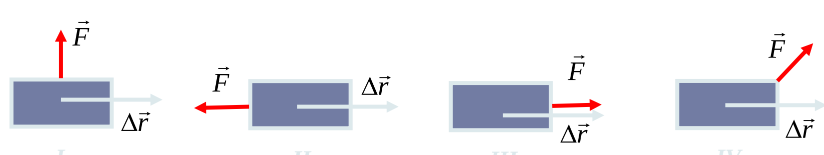
$$W = (F\cos\theta)\, d = 710 \cos 180^{o} \times 0.65 = -460\text{J}$$

[*Negative work*]

Work Done by a Constant Force

The work W done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δx of the displacement of the point of application of the force, and cosθ, where θ is the angle between the force and displacement vectors:

$$W = \vec{F}.\Delta\vec{x} = F\Delta x\cos\theta$$

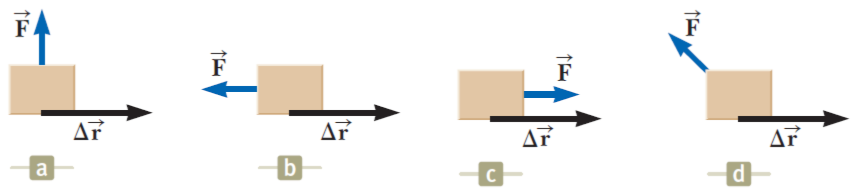


$W_I = 0$ $W_{II} = -F\Delta r$ $W_{III} = F\Delta r$ $W_{IV} = F\Delta r\cos\theta$

Quiz

(1) The gravitational force exerted by the Sun on the Earth holds the Earth in an orbit around the Sun. Let us assume that the orbit is perfectly circular. The work done by this gravitational force during a short time interval in which the Earth moves through a displacement in its orbital path is **(a)** zero **(b)** positive **(c)** negative **(d)** impossible to determine

(2) Figure below shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.



Example 3

A particle moving in the *xy* plane undergoes a displacement given by $\Delta\vec{r} = (2.0i + 3.0j)$ m as a constant force $\vec{F} = (5.0i + 2.0j)$ N acts on the particle. Calculate the work done by $\vec{F}$ on the particle.

Solution

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = [(5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\ \mathrm{N}] \cdot [(2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\ \mathrm{m}]$$

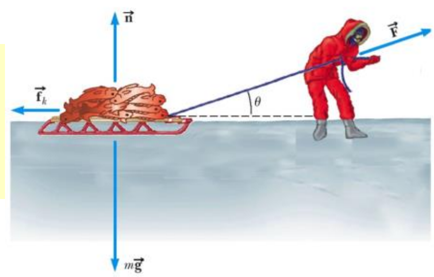
$$= (5.0\hat{\mathbf{i}} \cdot 2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{i}} \cdot 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{j}} \cdot 2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}} \cdot 3.0\hat{\mathbf{j}})\ \mathrm{N \cdot m}$$

$$= [10 + 0 + 0 + 6]\ \mathrm{N \cdot m} = 16\ \mathrm{J}$$

Example 4

An Eskimo pulls a sled as shown. The total mass of the sled is 50.0 kg, and he exerts a force of 1.20 × 10² N on the sled by pulling on the rope. How much work does he do on the sled if θ = 30° and he pulls the sled 5.0 m ?

$$W = (F\cos\theta)\Delta x$$
$$= (1.20\times10^2\,N)(\cos 30^\circ)(5.0m)$$
$$= 5.2\times10^2\,J$$



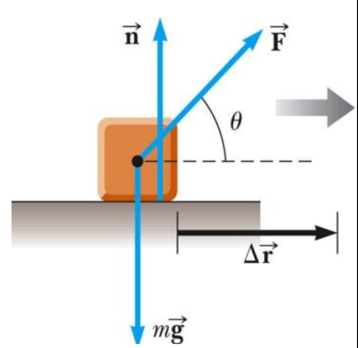
Work Done by Multiple Forces

If more than one force acts on an object, then the total work is equal to the algebraic sum of the work done by the individual forces

$$W_{net} = \sum W_{\text{by individual forces}}$$

- Remember work is a scalar, so this is the algebraic sum

$$W_{net} = W_g + W_N + W_F = (F\cos\theta)\Delta x$$



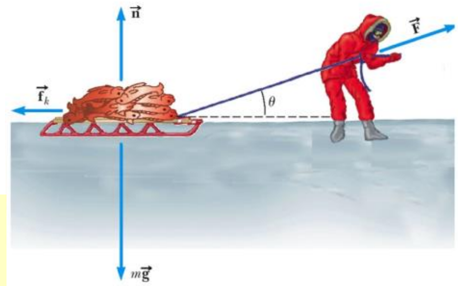
Example 5

In example 4 suppose μ_k = 0.200, How much work done on the sled by friction, and the net work if θ = 30° and he pulls the sled 5.0 m?

$$W_{net} = W_F + W_{fric} + W_N + W_g$$

$$F_{net,y} = N - mg + F\sin\theta = 0$$
$$N = mg - F\sin\theta$$

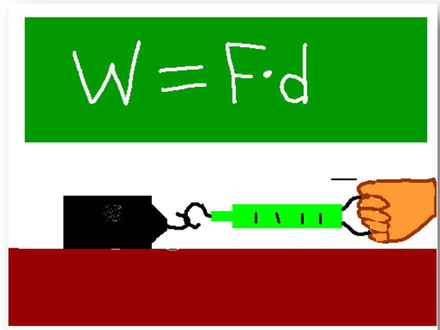
$$W_{fric} = (f_k \cos 180^\circ)\Delta x = -f_k \Delta x$$
$$= -\mu_k N \Delta x = -\mu_k (mg - F\sin\theta)\Delta x$$
$$= -(0.200)(50.0kg \cdot 9.8m/s^2$$
$$-1.2\times10^2 N \sin 30^\circ)(5.0m)$$
$$= -4.3\times10^2 J$$



$$W_{net} = W_F + W_{fric} + W_N + W_g$$
$$= 5.2\times10^2 J - 4.3\times10^2 J + 0 + 0$$
$$= 90.0 J$$

Problems to be solved by yourself

1. If a man lifts a 20-kg bucket from a well and does 6 kJ of work, how deep is the well? Assume the speed of the bucket remains constant as it is lifted.
2. A 65kg woman climbs a flight of 20 stairs, each 23 cm high. How much work was done against the force of gravity in the process?
3. A horizontal force of 150 N is used to push a 40-kg box on a rough horizontal surface through a distance of 6m. If the box moves at constant speed, find (a) the work done by the 150-N force, (h) the work done by friction.
4. If an applied force varies with position according to $F_{.} = 3x^3 - 5$, where x is in m, how much work is done by this force on an object that moves from x = 4 m to x = 7 m?



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Work done by varying force

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General Physics I

Mechanics: Principles & Applications

Unit4: Work and Energy

Lecture (14) Work Done by a Varying Force

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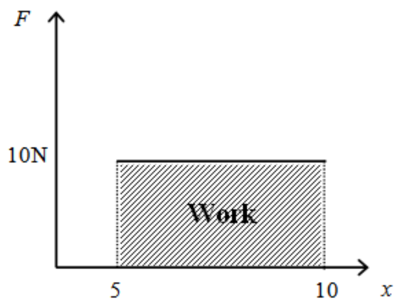
Unit4: Work and Energy

Work Done by a Varying Force

ذكرنا سابقا أن استخدام مفهوم الشغل سوف يساعدنا في التعامل مع الحركة عندما تكون القوة غير منتظمة، ولتوضيح ذلك دعنا نفترض قوة منتظمة قدرها **10N** تؤثر على جسم ليتحرك مسافة من x_i=5m إلى x_f=25m وبالتالي فإن الإزاحة مقدارها **20m**، ولتمثيل ذلك بيانياً نرسم محور القوة ومحور الإزاحة كما في الشكل، وبالتالي تكون القوة هي خط مستقيم يوازي محور *x*.

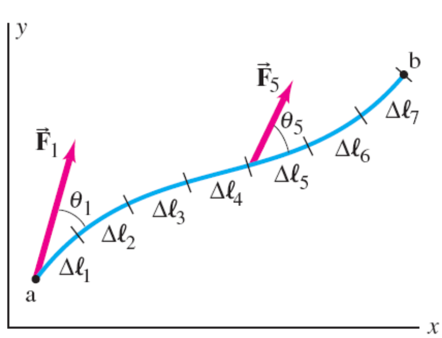


Work = *F s* = 10×(25-5) = 200J

وهذا عبارة عن المساحة تحت المنحنى وهي مساحة المستطيل الذي عرضه **10N** وطوله **20m**.

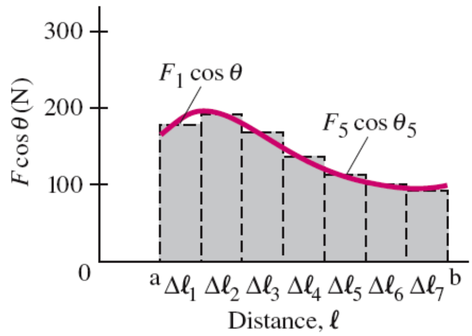
حالة كون القوة متغيرة خلال الإزاحة

Particle acted on by a varying force. Clearly, $\vec{F}.d$ ($F\cos\theta\, d$) is not constant!



A particle acted on by a variable force, F, moves along the path shown from point a to point b

For a force that varies, the work can be approximated by dividing the distance up into small pieces, finding the work done during each, and adding them up.



$$\Delta W = F_i \cos\theta_i \Delta l_i$$

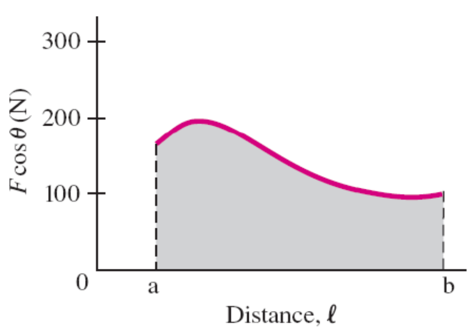
$$W = \sum_{i=1}^{7} \Delta W$$

$$W = \sum_{i=1}^{7} F_i \cos\theta_i \Delta l_i$$

Work done by a force *F* is approximately equal to the sum of the areas of the rectangles.

In the limit that the pieces become infinitesimally narrow, the work is the area under the curve:

$$W = \lim_{\Delta\ell_i \to 0} \Sigma F_i \cos\theta_i \, \Delta\ell_i = \int_a^b F \cos\theta \, d\ell.$$



$$W = \int_a^b \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}}.$$

Work done by a force *F* is exactly equal to the area under the curve of F cos θ vs. *l*.

Example 1

If an applied force varies with position according to $F_x = 3x^3 - 5$, where x is in m, how much work is done by this force on an object that moves from x=4m to x=7m?

Solution

$$F = 3x^3 - 5$$

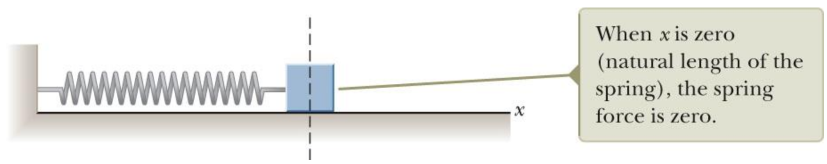
$$W = \int_{x_1}^{x_2} F\,dx \quad \Rightarrow \quad W = \int_4^7 (3x^3 - 5)\,dx$$

$$W = \left[\frac{3}{4}x^4 - 5x\right]_4^7$$

$$W = 1590 \text{ J}$$

Force of a Spring

a common physical system on which the force varies with position is a block on a frictionless, horizontal surface and connected to a spring.



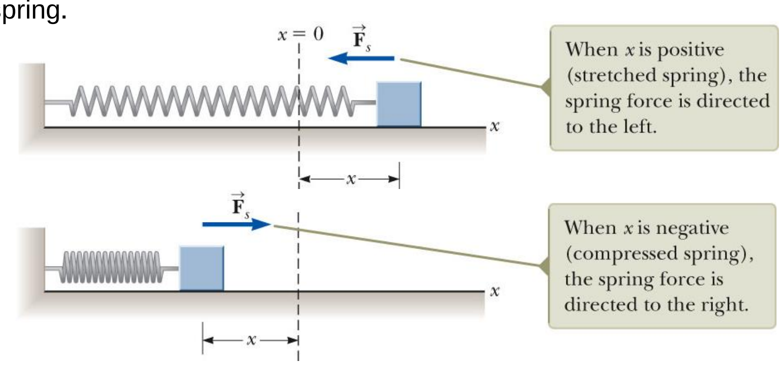
$$F = 0$$

Force of a Spring

If the spring is either stretched or compressed a small distance from its equilibrium position, it exerts on the block a force

$$F_x = -kx$$

where x is the position of the block relative to its equilibrium (x=0) position and k is a positive constant called the **spring constant** of the spring.



Hooke's Law

the force required to stretch or compress a spring is proportional to the amount of stretch or compression *x.* This force law for springs is known as **Hooke's law.**

$$F_x = -kx$$

The value of k is a measure of the *stiffness* of the spring. Stiff springs have large k values, and soft springs have small k values.

The negative sign in the equations signifies that the force exerted by the spring is always directed *opposite* the displacement from equilibrium.

Because the spring force always acts toward the equilibrium position (x = 0), it is sometimes called a *restoring force*.

Work Done by a Spring

Suppose the block has been pushed to the left to a position $-x_{max}$ and is then released.

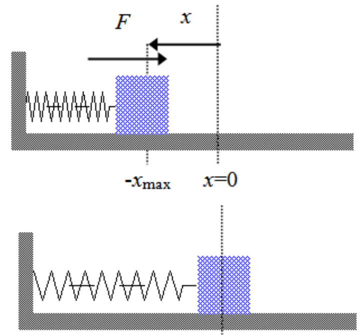
The work W_s done by the spring force on the block as the block moves from $x_i = -x_{max}$ to $x_f = 0$.

Applying equation

$$W = \int_{x_i}^{x_f} \vec{F}.d\vec{x}$$

$$W = \int_{-x_{max}}^{0} (-kx)\,dx$$

$$W = \frac{1}{2}kx^2{}_{max}$$



The work done by the spring force is positive because the force is in the same direction as its displacement

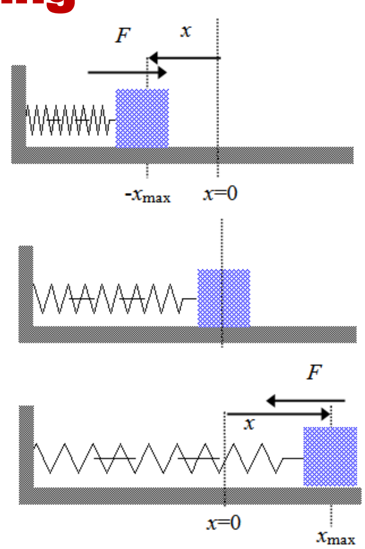
Work Done by a Spring

Because the block arrives at $x = 0$ with some speed, it will continue moving until it reaches a position $+x_{max}$.

The work done by the spring force on the block as it moves from $x_i = 0$ to $x_f = x_{max}$

$$W = -\frac{1}{2}kx^2{}_{max}$$

The work is negative because for this part of the motion the spring force is to the left and its displacement is to the right.



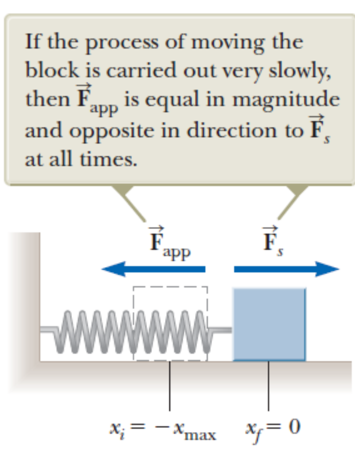
Net work done by a spring

If the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$, the work done by the spring force on the block is

$$W_s = \int_{x_i}^{x_f} (-kx)\, dx = \tfrac{1}{2}k{x_i}^2 - \tfrac{1}{2}k{x_f}^2$$

$$W_s = W_{-xmax \to 0} + W_{0 \to xmax} = \text{zero}$$

The work done on the block by an *external agent*



الشغل المبذول بواسطة مؤثر خارجي لتحريك الجسم المتصل بزنبرك ببطء من x_i=0 إلى x_f=-x_{max}

نعتبر أن القوة الخارجية F_{app} تساوي قوة الزنبرك F_s لان القوة الخارجية المطبقة تحرك الكتلة ببطء أي ان

$$F_{app} = -(-kx) = kx$$

الشغل المبذول بواسطة القوة الخارجية المطبقة على الكتلة:

$$W_{F_{app}} = \int_0^{x_m} F_{app}\, dx = \int_0^{x_m} kx\, dx = \frac{1}{2}k{x_m}^2$$

$$W_{F_{app}} = \frac{1}{2}k{x_m}^2$$

لاحظ أن الشغل المبذول بواسطة قوة خارجية تساوي سالب الشغل المبذول بواسطة قوة شد الزنبرك.

Measuring Spring Constant

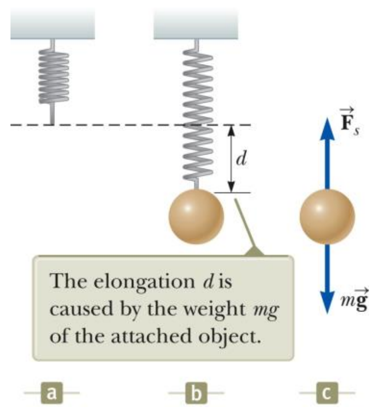
Start with spring at its natural equilibrium length.

Hang a mass on spring and let it hang to distance d From

$$F_x = kx - mg = 0$$

$$k = \frac{mg}{d}$$

so we can get spring constant.



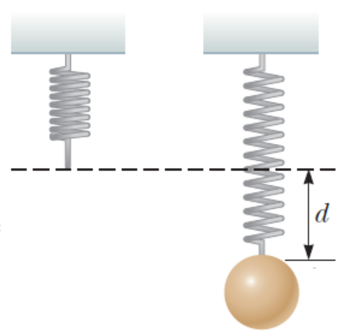
Example 2

The spring is hung vertically, and an object of mass *m* is attached to its lower end. Under the action of the "load" *mg,* the spring stretches a distance *d* from its equilibrium position

(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

(B) How much work is done by the spring on the object as it stretches through this distance?

(C) Evaluate the work done by the gravitational force on the object.



Solution

(A) Because the object is in equilibrium, the net force on it is zero and the upward spring force balances the downward gravitational force mg

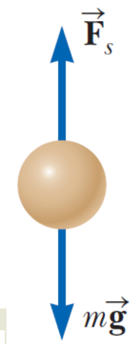
Apply the particle in equilibrium model to the object:

$$F_s - mg = 0 \quad \rightarrow \quad F_s = mg$$

Apply Hooke's law to give $F_s = kd$ and solve for k:

$$k = \frac{F_s}{d} \quad \rightarrow \quad k = \frac{mg}{d}$$

$$k = \frac{(0.55\ \text{kg})(9.80\ \text{m/s}^2)}{2.0 \times 10^{-2}\ \text{m}} = 2.7 \times 10^2\ \text{N/m}$$



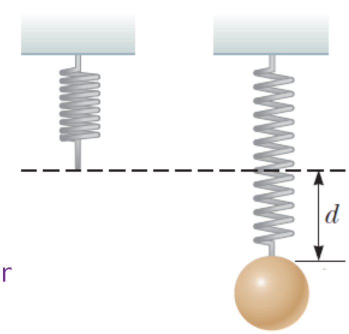
Solution

(B) How much work is done by the spring on the object as it stretches through this distance?

$$W_s = -\frac{1}{2}kd^2$$

$$W_s = -\frac{1}{2}(2.7 \times 10^2)(2 \times 10^{-2})^2$$

$$W_s = -5.4 \times 10^{-2}\text{J}$$



(B) Evaluate the work done by the gravitatior force on the object

$$W = \vec{F} \cdot \Delta\vec{r} = (mg)(d)\cos 0 = mgd$$

$$= (0.55\ \text{kg})(9.80\ \text{m/s}^2)(2.0 \times 10^{-2}\ \text{m}) = 1.1 \times 10^{-1}\text{J}$$

Problems to be solved by yourself

1. When a 4.00-kg object is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg object is removed, (a) how far will the spring stretch if a 1.50-kg block is hung on it? (b) How much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?

2. Hooke's law describes a certain light spring of unstretched length 35.0 cm. When one end is attached to the top of a doorframe and a 7.50-kg object is hung from the other end, the length of the spring is 41.5 cm. (a) Find its spring constant. (b) The load and the spring are taken down. Two people pull in opposite directions on the ends of the spring, each with a force of 190 N. Find the length of the spring in this situation.

المحاضرة القادمة

Work and Kinetic Energy

Physics Academy

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General Physics I

Mechanics: Principles & Applications

Unit4: Work and Energy

Lecture (15) Work and Kinetic Energy

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Unit4: Work and Energy

Introduction

- We have investigated work and identified it as a mechanism for transferring energy into a system.
- We have stated that work is an influence on a system from the environment, but we have not yet discussed the *result* of this influence on the system.
- One possible result of doing work on a system is that the system changes its speed.
- In this Lecture, we investigate this situation and introduce our first type of energy that a system can possess, called *kinetic energy*.

Work And Kinetic Energy

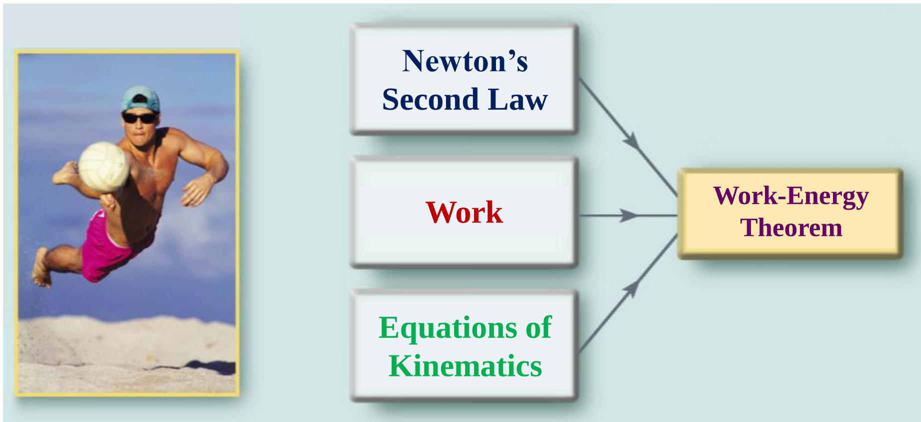
Energy: the ability to do work.

Kinetic Energy ≡ The energy of motion.

"Kinetic" ≡ Greek word for motion.

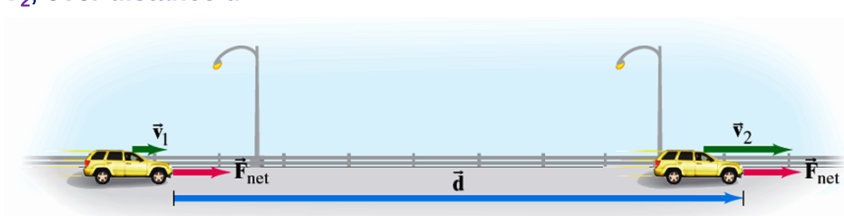
An object in motion has the ability to do work.

Work-Energy Theorem



Work and Kinetic Energy

Consider an object moving in straight line. Starts at speed v_1. Due to the presence of a net force F_{net}, it accelerates (uniformly) to speed v_2, over distance d.



$$F_{net} = ma \qquad (1)$$

At Constant acceleration

$$v_2{}^2 = v_1{}^2 + 2ad \quad \Rightarrow \quad v_2{}^2 - v_1{}^2 = 2ad$$

$$a = \frac{v_2{}^2 - v_1{}^2}{2d} \qquad (2)$$

Work done: $W_{net} = F_{net}d \quad (3)$

Combine (1), (2), (3):

$$F_{net} = ma \quad (1) \qquad a = \frac{v_2{}^2 - v_1{}^2}{2d} \quad (2) \qquad W_{net} = F_{net}d \quad (3)$$

$$W_{net} = mad \rightarrow W_{net} = md\left[\frac{v_2{}^2 - v_1{}^2}{2d}\right]$$

$$W_{net} = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2$$

The product of one half the mass and the square of the speed is defined as the ***kinetic energy*** of the particle and has a unit of J

$$k = \frac{1}{2}mv^2$$

The **SI Units** are **Joules**

The net work done by a constant force in accelerating an object of mass m from v_1 to v_2 is:

$$W_{net} = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2$$

$$W_{net} = K_f - K_i$$

This means that the work is the change of the kinetic energy of a particle.

$$W_{net} = \Delta K$$

The Work-Energy Theorem

The net work on an object = The change in kinetic energy

If the net work is positive, the kinetic energy increases. If the net work is negative, the kinetic energy decreases.

لاحظ أن طاقة الحركة K دائما موجبة ولكن التغير في طاقة الحركة يمكن أن يكون سالباً أو موجباً أو صفراً.

The Work-Energy Theorem

When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system.

$$W = KE_f - KE_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{net} = \Delta K$$

NOTE!

This is Newton's 2nd Law in

Work & Energy Language!

Example 1

How much net work is required to accelerate a 1000-kg car from 20 m/s to 30 m/s?

Find W_{net} to accelerate the 1000 kg car.

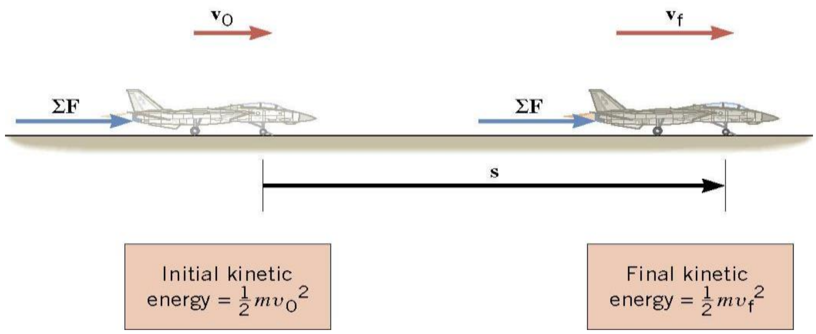
$W_{net} = \Delta K = K_2 - K_1 = \frac{1}{2}m\,v_2^2 - \frac{1}{2}m\,v_1^2$

$W_{net} = \frac{1}{2}(10^3 kg)(30 m/s)^2 - \frac{1}{2}(10^3 kg)(20 m/s)^2$

$W_{net} = 450{,}000 J - 200{,}000 J = 2.50\times10^5 J$

Example 2

A fighter-jet of mass 5×10^4kg is travelling at a speed of v_i=1.1×10^4m/s. The engine exerts a constant force of 4×10^5N for a displacement of 2.5×10^6m. Determine the final speed of the jet.



Solution

According to equation of work, the work done on the engine is

$$W = (F\cos\theta)\, s = 4\times10^5 \cos 0^o \times 2.5\times10^6 = 1\times10^{12}\text{J}$$

The work is positive, because the force and displacement are in the same direction.

Since $W = K_f - K_i$ the final kinetic energy of the fighter jet is

$$K_f = W + K_i$$

$$= (1\times10^{12}\text{J}) + ½\,(5\times10^4\text{kg})\,(1\times10^4\text{m/s})^2 = 4.031\times10^{12}\text{J}$$

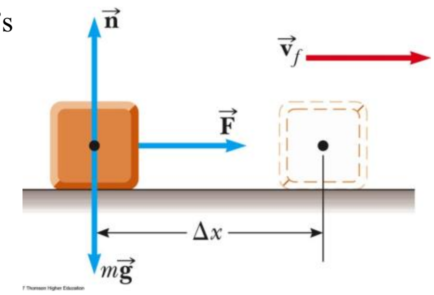
The final kinetic energy is $K_f = ½\, mv_f^2$, so the final speed is

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(4.03\times10^{12})}{5\times10^4}} = 1.27\times10^4\, m/s$$

حيث أن المحرك يبذل شغلاً موجباً لذا كانت السرعة النهائية أكبر من السرعة الابتدائية.

Example 3

A block of mass **m = 6 kg**, is pulled from rest ($v_0 = 0$) to the right by a constant horizontal force **F = 12 N**. After it has been pulled for **Δx = 3 m**, find it's final speed **v**.



Solution

$$W_{net} = \Delta K \equiv (½)[m(v)^2 - m(v_0)^2] \quad (1)$$

If **F = 12 N** is the only horizontal force,

then $$W_{net} = F\Delta x \quad (2)$$

Combine **(1)** & **(2):**

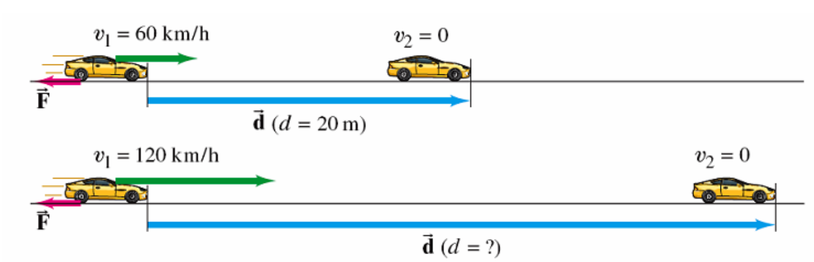
$$F\Delta x = (½)[m(v)^2 - 0]$$

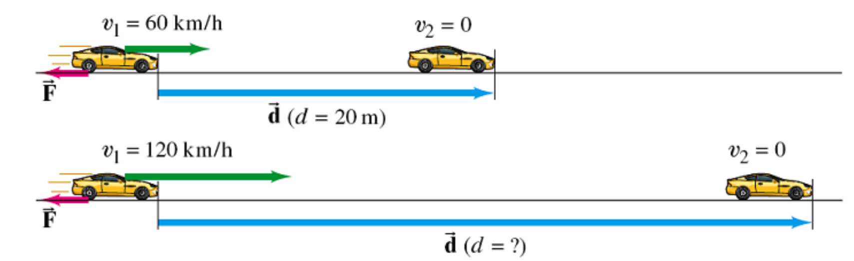
Solve for **v:** $(v)^2 = [2F\Delta x/m]$

$$v = [2F\Delta x/m]^{½} = 3.5 \text{ m/s}$$

Example 4

A car traveling at speed v_1 = 60 km/h can brake to a stop within a distance d = 20 m. If the car is going twice as fast, 120 km/h, what is its stopping distance? Assume that the maximum braking force is approximately independent of speed.





W_{net} = Fd cos (180°) = -Fd (from the definition of work)

W_{net} = ΔK = (½)m$(v_2)^2$ – (½)m$(v_1)^2$ (Work-Energy Principle)

but, **$(v_2)^2 = 0$** (the car has stopped) so **-Fd = ΔK = 0 - (½)m$(v_1)^2$**

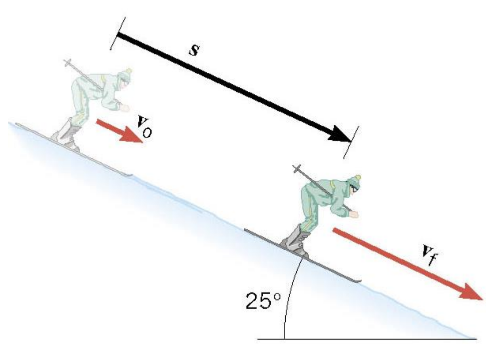
or **$d \propto (v_1)^2$**

So the stopping distance is proportional to the square of the initial speed! If the initial speed is doubled, the stopping distance quadruples!

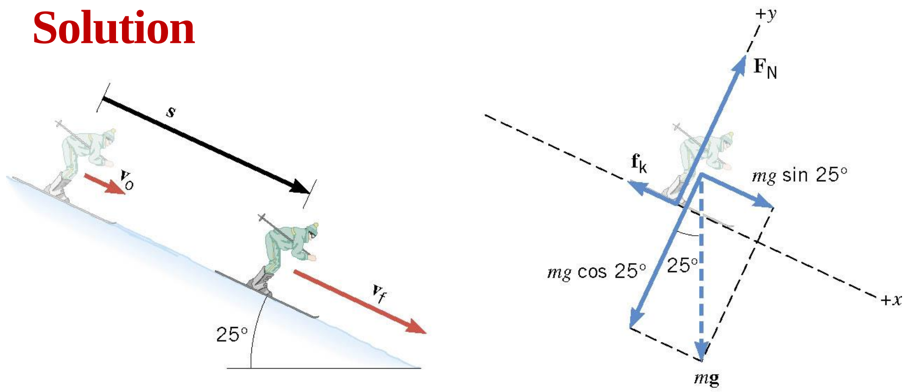
Then: d = 80 m

Example 5

A 58 kg skier is coasting down a 25^0 slope. A kinetic frictional force of magnitude f_k=70N opposes her motion. Near the top of the slope, the skier's speed is v_0=3.6m/s. Ignoring air resistance, determine the speed v_f at a point that is displaced 57m downhill.



Solution



(a) (b) Free-body diagram for the skier

$$\sum F = mg\sin 25° - f_k = (58kg)(9.80m/s^2)\sin 25° - 70N$$

F =+170 N

Solution

$$W = (\sum F\cos\theta)s = (170N)\cos 0°(57m) = 9700J$$

KE_f = W + KE_0

KE_f = 9700J + (1/2)(58kg)(3.6m/s)^2

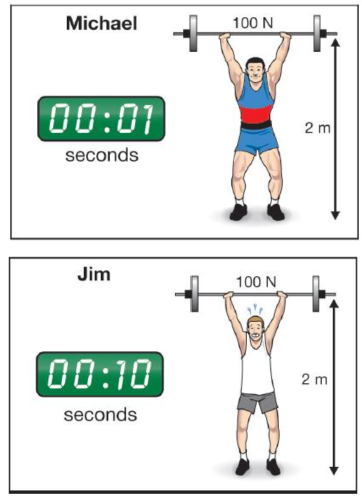
KE_f = 10100J

$$v_f = \sqrt{\frac{2(KE_f)}{m}} = \sqrt{\frac{2(10100J)}{58kg}} = 19m/s$$

Power

The power is defined as the time rate of energy transfer. **If an external force is applied to an object, and if the work done by this force is ΔW in the time interval Δt, then the average power is:**

$$P_{ave} = \frac{\Delta W}{\Delta t}$$



The instantaneous power is given by

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$P = \frac{dW}{dt} = F.\frac{dx}{dt} \qquad \therefore P = F.v$$

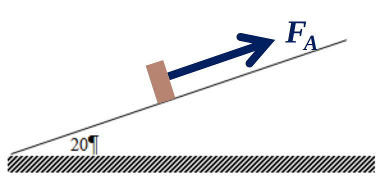
The unit of the power is J/s which is called watt (W).

Example 6

A 65-kg athlete runs at constant speed a distance of 600 m up a mountain inclined at 20º to the horizontal. He performs this feat in 80s. Assuming that air resistance is negligible,

(a) how much work does he perform and

(b) what is his power output during the run?



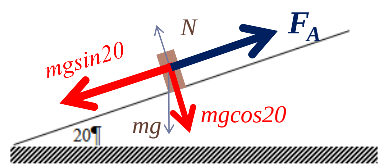
Solution

Since the athlete runs at constant speed, we have

$$W_A + W_g = 0$$

where W_A is the work done by the athlete and W_g is the work done by gravity. In this case,

$$W_g = -mgs(\sin\theta)$$



So

$$W_A = -W_g = + mgs(\sin\theta)$$

$$= (65kg)(9.80m/s^2)(600m)\sin 20^o$$

(b) His power output is given by

$$P_A = \frac{W_A}{\Delta t} = \frac{1.31\times10^5 J}{80s} = 1630W$$

Problems to be solved by yourself

1. A 200kg cart is pulled along a level surface by an engine. The coefficient of friction between the carte and surface is 0.4. (a) How much power must the engine deliver to move the carte at constant speed of 5m/s? (b) How much work is done by the engine in 3min?
2. A 1500-kg car accelerates uniformly from rest to a speed of 10 m/s in 3s. Find (a) the work done on the car in this time, (b) the average power delivered the engine in the first 3s, and (c) the instantaneous power delivered by the engine at t = 2s.
3. A woman raises a 10-kg flag front the ground to the top of a 10-m flagpole at constant velocity, 0.25 m/s. (a)Find the work done by the woman while raising the flag. (b) Find the work done by gravity. (c) What is the power output of the woman while raising the flag?
4. A 700-N marine in basic training climbs a 10-m vertical rope at uniform speed in 8 s. What is his power output?
5. A mechanic pushes a 2500kg car from rest to a speed v doing 5000J of work in the process. During this time, the car moves 25m. Neglecting friction between the car and the road, (a) What is the final speed, v, of the car? (b) What is the horizontal force exerted on the car?

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Work and Potential Energy

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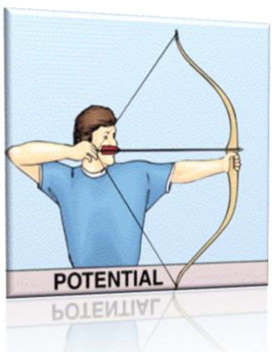
General Physics I

Mechanics: Principles & Applications



Unit5: Potential Energy and Conservation Energy

Lecture (16) Work and Potential Energy

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Unit 5: Potential Energy And Conservation Energy

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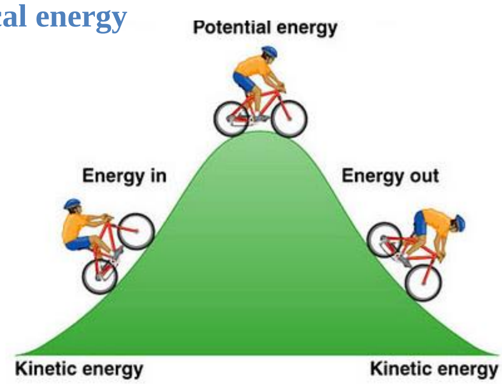


Potential Energy and Conservation Energy

❑ درسنا في المحاضرة السابقة مفهوم طاقة الحركة *Kinetic energy* لجسم متحرك ووجدنا أن طاقة حركة الجسم تتغير عندما يبذل شغل على الجسم.

❑ سندرس في هذه المحاضرة نوعاً آخر من أنواع الطاقة الميكانيكية وهو طاقة الوضع *Potential energy*. ويمكن لطاقة الوضع أن تتحول إلى طاقة حركة أو إلى بذل شغل.

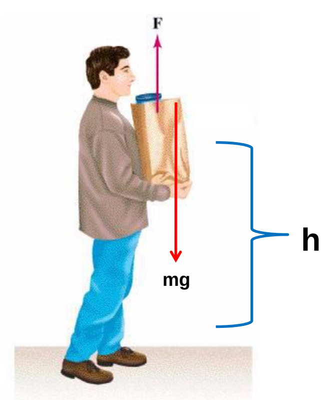
❑ **Potential energy is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.**

❑ **Some forms of potential energy:**

1. Gravitational Potential Energy,
2. Elastic Potential Energy

Lifting Mass at a Constant Speed

Suppose you lift a mass upward at a constant speed, $\Delta v = 0$ & $\Delta K=0$. What does the work equal now?

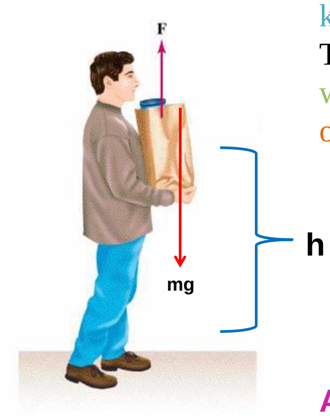


Since you are lifting at a constant speed, your APPLIED FORCE equals the WEIGHT of the object you are lifting.

$$F = mg$$

When you lift an object above the ground it is said to have POTENTIAL ENERGY!

Potential Energy



Since this man is lifting the package upward at a CONSTANT SPEED, the kinetic energy is NOT CHANGING. Therefore the work that he does goes into what is called the ENERGY OF POSITION or POTENTIAL ENERGY.

$$W = \vec{F}\vec{x}\cos\theta \quad F = mg; x = h$$

$$\theta = 0, \cos 0 = 1$$

$$W = mgh = PE$$

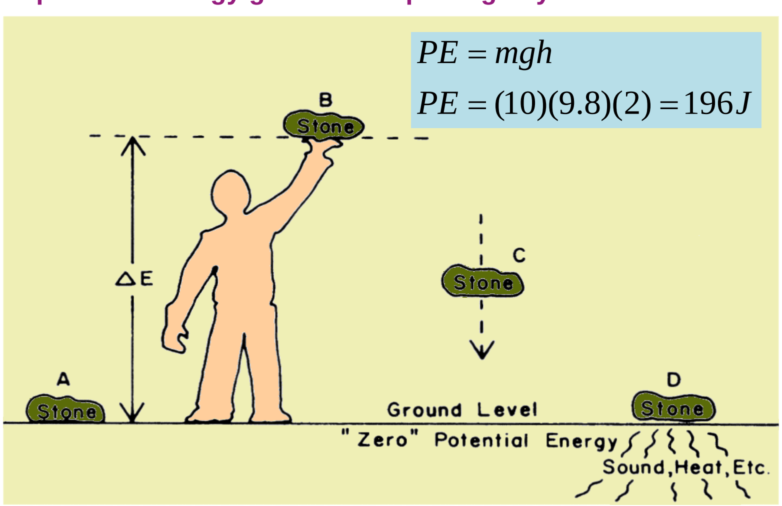
All Potential Energy is considering to be energy that is STORED!

Example 1

The man lifts a 10 kg package 2 meters above the ground. What is the potential energy given to the package by the man?

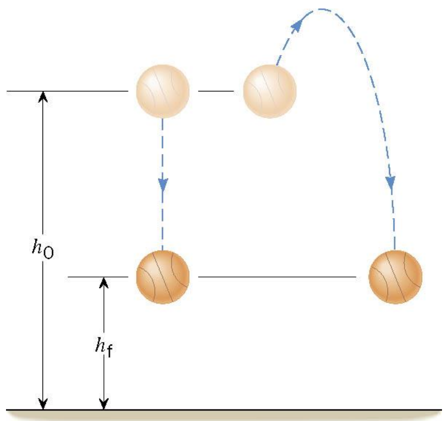
$$PE = mgh$$

$$PE = (10)(9.8)(2) = 196J$$



Conservative Forces

تجدر الإشارة هنا إلى أن أنواع القوى التي درسناها هي إما قوة عجلة الجاذبية الأرضية (F_g) أو قوة الاحتكاك (f) أو قوة الشد (T) أو القوة المؤثرة الخارجية (F_{app}).



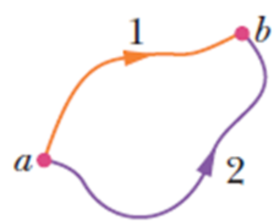
هذه القوى تقسم إلى نوعين، إما قوى محافظة conservative forces أو قوى غير محافظة non-conservative.

إذا كان الشغل الناتج عن قوة ما لا يعتمد على المسار فإن هذه القوة تكون محافظة.

أما إذا كان الشغل يعتمد على المسار فإن هذه القوة تكون غير محافظة.

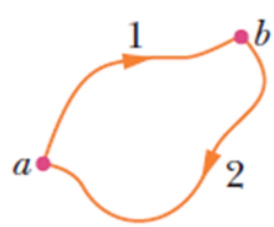
Conservative Forces

A force is conservative when the work done by that *force* acting on a particle moving between two points is *independents* of the path the particle takes between the points.



$$W_{ap}(along\ 1) = W_{ab}(along\ 2)$$

The total work done by a conservative force on a particle is zero when the particle moves around any closed path and returns to its initial position.



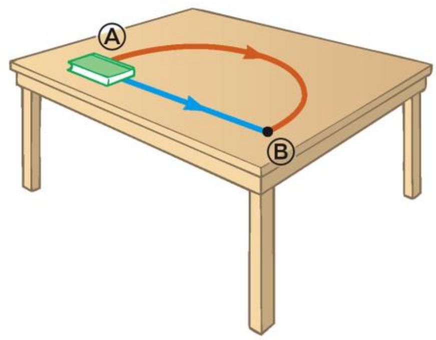
$$W_{ap}(along\ 1) = -W_{ab}(along\ 2)$$

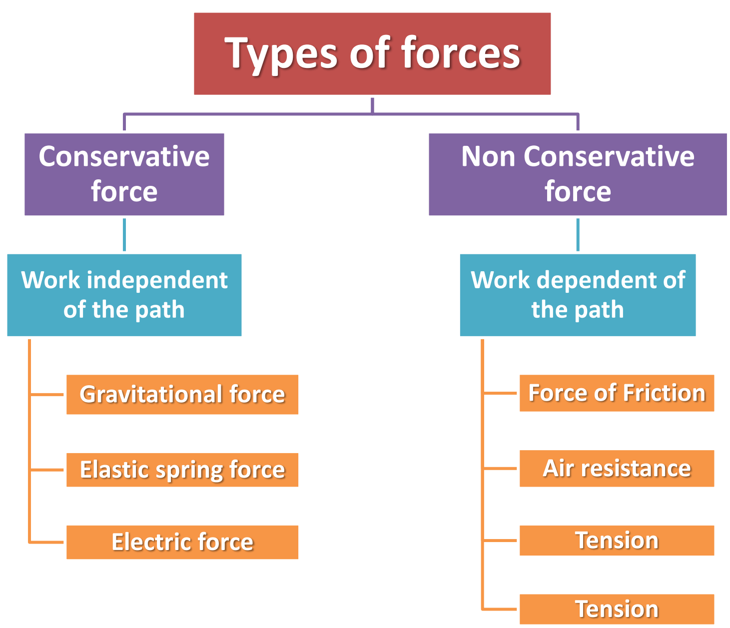
Non-conservative Forces

A non-conservative force is one where the work done depends on the path taken.

Friction is an example of a non-conservative force

- The work done depends on the path
- The red path will take more work than the blue path





أمثلة على أنواع القوى المحافظة

تعتبر **قوة الجاذبية الأرضية** مثالاً على القوة المحافظة، فعند نقل جسم من موضع إلى آخر فإن الشغل المبذول يعتمد على القوة mg وعلى الإزاحة بين نقطتي البداية والنهاية، ولا يعتمد الشغل على المسار فإذا كانت نقطة البداية والنهاية لها نفس الارتفاع عن سطح الأرض فإن الشغل يكون صفراً.

$$W_g = -mg(y_f - y_i)$$

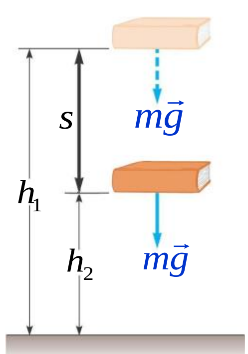
الشغل لا يعتمد على المسار عند نقل جسم من موضع آخر لأن قوة الجاذبية الأرضية قوة محافظة.

كما وأن **القوة الاسترجاعية للزنبرك** قوة محافظة حيث أن الشغل يعتمد على نقطتي البداية والنهاية فقط ولا يعتمد على المسار، وقد لاحظنا في الفصل السابق أن الشغل المبذول بواسطة الزنبرك يساوي صفراً في حركة الزنبرك دورة كاملة حيث يكون فيها نقطة النهاية هي العودة إلى نقطة البداية.

$$W_s = \tfrac{1}{2} kx^2$$

Work-gravitational potential energy theorem

Consider a book with mass, m is dropped from height, h_1 to height, h_2 as shown in the Figure. The work done by the gravitational force (weight) is



$$W_g = mgs = mg(h_1 - h_2)$$

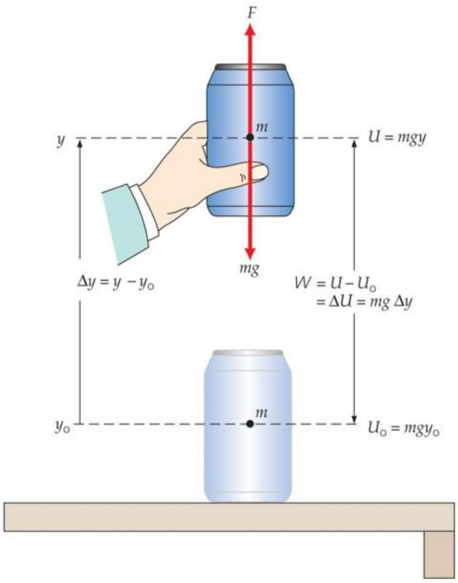
$$W_g = mgh_1 - mgh_2 = U_i - U_f$$

$$W_g = -(U_f - U_i) = -\Delta U$$

Therefore in general, $W = -\Delta U$

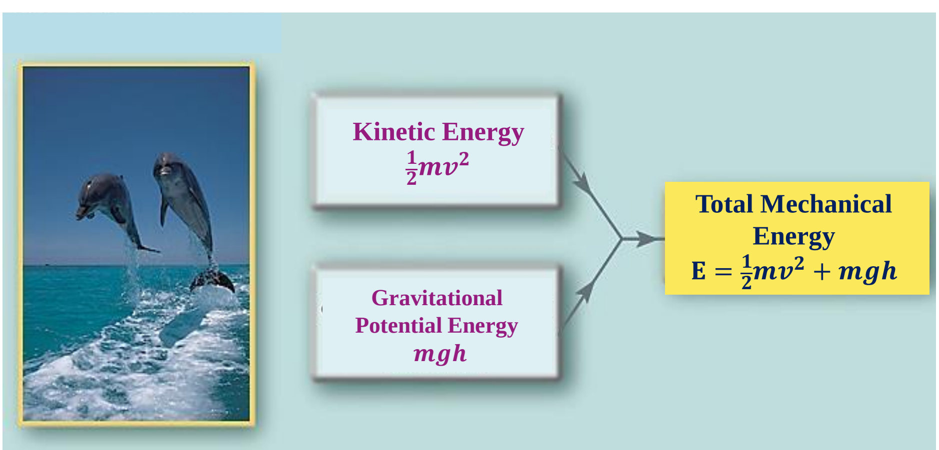
States "**the change in gravitational potential energy as the negative of the work done by the gravitational force**".

$$\Delta U = U_f - U_i = -W$$



- علمنا أن الشغل يساوى التغير في طاقة الحركة، ولكن إذا تحرك جسم تحت تأثير قوة محافظة مثل قوة عجلة الجاذبية الأرضية إزاحة محددة فإن الشغل هنا يعتمد على نقطتي البداية والنهاية ولا يعتمد على المسار.
- في هذه الحالة لا نستطيع القول أن الشغل يساوى التغير في طاقة الحركة.
- إذا حاول شخص رفع كتلة ما من سطح الأرض إلى ارتفاع معين قدره h فإن هذا الشخص سيبذل شغلاً موجباً مساوياً لـ mgh لان القوة التي بذلها في اتجاه الحركة، ولكن من وجهة نظر الجسم فإنه بذل شغلاً سالباً قدره $-mgh$ وذلك لأن قوته (وزنه) في عكس اتجاه الإزاحة،
- هذا الشغل السالب يدعى طاقة الوضع التي اكتسبها الجسم عند تحريكه من نقطة إلى أخرى تحت تأثير قوة محافظة (قوة عجلة الجاذبية الأرضية).

The Conservation of Mechanical Energy



Conservation of Mechanical Energy

لنفترض وجود جسم يتحرك في بعد واحد x تحت تأثير قوة محافظة F_x, فإن الشغل المبذول بواسطة القوة يساوي التغير في طاقة حركة الجسم.

$$W = \Delta K = -\Delta U$$

$$\Delta K = -\Delta U$$

$$\Delta K + \Delta U = \Delta(K + U) = 0$$

$$\Delta E = 0$$

This is the law of conservation of mechanical energy, which can be written as

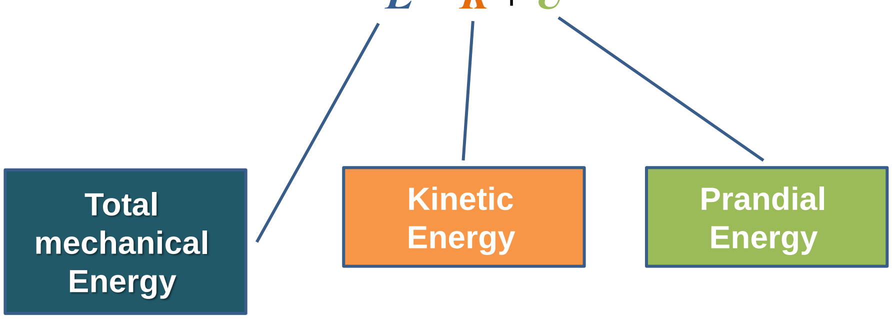
$$K_i + U_i = K_f + K_f$$

Law of conservation mechanical energy

Total Mechanical Energy

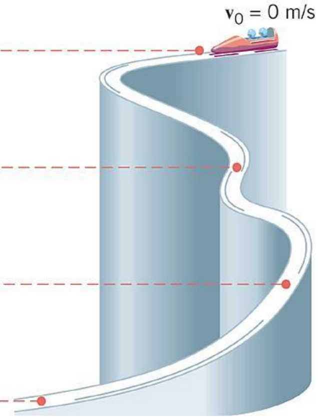
لنعرف الطاقة الميكانيكية الكلية ***Total mechanical energy*** بحاصل جمع طاقة الحركة وطاقة الوضع للجسم.

$$E = K + U$$



The Principle of Conservation of Mechanical Energy

KE	PE	E = KE + PE
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J



The total mechanical energy (E = KE + PE) of an object remains constant for conservative force only.

Law of Conservation Mechanical Energy

$$\Delta E = 0$$

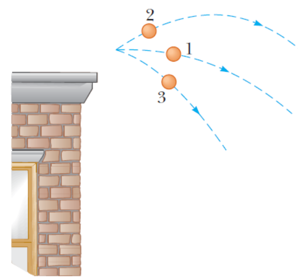
$$E_f - E_i = 0$$

$$E_i = E_f$$

Law of conservation mechanical energy

The law of conservation of mechanical energy states that the total mechanical energy of a system remains constant for conservative force only. This means that when the kinetic energy increased the potential energy decrease.

Quiz

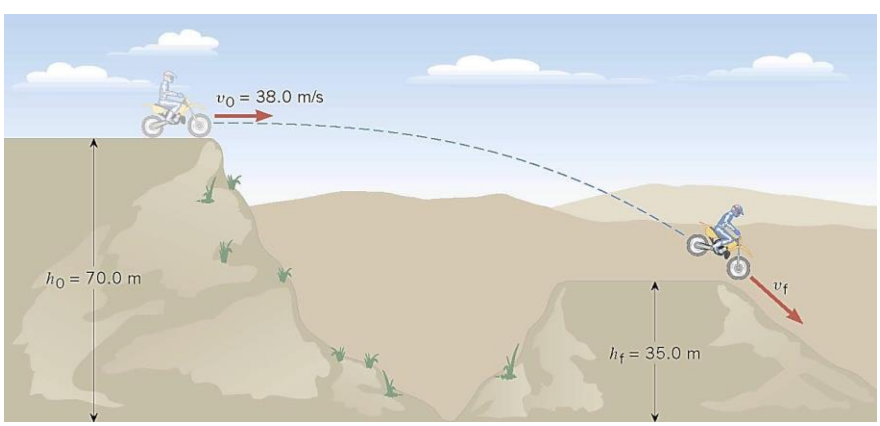


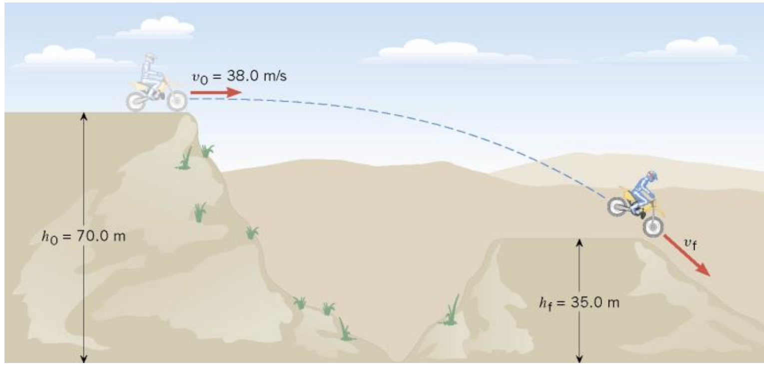
(1) Three identical balls are thrown from the top of a building, all with the same initial speed. The first ball is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal, as in the Figure. Neglecting air resistance, rank the speeds of the balls as they reach the ground, from fastest to slowest. (a) 1, 2, 3 (b) 2, 1, 3 (c) 3, 1, 2 (d) all three balls strike the ground at the same speed.

(2) Ali, of mass *m*, drops from a tree branch at the same time that Salah, also of mass *m*, begins his descent down a frictionless slide. If they both start at the same height above the ground, which of the following is true about their kinetic energies as they reach the ground?
(a) Ali's kinetic energy is greater than Salah's.
(b) Salah's kinetic energy is greater than Ali's.
(c) They have the same kinetic energy.
(d) The answer depends on the shape of the slide.

Example 2

A motorcyclist is trying to leap across the canyon by driving horizontally off a cliff at 38.0 m/s. Ignoring air resistance, find the speed with which the cycle strikes the ground on the other side.





$$E_f = E_o$$

$$mgh_f + \tfrac{1}{2}mv_f^2 = mgh_o + \tfrac{1}{2}mv_o^2$$

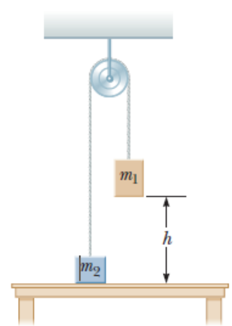
$$gh_f + \tfrac{1}{2}v_f^2 = gh_o + \tfrac{1}{2}v_o^2$$

$$v_f = \sqrt{2g(h_o - h_f) + v_o^2}$$

$$v_f = \sqrt{2(9.8)(35) + (38)^2}$$
$$= 46.2\,\text{m/s}$$

Problems to be solved by yourself

1. Use conservation of energy to determine the final speed of a mass of 5.0kg attached to a light cord over a massless, frictionless pulley and attached to another mass of 3.5 kg when the 5.0 kg mass has fallen (starting from rest) a distance of 2.5 m as shown in the Figure



2. A 0.5-kg ball is thrown vertically upward with an initial speed of 16 m/s. Assuming its initial potential energy is zero, find its kinetic energy, potential energy, and total mechanical energy (a) at its initial position, (b) when its height is 5m, and (c) when it reaches the top of its flight. (d) Find its maximum height using the law of conservation of energy.

المحاضرة القادمة

Examples on Work and Potential Energy

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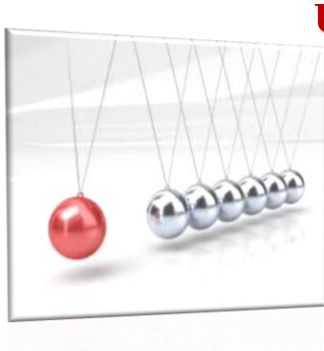
Mechanics: Principles & Applications



Unit5: Potential Energy and Conservation Energy

Lecture (17) Examples on Work and Potential Energy

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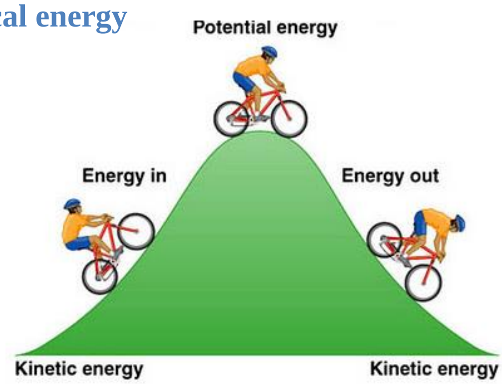
Unit 5: Potential Energy And Conservation Energy

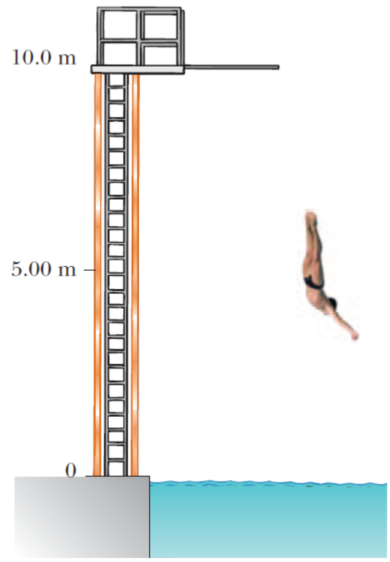


Example 1

A diver of mass *m* drops from a board 10.0 m above the water's surface, as in the Figure. Neglect air resistance.

(a) Use conservation of mechanical energy to find his speed 5.00 m above the water's surface.

(b) Find his speed as he hits the water.



Solution

a) find his speed 5.00 m above the water's surface.

$$KE_i + PE_i = KE_f + PE_f$$

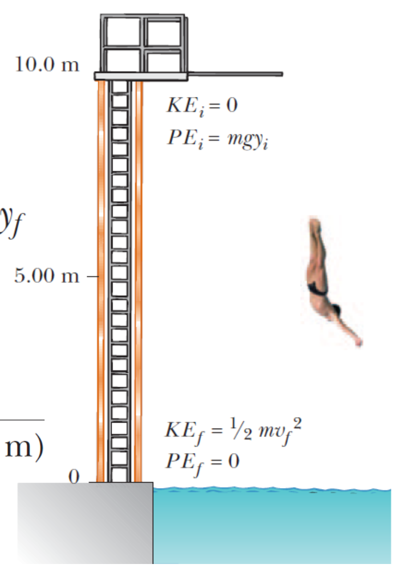
$$\tfrac{1}{2}mv_i^2 + mgy_i = \tfrac{1}{2}mv_f^2 + mgy_f$$

$$0 + gy_i = \tfrac{1}{2}v_f^2 + gy_f$$

$$v_f = \sqrt{2g(y_i - y_f)}$$

$$= \sqrt{2(9.80\ \mathrm{m/s^2})(10.0\,\mathrm{m} - 5.00\ \mathrm{m})}$$

$$v_f = 9.90\ \mathrm{m/s}$$



Solution

b) Find his speed as he hits the water.

$$KE_i + PE_i = KE_f + PE_f$$

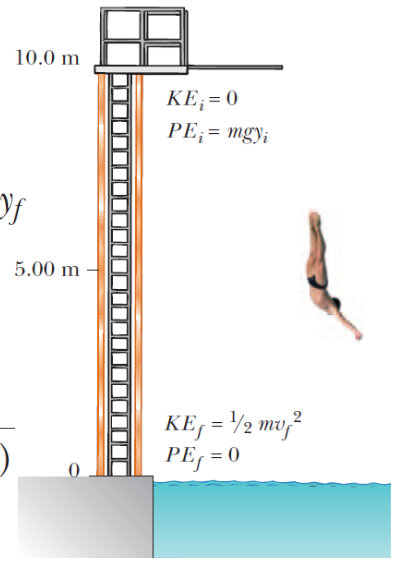
$$\tfrac{1}{2}mv_i^2 + mgy_i = \tfrac{1}{2}mv_f^2 + mgy_f$$

$$0 + mgy_i = \tfrac{1}{2}mv_f^2 + 0$$

$$v_f = \sqrt{2gy_i}$$

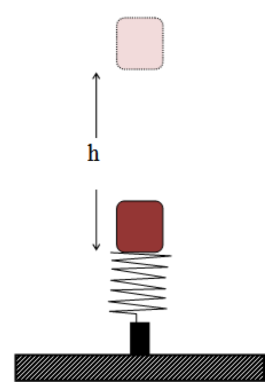
$$= \sqrt{2(9.80\ \mathrm{m/s^2})(10.0\ \mathrm{m})}$$

$$= 14.0\ \mathrm{m/s}$$



Example 2

A block of mass 0.25kg is placed on a vertical spring of constant k = 5000N/m, and is pushed downward compressing the spring a distance of 0.1 m. As the block is released, it leaves the spring and continues to travel upward. To what maximum height above the point of release does the block rise?



Solution

Taking $U_g = 0$ to be at the point of release, and noting that = $v_i = 0$, gives

$$E_i = K_i + U_i = 0 + (U_s + U_g)_i$$

$$E_i = \frac{1}{2} k x^2 + 0 = 25J$$

When the mass reaches its maximum height h, $v_f = 0$, and the spring is unstretched, so $U_S = 0$.

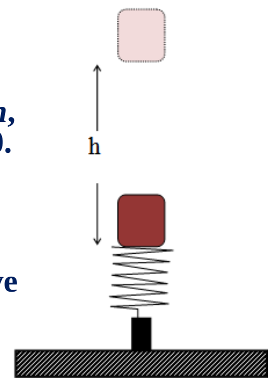
$$E_f = K_f + U_f = 0 + mgh$$

$$E_f = (0.25 \text{ kg})(9.80 \text{ m/s}^2)\, h$$

Since mechanical energy is conserved, we have $E_f = E_i$, or

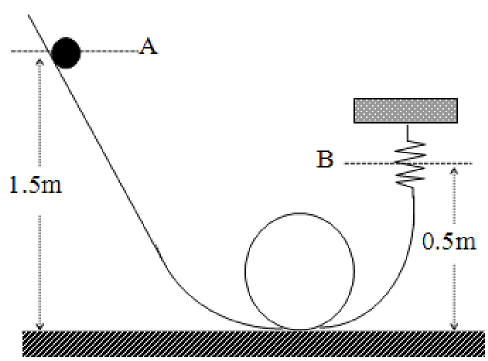
$$(0.25 \text{ kg})(9.80 \text{ m/s}^2)\, h = 25J$$

$$h = 10.2\text{m}$$



Example 3

A 0.2 kg ball is forced to slide on a frictionless wire as in the Figure. The ball starts from rest at A and ends up at B after colliding with a light spring of force constant k. If the spring compresses a distance of 0.1 m, what is the force constant of the spring?



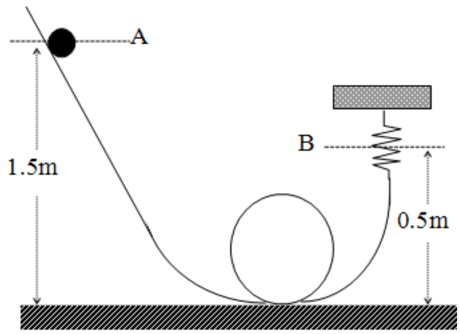
Solution

$$K_i + U_i = K_f + U_f$$

The gravitational potential energy of the ball at A -with respect to the lowest point is

$$U_{gi} = mgh_i = (0.2\ kg)\ (9.8\ m/s^2)\ (1.5\ m) = 2.94\ J$$

The kinetic energy of the bead at A is zero since it starts from rest.



The gravitational potential energy of the ball at B is

$$U_{gf} = mgh_f = (0.2\ kg)\ (9.8\ m/s^2)\ (0.5\ m) = 0.98\ J$$

Solution

Since the spring is part of the system, we must also take into account the energy stored in the spring at B. Since the spring compresses a distance $x_m = 0.1$m, we have

$$U_S = \tfrac{1}{2}\, k\, xm^2 = \tfrac{1}{2}\, k\, (0.1)^2$$

Using the principle of energy conservation gives

$$E_i = E_f$$

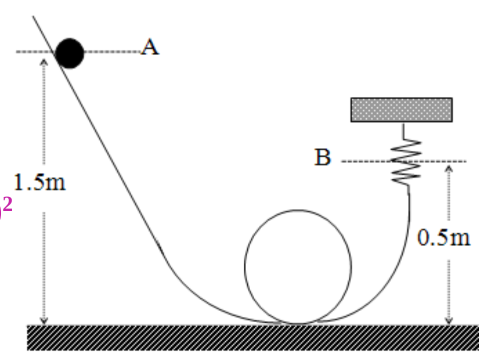
$$K_i + U_i = K_f + U_f$$

$$0 + U_{gi} = 0 + U_{gf} + U_s$$

$$U_{gi} = U_{gf} + U_s$$

$$2.94\ J = 0.98\ J + \tfrac{1}{2}\, k\, (0.1)^2$$

$$k = 392\ N/m$$

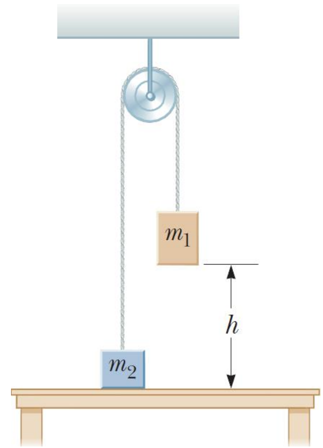


Example 4

Two masses are connected by a light string passing over a light frictionless pulley as shown in Figure. The 5-kg mass is released from rest. Using the law of conservation of energy,

(a) determine the velocity of the 3kg mass just as the 5kg mass hits the ground.

(b) Find the maximum height to which the 3kg mass will rise.



Solution

من قانون الحفاظ على الطاقة الميكانيكية

$$E_f = E_i$$

$$E_i = K_i + U_i = 0 + Mgh$$

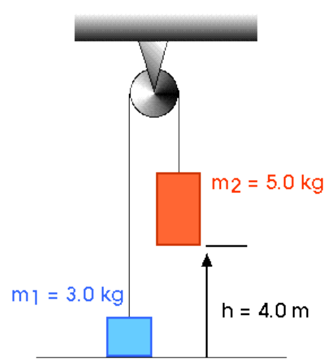
$$E_f = K_f + U_f = \frac{1}{2} Mv^2 + \frac{1}{2} mv^2 + mgh$$

$$\frac{1}{2} (m+M)v^2 + mgh = Mgh$$

$$\frac{1}{2} (m+M)v^2 = (M-m)gh$$

$$v^2 = \frac{2gh(M-m)}{(M+m)}$$

$$v = 4.4 \text{ m/s}$$



Solution

عندما تصل الكتلة الكبيرة الأرض تكون قطعت مسافة 4 متر في
حين ان الكتلة الصغيرة ترتفع 4 متر وتمتلك طاقة حركة تجعلها
ترتفع مسافة إضافية إلى ان تتحول الطاقة الحركية إلى طاقة
وضع بالكامل

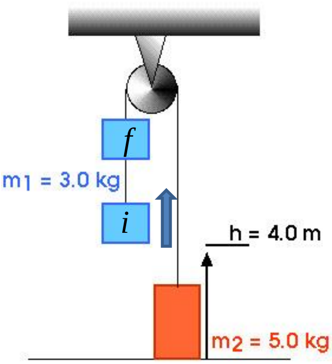
$$E_i = E_f$$

$$0 + \tfrac{1}{2} mv^2 = mgh + 0$$

$$h = \frac{v^2}{2g} = \frac{(4.4)^2}{2 \times 9.8} = 1m$$

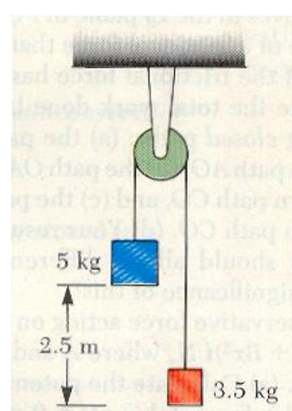
وهذه المسافة التي تحركتها الكتلة الصغيرة للأعلى بعد وصول
الكتلة الكبيرة إلى الأرض وبالتالي يكون اقصى ارتفاع للكتلة
الصغيرة عن الأرض هو

$$y_{max} = 1 + h = 1 + 4 = 5m$$



Example 4

Use conservation of energy to determine the final speed of a mass of 5.0kg attached to a light cord over a massless, frictionless pulley and attached to another mass of 3.5 kg when the 5.0 kg mass has fallen (starting from rest) a distance of 2.5 m as shown in Figure



Solution

من قانون الحفاظ على الطاقة الميكانيكية

$$E_f = E_i$$

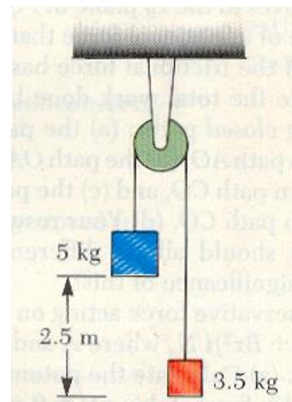
$$E_i = K_i + U_i = 0 + Mgh$$

$$E_f = K_f + U_f$$

$$E_f = \frac{1}{2} Mv^2 + \frac{1}{2} mv^2 + mgh$$

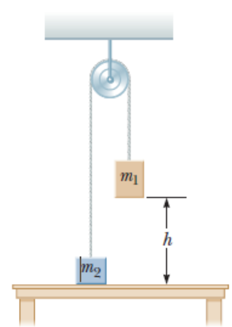
$$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} mv^2 + mgh$$

من هذه المعادلة يمكن إيجاد السرعة المطلوبة

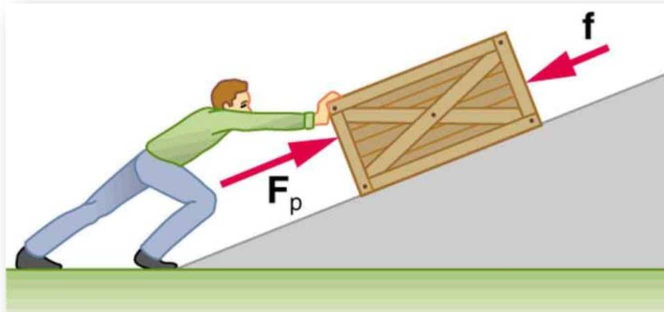


Problems to be solved by yourself

1. Use conservation of energy to determine the final speed of a mass of 5.0kg attached to a light cord over a massless, frictionless pulley and attached to another mass of 3.5 kg when the 5.0 kg mass has fallen (starting from rest) a distance of 2.5 m as shown in the Figure



2. A 0.5-kg ball is thrown vertically upward with an initial speed of 16 m/s. Assuming its initial potential energy is zero, find its kinetic energy, potential energy, and total mechanical energy (a) at its initial position, (b) when its height is 5m, and (c) when it reaches the top of its flight. (d) Find its maximum height using the law of conservation of energy.



المحاضرة القادمة

Non-conservative forces and the work-energy theorem

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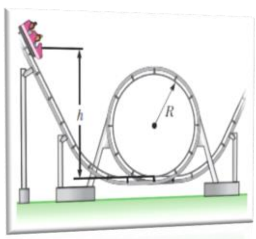
General Physics I

Mechanics: Principles & Applications

Unit5: Potential Energy and Conservation Energy



Lecture (18) Non-conservative forces and the work-energy theorem

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Unit 5: Potential Energy And Conservation Energy

1 Potential energy and conservation energy

2 Conservative forces

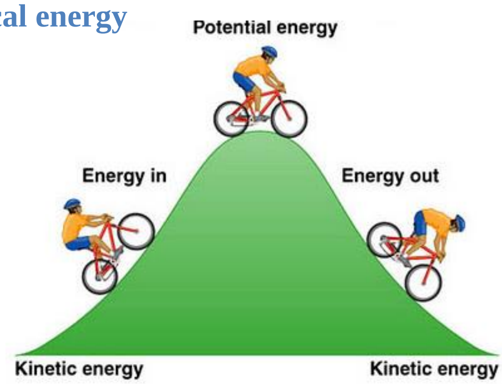
3 Conservation of mechanical energy

4 Total mechanical energy

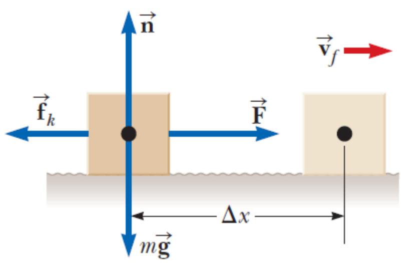
5 Non-conservative forces

6 Work-energy theorem

6 Problems



Non-Conservative Forces and The Work-Energy Theorem



في حالة التعامل مع قوة غير محافظة مثل قوة الاحتكاك بالإضافة إلى قوى محافظة، فإننا لا نستطيع أن نستخدم القانون السابق

$$\Delta E = 0$$

الذي ينص على أن التغير في الطاقة الميكانيكية الكلية يساوى صفراً لأن هناك جزءً من الطاقة يضيع على شكل حرارة بواسطة الشغل المبذول نتيجة لقوة الاحتكاك.

لذلك نحتاج إلى قانون أشمل وأعم ليشمل جميع أنواع القوى.

The Work-energy Theorem

نعلم سابقا أن الشغل يساوى التغير في طاقة الحركة

$$W = \Delta K$$

وحيث أن الشغل قد يكون مبذولاً بواسطة قوى محافظة *Wc* وأحياناً يكون الشغل مبذولاً بواسطة قوى غير محافظة يرمز له بالرمز *Wnc*.

$$W_{nc} + W_c = \Delta K$$

وحيث أن الشغل بواسطة قوة محافظة *Wc* يساوى سالب التغير في طاقة الوضع.

$$W_c = -\Delta U$$

أي ان

$$W_{nc} + -\Delta U = \Delta K \quad \Rightarrow \quad W_{nc} = \Delta K + \Delta U$$

The Work-energy Theorem

$$W_{nc} = \Delta K + \Delta U$$

وهذا يعنى أن الشغل المبذول بواسطة قوة غير محافظة يساوى التغير طاقة الحركة بالإضافة إلى التغير في طاقة الوضع.

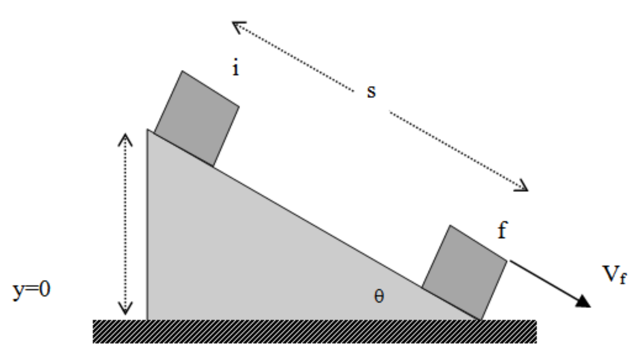
$$W_{nc} = (K_f + U_f) - (K_i + U_i)$$

$$W_{nc} = E_f - E_i$$

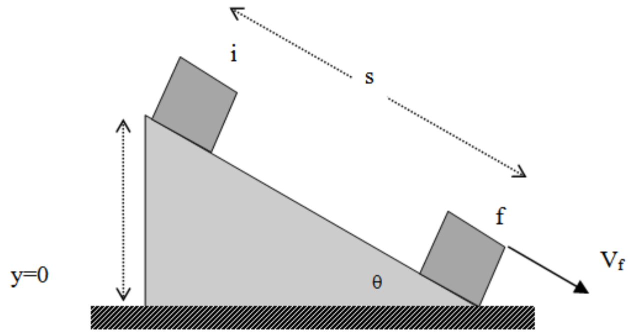
وهذا يمثل القانون العام للعلاقة بين الشغل والطاقة والذي ينص على أن الشغل المبذول بواسطة قوة غير محافظة يساوى التغير الكلى في الطاقة الميكانيكية.

Example 1

A 3kg block slides down a rough incline 1m in length as shown in the figure. The block starts from rest at the top and experience a constant force of friction of 5N. The angle of inclination is 30º. (a) Use energy methods to determine the speed of the block when it reach the bottom of the incline.



Solution



$W_{nc} = E_f - E_i$

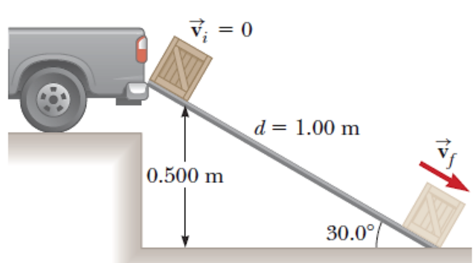
$W_{nc} = (K_f + U_f) - (K_i + U_i)$

$- f s = (1/2\ mv_f^2 + 0) - (0 + mgh)$

ومن هذه المعادلة يمكن إيجاد السرعة النهائية للجسم المنزلق. كذلك لاحظ يمكن إيجاد السرعة النهائية باستخدام قانون نيوتن الثاني.

Example 2

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.08 as shown in the Figure. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp.



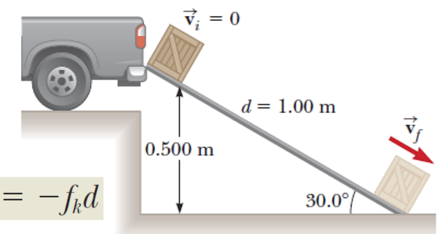
(A) Use energy methods to determine the speed of the crate at the bottom of the ramp.

(B) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 5.00 N?

Solution

(A) Determine the speed of the crate at the bottom of the ramp.

Because v_i=0, the initial kinetic energy of the system when the crate is at the top of the ramp is zero. If the *y* coordinate is measured from the bottom of the ramp with the upward direction being positive, then y_i=0.500 m.



$$E_i = K_i + U_i = 0 + U_i = mgy_i$$

$$E_f = K_f + U_f = \tfrac{1}{2}mv_f^2 + 0$$

$$\Delta E_{\text{mech}} = E_f - E_i = \tfrac{1}{2}mv_f^2 - mgy_i = -f_k d$$

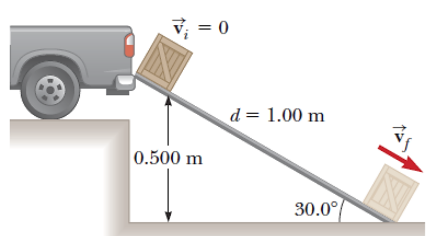
$$v_f = \sqrt{\frac{2}{m}(mgy_i - f_k d)}$$

$$v_f = \sqrt{\frac{2}{3.00\text{ kg}}[(3.00\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m}) - (5.00\text{ N})(1.00\text{ m})]} = 2.54\text{ m/s}$$

Solution

(B) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 5.00 N?

In this case we can consider the mechanical energy of the system to consist only of kinetic energy because the potential energy of the system remains fixed.



$$E_i = K_i = \tfrac{1}{2}mv_i^2$$

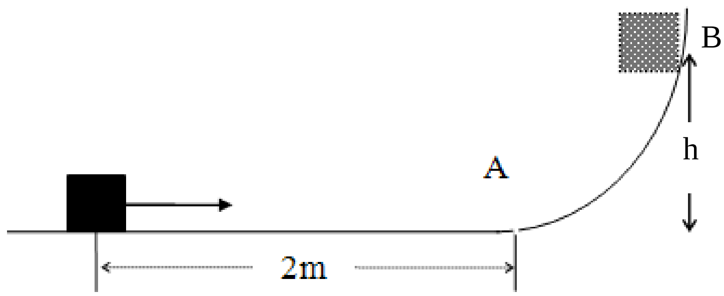
$$E_f - E_i = 0 - \tfrac{1}{2}mv^2 = -f_k d$$

$$\tfrac{1}{2}mv^2 = f_k d$$

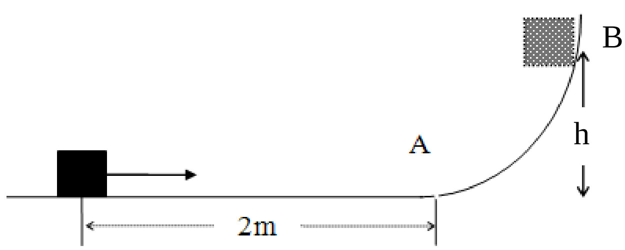
$$d = \frac{mv^2}{2f_k} = \frac{(3.00\text{ kg})(2.54\text{ m/s})^2}{2(5.00\text{ N})} = 1.94\text{ m}$$

Example 3

A block of mass 0.2kg is given an initial speed v_o=5m/s on a horizontal, rough surface of length 2m as in Figure. The coefficient of kinetic friction on the horizontal surface is 0.30. If the curved part of the track is frictionless, how high does the block rise before coming to rest at B?



Solution



The initial kinetic energy of the block is

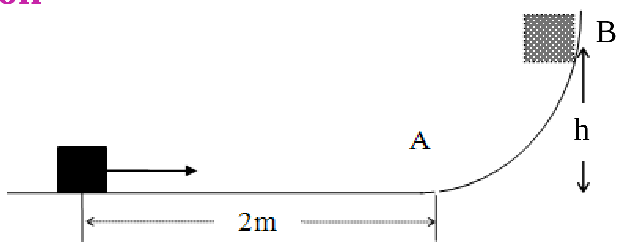
$K_o = \frac{1}{2} mv^2 = \frac{1}{2}$ (0.2kg) (5m/s)^2

= 2.50 J

The work done by friction along the horizontal track is

$W_f = -fd = -\mu mgd$ = -(0.30) (0.2) (9.8) (2) = -1.18 J

Solution



Using the work-energy theorem, we can find the kinetic energy at A

$$W_f = K_A - K_o = K_A - 2.50$$

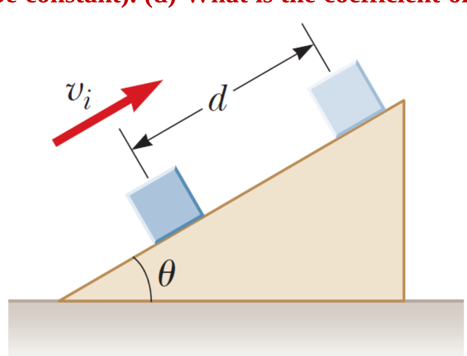
$$K_A = 2.50 + W_f = 2.50 - 1.18 = 1.32\ \text{J}$$

Since the curved track is frictionless, we can equate the kinetic energy of the block at A to its gravitational potential energy at B.

$$mgh = K_A = 1.32\ \text{J} \Rightarrow h = 0.67\ \text{m}$$

Example 4

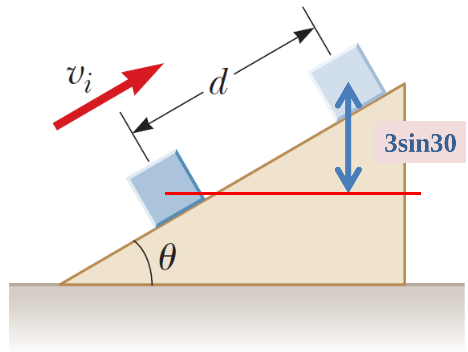
A 5-kg block is set into motion up an inclined plane as in Figure with an initial speed of 8 m/s. The block comes to rest after travelling 3 m along the plane, as shown in the diagram. The plane is inclined at an angle of 30' to the horizontal. (a) Determine the change in kinetic energy. (b) Determine the change in potential energy. (c) Determine the frictional force on the block (assumed to be constant). (d) What is the coefficient of kinetic friction?



Solution

$K_i = 1/2\, mv_i^2$ & $U_i = 0$

$K_f = 0$ & $U_f = mgh$



(a) The change in kinetic energy

$\Delta K = K_f - K_i$

$\Delta K = -\ 1/2\, mv_i^2 = -160$ J

(b) The change in potential energy

$\Delta U = U_f - U_i$

$\Delta U = mgh = 5 \times 9.8 \times 3\sin 30 = 73.5$ J

(c) The force of friction

$-f\,s = (K_f + U_f) - (K_i + U_i)$

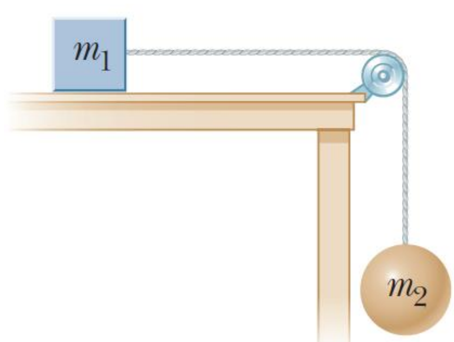
$-3\,f = (0 + 73.5) - (160 + 0) = 28.8$ N

(d) the coefficient of kinetic friction u_k

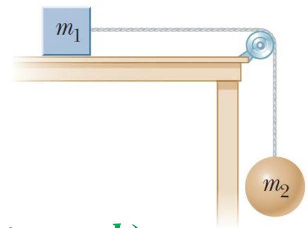
$f = u_k\, mg \cos 30$ ⇨ $$\mu_k = \frac{28.8\ \text{N}}{(5.00\ \text{kg})(9.80\ \text{m/s}^2)\cos 30.0°} = \boxed{0.679}$$

Example 5

The coefficient of friction between the 3.0-kg object and the surface in the Figure is 0.40. What is the speed of the 5.0-kg mass when it has fallen a vertical distance of 1.5 m?



Solution



$$W_{nc} = E_f - E_i$$

$$W_{nc} = (K_f + U_f) - (K_i + U_i)$$

$$-f\,h = \left(\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2\right) - (0 + m_2gh)$$

$$-\mu m_1 g\,h = \frac{1}{2}(m_1+m_2)v^2 - m_2gh$$

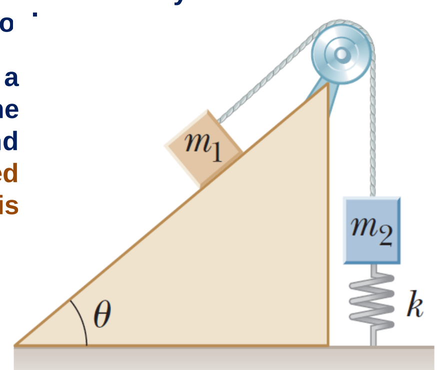
$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\ \text{m/s}^2)(1.50\ \text{m})[5.00\ \text{kg} - 0.400(3.00\ \text{kg})]}{8.00\ \text{kg}}} = \boxed{3.74\ \text{m/s}}$$

Example 6

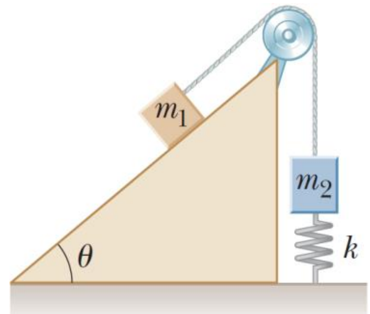
A block of mass m_1 = 20.0 kg is connected to a block of mass m_2 = 30.0 kg by a massless string that passes over a light, frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of k = 250 N/m as shown in the Figure. The spring is unstretched when the system is as shown in the figure, and the incline is frictio .

The 20.0-kg block is pulled a distance h = 20.0 cm down the incline of angle θ = 40.0º and released from rest. Find the speed of each block when the spring is again unstretched.



Solution

لحل هذا السؤال نفرض أن x هي المسافة التي استطال بها الزنبرك نتيجة لسحب الكتلة 20kg مسافة محددة وبالتالي فإن $x = 0.2$m. وكذلك نفرض أن طاقة الوضع $U_g = 0$ مقاسه عند أدنى قيمة للكتلة 20kg قبل تركها. فإذا كانت v هي سرعة الكتلتين عند مرورهما بموضع الاتزان قبل استطالة الزنبرك.



$$(K+U)_i = (K+U)_f$$

$$0 + (30.0\ \text{kg})(9.80\ \text{m/s}^2)(0.200\ \text{m}) + \frac{1}{2}(250\ \text{N/m})(0.200\ \text{m})^2$$

$$= \frac{1}{2}(50.0\ \text{kg})v^2 + (20.0\ \text{kg})(9.80\ \text{m/s}^2)(0.200\ \text{m})\sin 40.0°$$

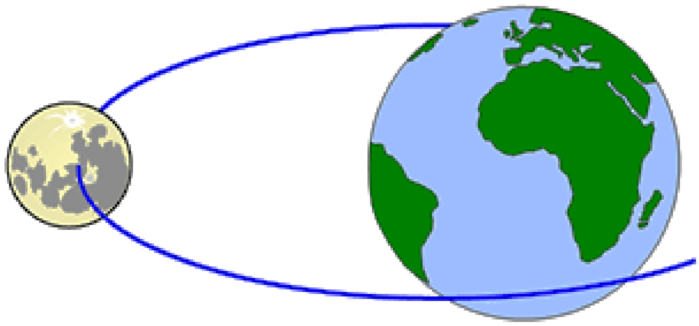
$$58.8\ \text{J} + 5.00\ \text{J} = (25.0\ \text{kg})v^2 + 25.2\ \text{J}$$

نعوض في المعادلة السابقة بالقيم ونحل المعادلة لحساب قيمة السرعة.

$v = 1.24$m/s

Problems to be solved by yourself

1. **A mass of 2.5 kg is attached to a light spring with *k* = 65 N/m. The spring is stretched and allowed to oscillate freely on a frictionless horizontal surface, When the spring is stretched 1 0 cm, the kinetic energy of the attached mass and the elastic potential energy are equal. What is the maximum speed of the mass?**
2. **A block of mass 2 kg is kept at rest by compressing a horizontal massless spring having a spring constant *k* = 100 N/m by 10 cm. As the block is released it travels on a rough horizontal surface a distance of 0.25 m before it stops. Calculate the coefficient of kinetic friction between the horizontal surface and the block.**
3. **A block of mass 0.25 kg is placed on a vertical spring of constant k = 5000 N/m and is pushed downward, compressing the spring a distance of 0.1 m. As the block is released it leaves the spring and continues to travel upward. To what maximum height above the point of release does the block rise?**



المحاضرة القادمة

The law of universal gravitation

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General Physics I

Mechanics: Principles & Applications

Unit6: The Law of Universal Gravitation

Lecture (19) Newton's Universal Law of Gravity

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Unit 6: The Law of Universal Gravitation

1 Introduction

2 Newton's universal law of gravity

3 Weight and gravitational force

4 Gravitational potential energy

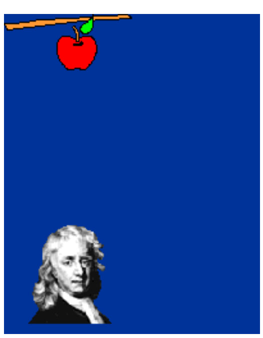
5 Total Energy for circular orbital motion

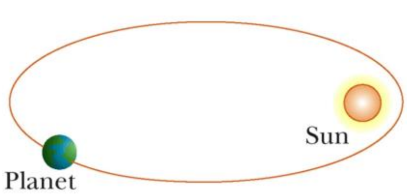
6 Escape velocity

7 Problems

Newton's Universal Law of Gravity

Newton's Universal Law of Gravity





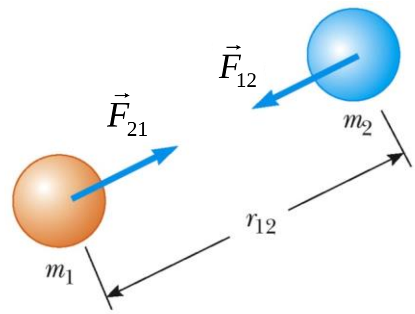
وضع العالم نيوتن قانون الجاذبية العام بعد الرواية المشهورة عنه وهي سقوط التفاحة على رأسه بينما كان نائماً تحت شجرة، فتوصل إلى أن القوة التي أثرت على التفاحة لتسقط على الأرض هي نفس القوة التي تجذب القمر إلى الأرض. وتبين أيضاً أن قانون الجذب العام لنيوتن ينطبق على القوة المتبادلة بين الكواكب والأجسام المادية على حد سواء.

- **The apple was attracted to the Earth**
- **All objects in the Universe were attracted to each other in the same way the apple was attracted to the Earth**

Newton's Universal Law of Gravity

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

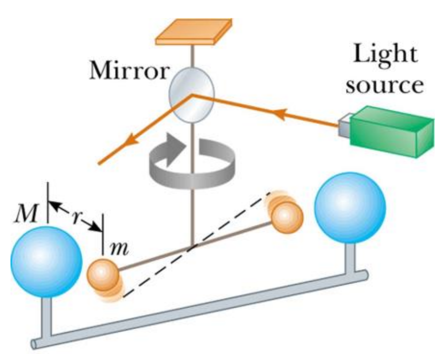
$$F = G\frac{m_1 m_2}{r^2}$$



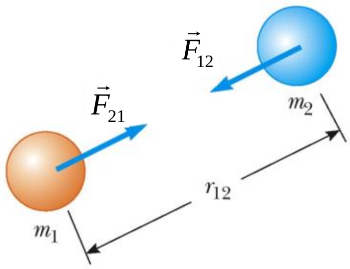
Universal Gravitation

- G is the constant of universal gravitation
- G = 6.673 x 10^{-11} N m² /kg²
- This is an example of an *inverse square law*
- Determined experimentally
- Henry Cavendish in 1798

$$F = G\frac{m_1 m_2}{r^2}$$



Universal Gravitation



القوة المتبادلة بين كتلتين m_1 و m_2 هي ناتجة عن التأثير المتبادل بينهما وعليه فإن F_{21} هي قوة الجذب على الكتلة الثانية من تأثير الكتلة الأولى. كذلك فإن القوة F_{12} هي قوة الجذب على الكتلة الأولى من تأثير الكتلة الثانية وفي كلا الحالتين فإن القوتين متساويتان في المقدار ومتعاكستان في الاتجاه. ويعبر عن ذلك بالمعادلة التالية:

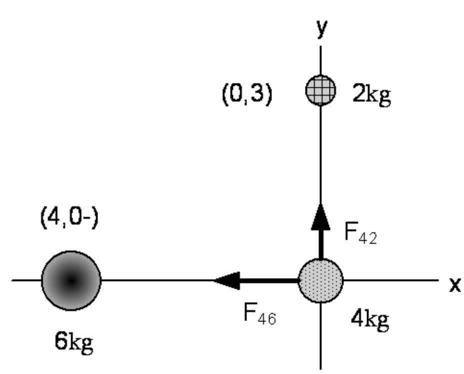
$$\vec{F}_{21} = -\vec{F}_{12}$$

يمكن استخدام قانون الجذب العام لنيوتن لإيجاد القوة المتبادلة بين جسم كتلته m والكرة الأرضية، وهنا يتم التعامل مع كتلة الكرة الأرضية على أنها مركزة في المركز وتحسب المسافة من مركز الأرض إلى الجسم ويكون قانون الجذب العام هو

$$F = G\frac{M_e m}{R_e^{\ 2}}$$

Example 1

Three uniform spheres of mass 2kg, 4kg, and 6kg are placed at the corners of a right triangle as shown in Figure. Calculate the resultant gravitational force on the 4kg mass.



Solution

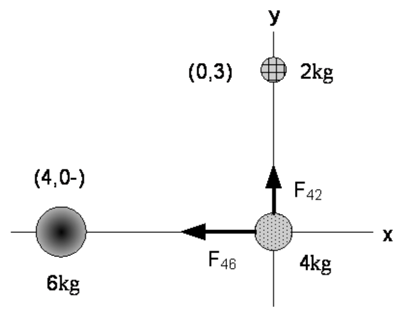
$$\vec{F}_4 = \vec{F}_{42} + \vec{F}_{46}$$

The force on the 4kg mass due to the 2kg mass is

$$\vec{F}_{42} = G\frac{m_4 m_2}{r_{42}^{\;2}}\, j$$

$$\vec{F}_{42} = (6.67\times10^{-11})\frac{4\times2}{3^2}\, j$$

$$\vec{F}_{42} = 5.93\times10^{-11}\, j \quad N$$



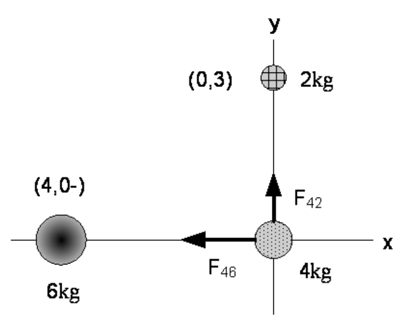
Solution

The force on the 4kg mass due to the 6kg mass is

$$\vec{F}_{46} = G\frac{m_4 m_6}{r_{46}^{\;2}}(-i)$$

$$\vec{F}_{46} = -(6.67\times10^{-11})\frac{4\times6}{4^2}\, i$$

$$\vec{F}_{46} = -10\times10^{-11} i \quad N$$



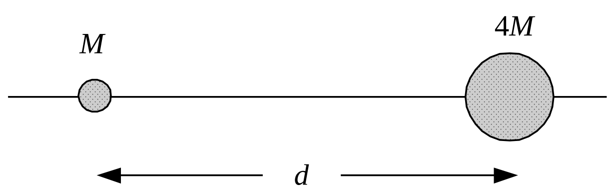
hence,

$$\vec{F}_4 = (-10i + 5.93j)\times10^{-11} \quad N$$

$$F_4 = 11.6\times10^{-11} \; N \qquad \theta = 149^o$$

Example 2

Two stars of masses *M* and 4*M* are separated by distance *d*. Determine the location of a point measured from *M* at which the net force on a third mass would be zero.



Solution

حتى تكون القوى المؤثرة على الكتلة الثالثة **m** فإن القوتين المؤثرتين على الكتلة الثالثة يجب أن تكونا متساويتين في المقدار ومتعاكستين في الاتجاه. وهذا يتحقق عندما يكون موضع الكتلة الثالثة بين الكتلتين **M** و **4M** وبالقرب من الكتلة الأصغر كما في الشكل.

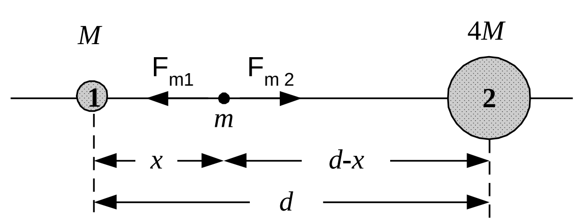
$$\vec{F}_{m2} = -\vec{F}_{m1}$$

$$G\frac{m4M}{(d-x)^2} = G\frac{mM}{(x)^2}$$

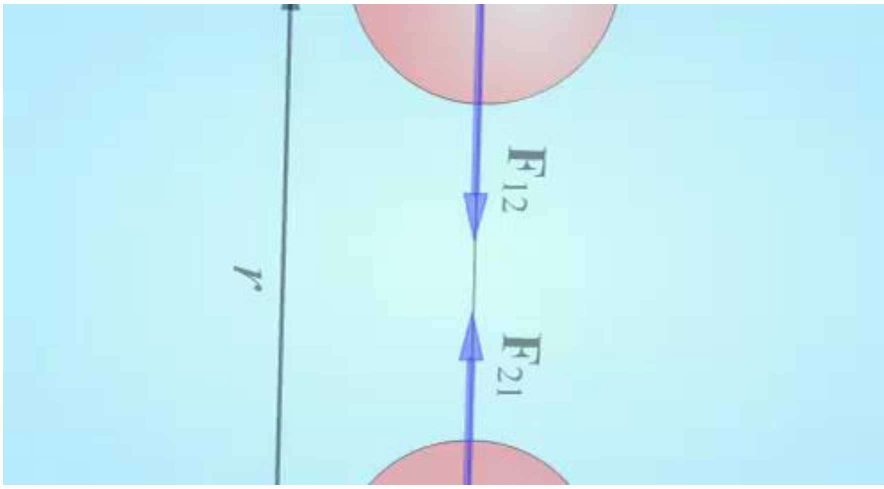
$$\frac{4}{(d-x)^2} = \frac{1}{(x)^2}$$

Solving for x then,

$$x = \frac{d}{3}$$



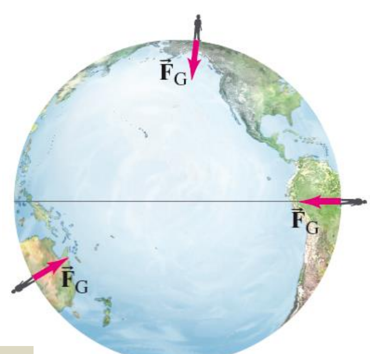
Gravitational Force



Free-fall Acceleration and Gravitational Force

Consider an object of mass m near the Earth's surface

$$F = G\frac{m_1 m_2}{r^2} = G\frac{mM_E}{R_E{}^2}$$



Acceleration g due to gravity

$$F = G\frac{mM_E}{R_E{}^2} = mg$$

$$\therefore g = G\frac{M_E}{R_E{}^2}$$

Since

$M_E = 5.9742\times10^{23}\ \text{kg}$ $R_E = 6378.1\ \text{km}$

Near the Earth's surface

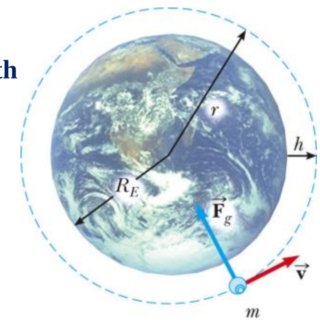
$$\therefore g = G\frac{M_E}{R_E{}^2} = 6.67\times10^{11}\frac{5.98\times10^{24}}{6.38\times10^{6}} = 9.8m/s^2$$

Weight and Gravitational Force

هنا يجب أن نذكر أن قوة الجاذبية بين كتلتين m_1 و m_2 هي من القوى ذات التأثير عن بعد action-at-a-distance وبالتالي يمكن أن نعتبر عجلة الجاذبية الأرضية على أنها مجال الجاذبية *gravitational field* ويمكن تعريف مجال الجاذبية الأرضية بأنها القوة المؤثرة على كتلة الجسم الموجود في مجال الجاذبية.

$$\vec{g} = \frac{\vec{F}}{m}$$

For a body of mass *m* a distance *h* above the earth then the distance *r* in the equation of the law of gravity is $r = R_e + h$



$$F = G\frac{m_1 m_2}{r^2} = G\frac{mM_E}{(R_E + h)^2}$$

$$g = G\frac{M_E}{(R_E + h)^2}$$

نستنتج من ذلك أن عجلة الجاذبية الأرضية تقل مع زيادة الارتفاع عن سطح الأرض وتكون صفراً عندما تكون *r* في اللانهاية.

Example 3

Determine the magnitude of the acceleration of gravity at an altitude of 500km.

Solution

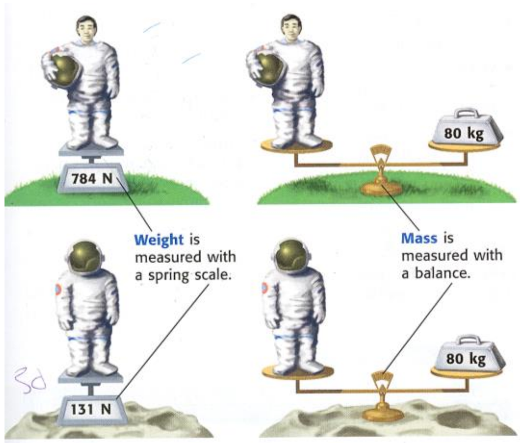
$$g' = G\frac{M_e}{(R_e + h)^2}$$

$$g' = 6.67 \times 10^{-11} \frac{5.98 \times 10^{24}}{(6.38 \times 10^6 + 0.5 \times 10^6)^2}$$

$$= 8.43 m/s^2$$

Weight and Gravitational Force

From Newton's second law we define the weight as a kind of force equal to *mg* where *m* is the mass of the particle and *g* the acceleration due to gravity, we can define the weight using the Newton's universal law of gravity as follow



$$W = mg = G\frac{M_e m}{R_e^{\ 2}}$$

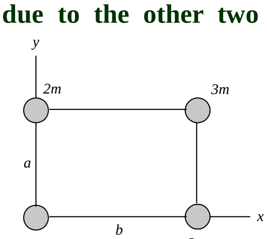
Problems to be solved by yourself

1. Two identical, isolated particles, each of mass 2 kg, are separated by a distance of 30 cm. What is the magnitude of the gravitational force of one particle on the other?

2. A 200-kg mass and a 500-kg mass are separated by a distance of 0.40 m. (a) Find the net gravitational force due to these masses acting on a 50-kg mass placed midway between them. (b) At what position (other than infinitely remote ones) would the 50-kg mass experience a net force of zero?

3. Three 5-kg masses are located at the corners of an equilateral triangle having sides 0.25 m in length. Determine the magnitude and direction of the resultant gravitational force on one of the masses due to the other two masses.

4. Four particles are located at the corners of a rectangle as in the Figure. Determine the x and y components of the resultant force acting on the particle of mass *m*.



المحاضرة القادمة

Gravitational Potential Energy

Physics Academy

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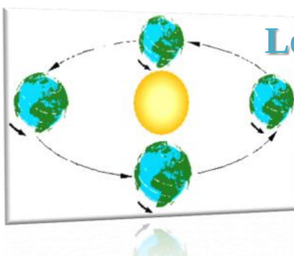
General Physics I

Mechanics: Principles & Applications

Unit6: The Law of Universal Gravitation



Lecture (20) Gravitational potential energy

Dr. Hazem Falah Sakeek

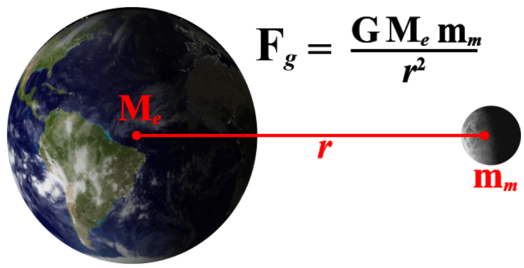
Al-Azhar University of Gaza

Unit 6: The Law of Universal Gravitation

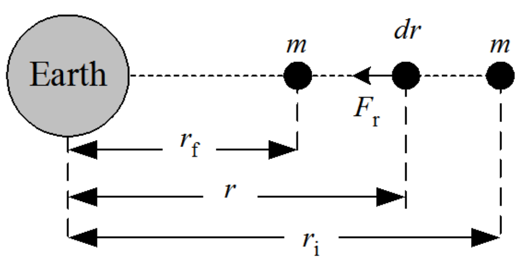
Gravitational Potential Energy

درسنا في محاضرة سابقة أن طاقة الوضع لجسم على سطح الأرض أو على ارتفاع *h* من سطح الأرض تساوي *mgh* وهذا عندما تكون *h* على مسافات قريبة من سطح الأرض أو عندما تكون *h* أصغر بكثير من نصف قطر الأرض.

سندرس الآن طاقة الوضع في مجال الجاذبية الأرضية عندما يتغير موضع الجسم من مكان إلى آخر بالنسبة لمركز الأرض كما في الشكل التالي.



To move the particle of mass *m* from r_i to r_f in the gravitational field *g* a negative work *W* is done by an external agent since the external force F_{ex} is in opposite direction of the displacement. Therefore the change in gravitational potential energy associated with a given displacement *dr* is defined as the negative work done by the gravitational force during the displacement,

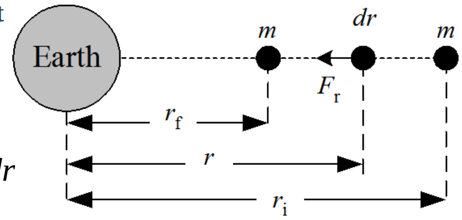


$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$$

When the particle move from r_i to r_f, it will be subjected to gravitational force given by

$$\vec{F} = -\frac{GM_e m}{r^2}$$

where the negative sign indicates that the force is attractive.



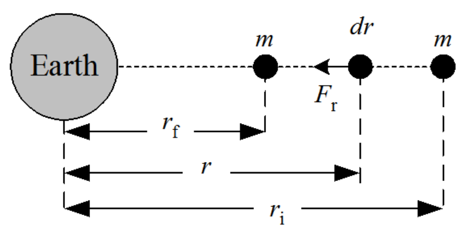
Substitute in equation

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$$

$$U_f - U_i = GM_e m \int_{r_i}^{r_f} \frac{dr}{r^2} = GM_e m \left[-\frac{1}{r} \right]_{r_i}^{r_f}$$

$$U_f - U_i = -GM_e m \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

$$U_f - U_i = -GM_e m \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$



Take U_i=0 at r_i=∞ we obtain the potential energy as a function of r from the centre of the earth

$$U(r) = -\frac{GM_e m}{r}$$

The potential energy between any two particles m_1 and m_2 is given by

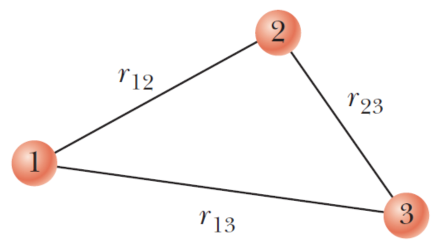
$$U = -G\frac{m_1 m_2}{r}$$

❑ نستنتج من المعادلة الأخيرة أن طاقة الوضع المتبادلة بين جسمين تتناسب عكسيا مع المسافة الفاصلة بينهما في حين أن قوة الجاذبية تتناسب عكسياً مع مربع المسافة بينهما.

❑ تكون طاقة الوضع بين جسمين سالبة لأن القوة المتبادلة بينهما دائماً قوى تجاذبية.

Potential Energy for more than Two Particles

For more than two particles the potential energy can be evaluated by the algebraic sum of the potential energy between any two particles.

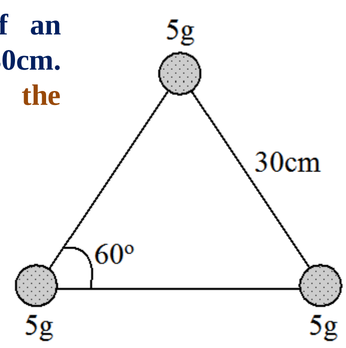


$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$U_{total} = -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}}\right)$$

Example 1

A system consists of three particles, each of mass 5g, located at the corner of an equilateral triangle with sides of 30cm. Calculate the potential energy of the system.



Solution

$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$U_{total} = -G\left(\frac{m^2}{r} + \frac{m^2}{r} + \frac{m^2}{r}\right) = -\frac{3GM^2}{r}$$

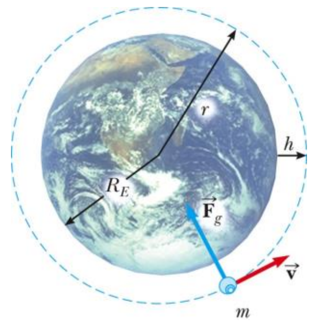
$$U_{total} = -\frac{3 \times 6.67 \times 10^{-11} \times (0.005)^2}{0.3} = -1.67 \times 10^{-14}\,J$$

Total Energy for Circular Orbital Motion

When a body of mass *m* moving with speed *v* in circular orbit around another body of mass *M* where *M* >> *m* as the earth around the sun or satellite around the earth, the body of mass *M* is at rest with respect to the frame of reference. ***The total energy of the two body system is the sum of the kinetic energy and the potential energy.***

$$E = K + U$$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$



As the mass *m* moves from initial point *i* to a final point *f*, the total energy remains constant, therefore the total energy equation become,

$$E = \frac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f}$$

From Newton's second law F = ma where *a* is the is the radial acceleration therefore,

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Multiply both sides by *r*/2

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$E = \frac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f}$$

$$E = \frac{GMm}{2r} - \frac{GMm}{r}$$

The total energy for circular orbit ⇒ $E = -G\frac{Mm}{2r}$

Note that the total energy is negative in a circular orbit. And the kinetic energy is positive and equal to one half the magnitude of the potential energy. The total energy called the binding energy for the system.

Example 2

A space transportation vehicle releases a 470-kg communications satellite while in an orbit 280 km above the surface of the Earth. A rocket engine on the satellite boosts it into a geosynchronous orbit (المدار الأرضي الجغرافي المتزامن) 36,000 km above the surface of the Earth. How much energy does the engine have to provide?

Solution

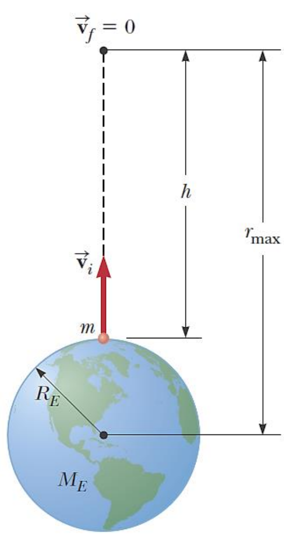
$$r_i = R_E + 280 \text{ km} = 6.65 \times 10^6 \text{ m}$$

$$\Delta E = E_f - E_i = -\frac{GM_E m}{2r_f} - \left(-\frac{GM_E m}{2r_i}\right) = -\frac{GM_E m}{2}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$\Delta E = -\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})(470 \text{ kg})}{2} \times \left(\frac{1}{4.22 \times 10^7 \text{ m}} - \frac{1}{6.65 \times 10^6 \text{ m}}\right)$$

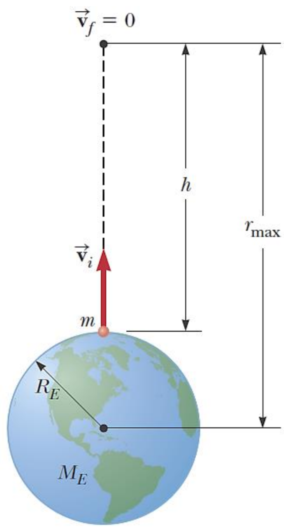
$$= 1.19 \times 10^{10} \text{ J}$$

Escape Velocity



باستخدام مفهوم الطاقة الكلية سنقوم بحساب سرعة الإفلات *escape velocity* من الجاذبية الأرضية. وسرعة الإفلات هي أقل سرعة ابتدائية لجسم يقذف رأسياً ليتمكن الجسم من الإفلات من مجال الجاذبية الأرضية.

Suppose an object of mass m is projected vertically upward from the earth with initial speed $v_i = v$ and $r_i = Re$. When the object is at maximum altitude, $v_f = 0$ and $r_f = r_{max}$.



In this case the total energy of the system (Earth & object) is conserved, we can use the equation

$$\frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e} = -\frac{GM_e m}{r_{max}}$$

solving for v_i^2 we get,

$$v_i^2 = 2GM_e\left(\frac{1}{R_e} - \frac{1}{r_{max}}\right)$$

من هذه المعادلة إذا علمنا قيمة السرعة الابتدائية لانطلاق الجسم v_i يمكن حساب أقصى ارتفاع يمكن أن يصل إليه الجسم h حيث أن $h = r_{max} - Re$.

لحساب سرعة الإفلات للجسم من مجال الجاذبية الأرضية مثل ما هو الحال عند إطلاق صاروخ فضائي أو مكوك من سطح الأرض إلى الفضاء الخارجي فإن سرعة الانطلاق الابتدائية التي يجب أن ينطلق بها المكوك يجب أن لا تقل عن سرعة الإفلات وإلا فإن المكوك سوف لن يصل إلى هدفه نتيجة لتأثير قوة الجاذبية. ولإيجاد سرعة الإفلات المطلوبة فإن ......

$$v_i^2 = 2GM_e\left(\frac{1}{R_e} - \frac{1}{r_{max}}\right)$$

For the escape velocity the object will reach a final speed of $v_f = 0$ when $r_{max} = \infty$, therefore we substitute for $v_i = v_{esc}$ and we get

$$v_{esc} = \sqrt{\frac{2GM_e}{R_e}}$$

Note that the escape velocity does not depends on the mass of the object projected from the earth.

This equation can be used to evaluate the escape velocity from any planet in the universe if the mass and the radius of the planet are known.

$$v_{esc} = \sqrt{\frac{2GM_e}{R_e}}$$

Escape velocities for the planets	
Planet	v_{esc} (km/s)
Mercury	4.3
Venus	10.3
Earth	11.2
Moon	2.3
Mars	5.0
Jupiter	60
Saturn	36
Uranus	22
Neptune	24
Pluto	1.1
Sun	618

Example 2

Calculate the escape speed from the Earth for a 5000-kg spacecraft and **determine** the kinetic energy it must have at the Earth's surface to move infinitely far away from the Earth.

Solution

$$v_{\text{esc}} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.37 \times 10^6\ \text{m}}}$$

$$= 1.12 \times 10^4\ \text{m/s}$$

$$K = \tfrac{1}{2}mv_{\text{esc}}^2 = \tfrac{1}{2}(5.00 \times 10^3\ \text{kg})(1.12 \times 10^4\ \text{m/s})^2$$

$$= 3.13 \times 10^{11}\ \text{J}$$

Example 3

(a) Calculate the minimum energy required to send a 3000kg spacecraft from the earth to a distance point in space where earth's gravity is negligible. (b) If the journey is to take three weeks, what average power would the engine have to supply?

Solution

(a) $$v_{esc} = \sqrt{\frac{2GM_e}{R_e}} = 1.12\times10^4\,\text{m/s}$$

$$K = \frac{1}{2}mv_{esc}^2 = \frac{1}{2}\times3000\times(1.12\times10^4)^2$$
$$= 1.88\times10^{11}\,J$$

(b) $$P_{av} = \frac{K}{\Delta t} = \frac{1.88\times10^{11}}{21\text{days}\times8.64\times10^4\,s/\text{day}} = 103kW$$

Example 4

A spaceship is fired from the Earth's surface with an initial speed of 2×10^4 m/s. What will its speed when it is very far from the Earth? (Neglect friction.)

Solution

Energy is conserved between the surface and the distant point

$$(K+U_g)_i = (K+U_g)_f$$

$$\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = \frac{1}{2}mv_f^2 - \frac{GM_E m}{\infty}$$

$$v_f^2 = v_i^2 - \frac{2GM_E}{R_E} \Longrightarrow v_f^2 = v_i^2 - v_{esc}^2$$

$$v_f^2 = (2\times10^4)^2 - \frac{2(6.67\times10^{-11})^2(5.98\times10^{24})}{6.37\times10^6}$$

$$v_f = 1.66\times10^4 \quad m/s$$

Problems to be solved by yourself

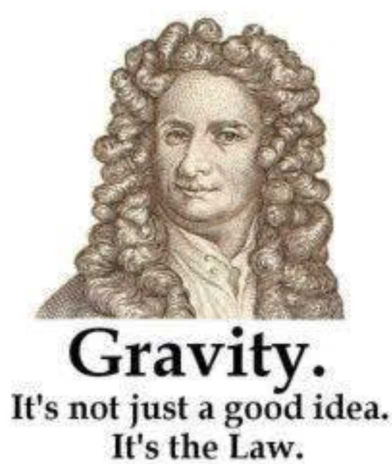
1. A satellite of the earth has a mass of 100 kg and is altitude of 2×10^6 m. (a) What is the potential energy of the satellite-earth system? (b) What is the magnitude of the force on the satellite?

2. A system consists of three particles, each of mass 5g, located at the corners of an equilateral triangle sides of 30 cm. (a) Calculate the potential energy of the system. (b) If the particles are released simultaneously, where will they collide?

3. How much energy is required to move a 1000-kg form the earth's surface to an altitude equal to twice the earth's radius?

4. Calculate the escape velocity from the moon, where $M_m=7.36\times10^{22}$kg, $R_m=1.74\times10^6$m

5. A spaceship is fired from the earth's surface with an initial speed of 2.0×10^4m/s. What will its speed when it is very far from the earth?



المحاضرة القادمة

Problems on Gravitational Potential Energy

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أستاذ الفيزياء المشارك
جامعة الأزهر - غزة

- رئيس قسم الفيزياء بجامعة الازهر - غزة في الفترة ١٩٩٣-١٩٩٨
- مؤسس وعميد كلية الدراسات المتوسطة بجامعة الازهر - غزة من الفترة ١٩٩٦-٢٠٠٥
- عميد القبول والتسجيل بجامعة الازهر - غزة في الفترتين ١٩٩٨-٢٠٠٠ و٢٠٠٧-٢٠٠٨
- مدير الحاسب الالي بجامعة الازهر - غزة في الفترة من ١٩٩٤-٢٠٠٠
- رئيس وحدة تكنولوجيا المعلومات بجامعة الازهر - غزة في الفترة من ٢٠٠٠-٢٠٠٥
- مؤسس موقع الفيزياء التعليمي
- مؤسس اكاديمية الفيزياء للتعليم الالكتروني
- مؤسس مركز الترجمة العلمي
- مؤسس قناة الفيزياء التعليمي على اليوتيوب
- مؤسس ورئيس تحرير مجلة الفيزياء العصرية

لمزيد من المعلومات يرجى زيارة
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